
Image modeling for precise astrometry

位置天文測定のための星像モデルの構築

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Effective PSF

Modern astrometry requires ~ 0.01 -pixel level measurement accuracies.

Precise point-source image modeling is required.

Point-spread function (PSF): $\varphi(x - x_0, y - y_0)$

A continuous function defined on a focal plane. x and y are focal-plane coordinates.
The shape is defined by optics and a source spectrum.

Pixel-response function (PRF): $\psi(i, j, x_0, y_0)$

A function providing the photon count at (i, j) pixel. x_0 and y_0 are stellar positions.
A PRF is derived by integrating the PSF and an intrapixel flat pattern within a (i, j) pixel.

Profile fitting with an appropriate PRF provides accurate (positional) measurements.

Effective PSF

Assuming that every pixel has the same intrapixel flat pattern, a PRF can be represented as a virtual two-dimensional image.

Effective PSF (ePSF): $\Psi(i - \delta_x, j - \delta_y)$

A two-dimensional continuous function. δ_x and δ_y are pixel phases.

$$\begin{aligned}\psi(i, j, x_0, y_0) &= \int_{i-0.5}^{i+0.5} \int_{j-0.5}^{j+0.5} \eta(x-i, y-j) \phi(x-x_0, y-y_0) dx dy \\ &= \Psi(i - i_0 - \delta_x, j - j_0 - \delta_y)\end{aligned}$$

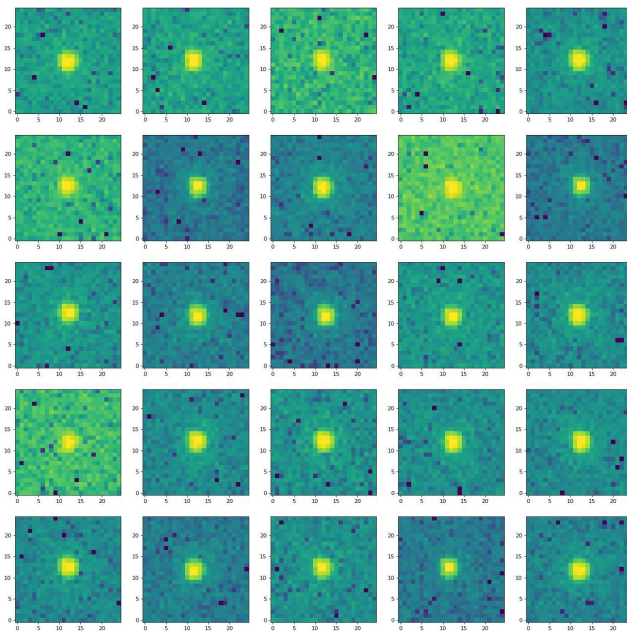
The next challenge is **estimating the ePSF for an exposure.**

ePSF method using photutils

Anderson & King (2000, **AK20**) provided a procedure to derive ePSF in case that all the stellar images are descended from the same PRF.

The procedure is (in part) implemented in `photutils` and has been widely used.

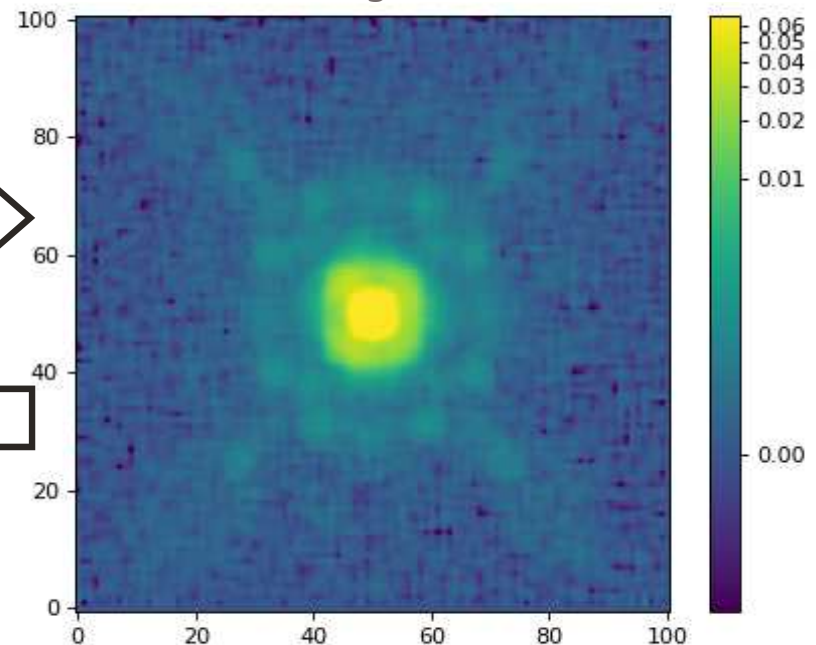
Multiple stellar image cutouts



estimate ePSF

update centroids

Estimated ePSF image



ePSF method using photutils

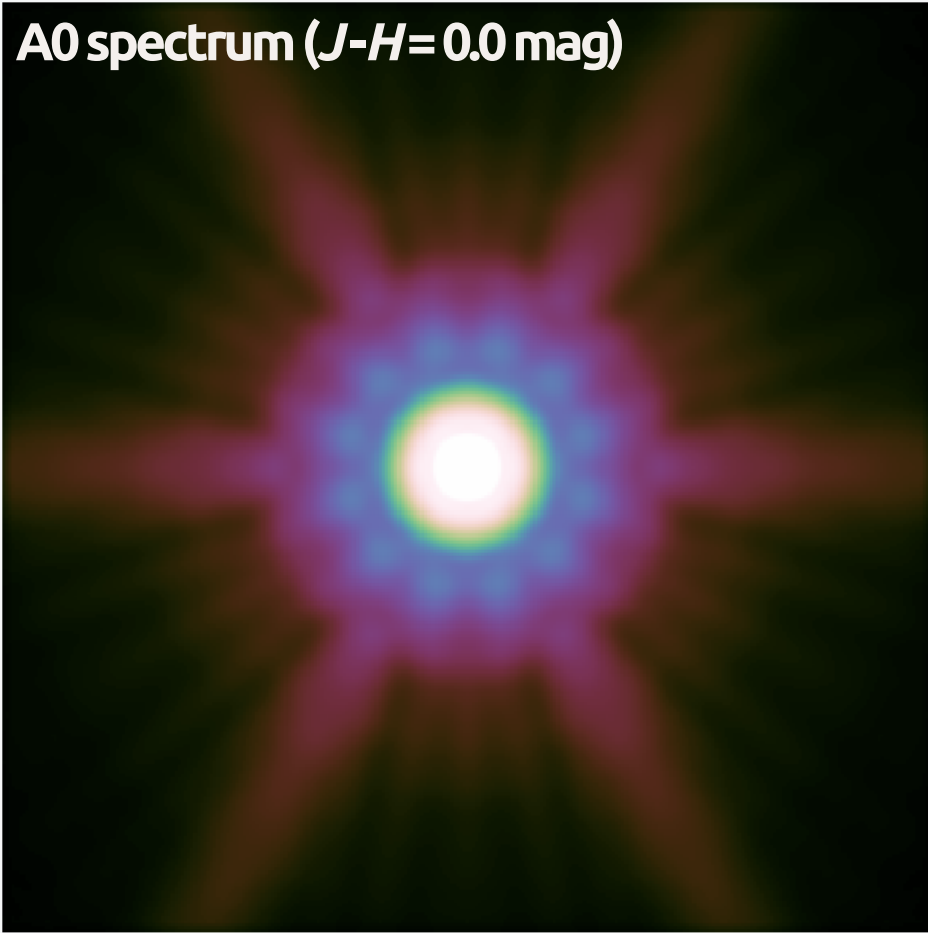
The ePSF method (`photutils`) is useful but has several drawbacks.

1. Excessive degrees of freedom
2. Pixel-phase errors
3. Single ePSF assumption

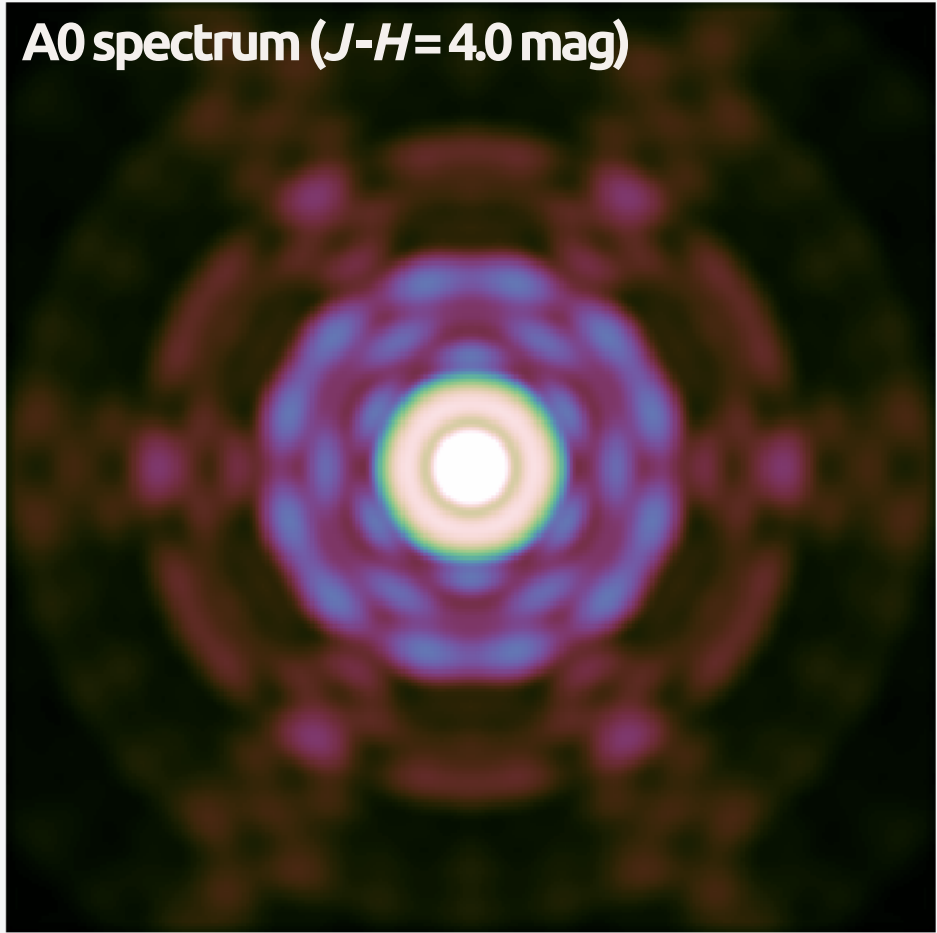
ePSF method using photutils

T ePSFs of a hypothetical infrared system (JASMINE)

A0 spectrum ($J-H=0.0$ mag)



A0 spectrum ($J-H=4.0$ mag)



ePSF method using photutils

The ePSF method (`photutils`) is useful but has several drawbacks.

1. Excessive degrees of freedom
2. Pixel-phase errors
3. Single ePSF assumption \Rightarrow interpolating multiple ePSFs (requiring more sources)

Purpose

Establish a method to construct a variable ePSF model from obtained images.

HePSF method

To construct a variable ePSF, we expand an ePSF using basis functions.

$$\Psi(x, y) \simeq \frac{1}{Z} \left[1 + \sum_{2 \leq k+l \leq M} \nu_{k,l} \mathcal{H}_k \left(\frac{x}{\sigma_x} \right) \mathcal{H}_l \left(\frac{y}{\sigma_y} \right) \right] \exp \left[-\frac{1}{2} \left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right) \right]$$

\mathcal{H}_k is the k-th order of Hermite polynomials function.

Refregier (2003) used a similar method to model galaxy shapes. Including a sufficient number of basis functions, this expression can approximate complicated ePSF.

The expansion coefficients $\nu_{k,l}$ can be a function of additional parameters (e.g., colors).

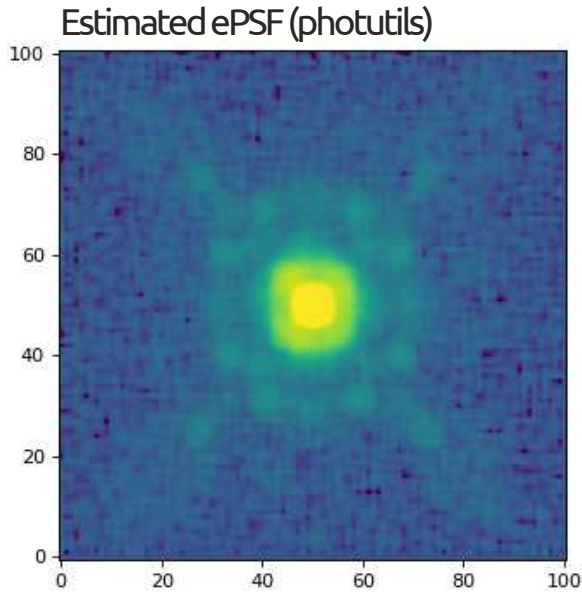
HePSF method

Characteristics of two approaches are compared below:

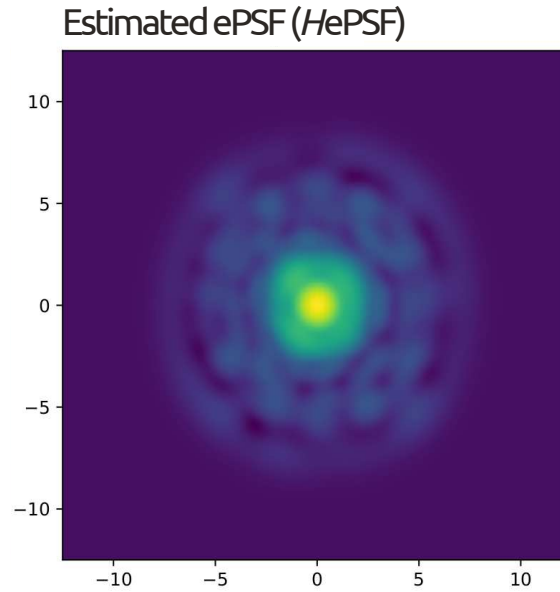
	ePSF (photutils)	HePSF
ePSF implementation	non-parametric	parametric
Degrees of freedom	large (> a few hundred) \propto image size ²	handful (~ 10–200)
Model optimization	alternatively optimized using an efficient iterative procedure	simultaneously optimized using gradient-based methods
variable ePSF modeling	interpolating multiple ePSFs large sample required	easily implemented by adding additional parameters
availability	implemented in photutils	not available yet

HePSF method

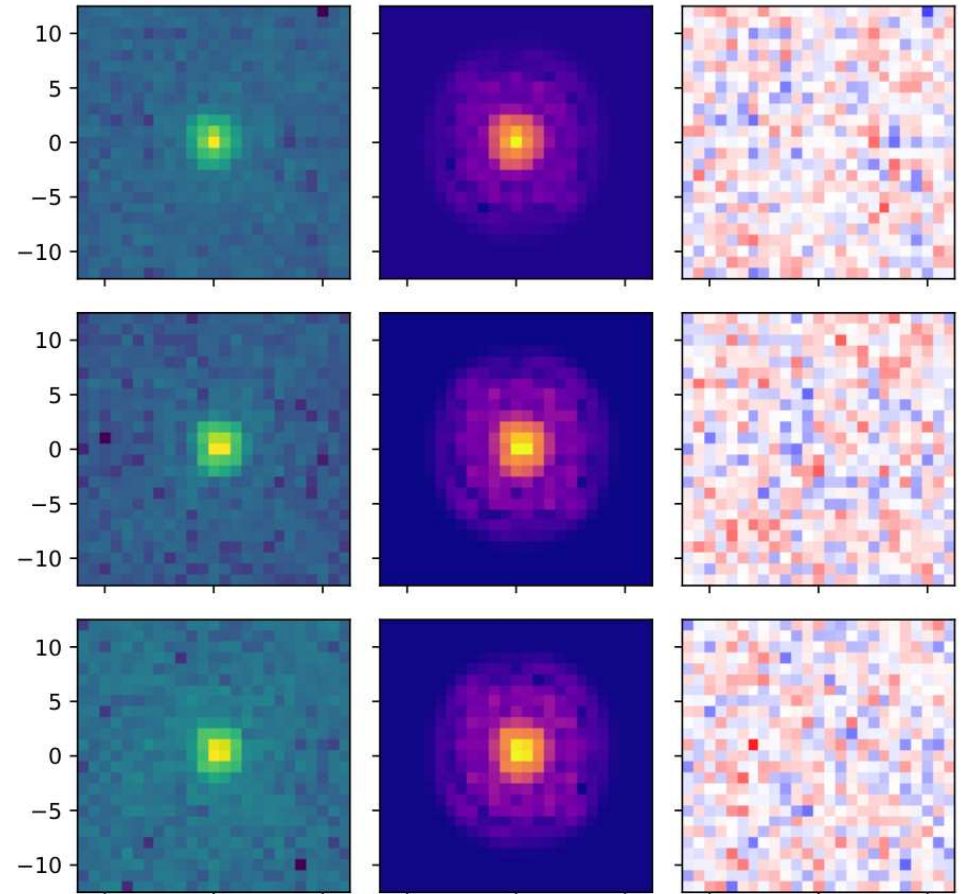
HePSF demonstration using a HST mock image



25×25 pixel (×4 oversampling)
degrees of freedom $\approx 10,000$
3 iterations using 403 stars
log-scale (99.0%)



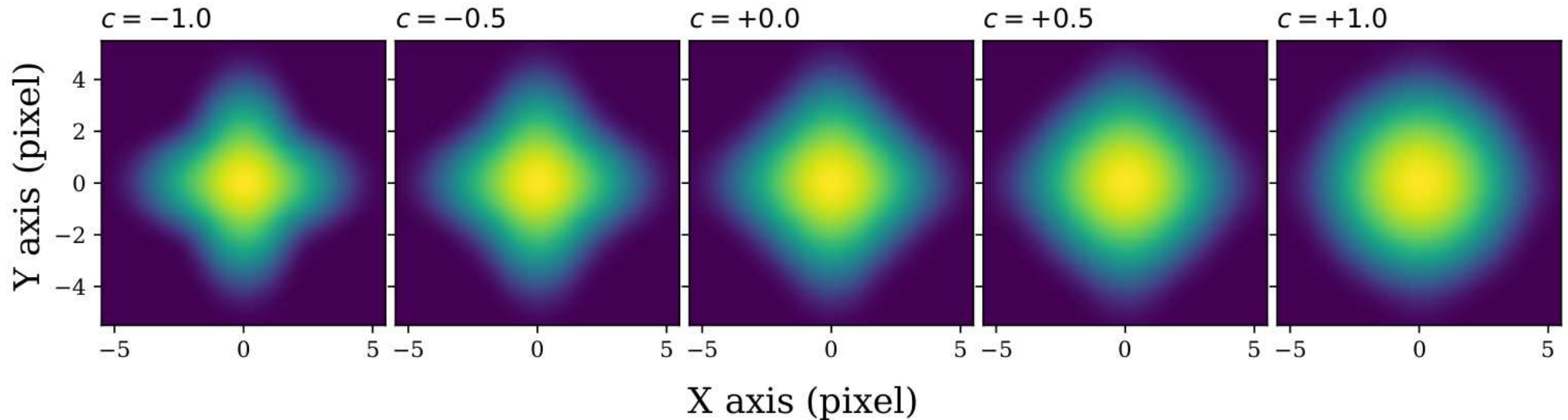
covering 21×21 pixel area
degrees of freedom ≈ 228
optimized using 50 stars
log-scale (99.0%)



ePSF from a tutorial in photutils docs;

Experiments

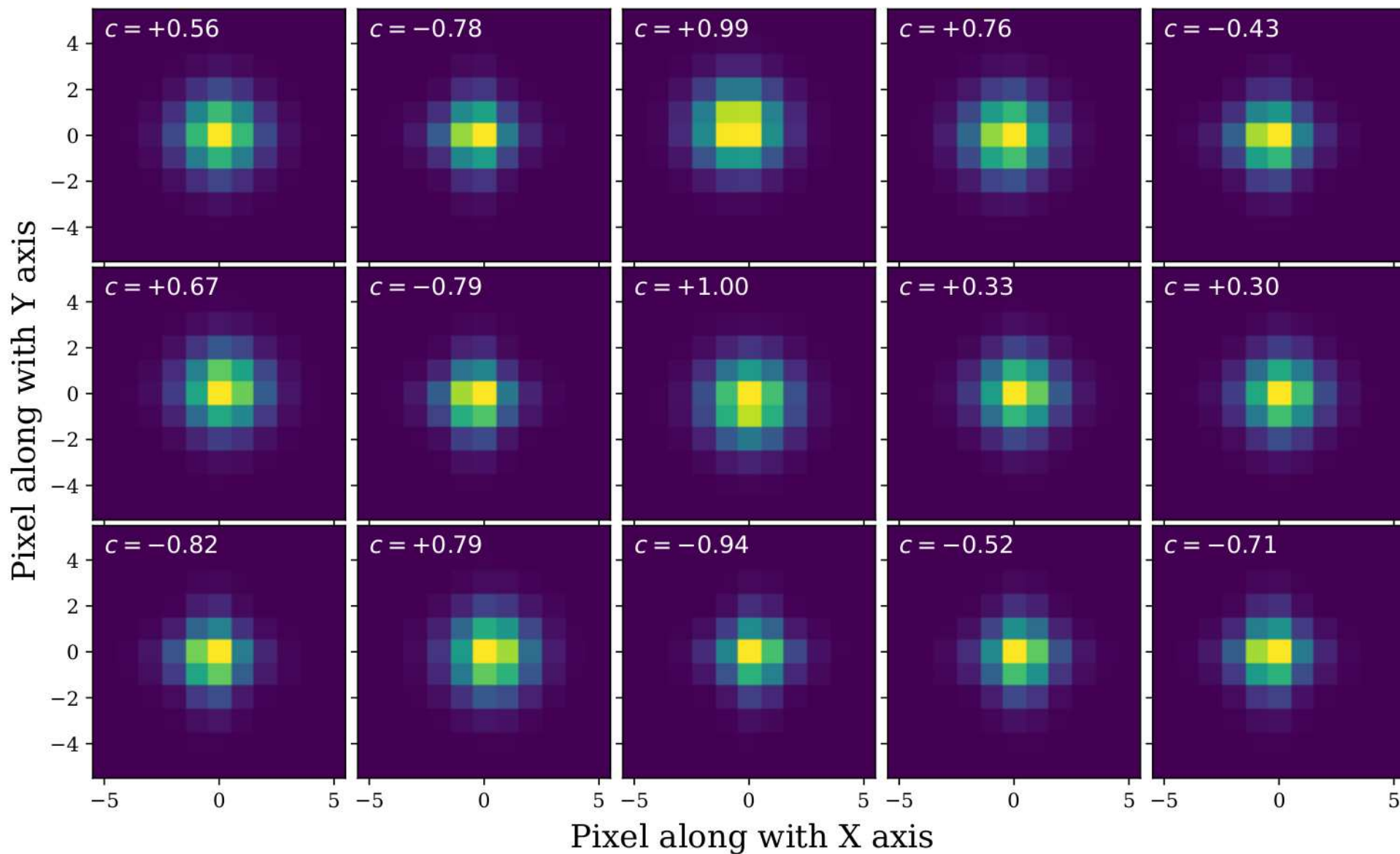
As a concept verification, we applied *HePSF* method to simulated data.
An artificial ePSF with a color parameter was used to generate stellar images.



Color parameters were sampled from a uniform distribution $(-1, 1)$.

100 stellar images were generated and their positions were measured.

Exapmls of generated images



Experiments

Details of simulated data are listed in the table below.

For simplicity, solely photon and background noise is considered.

	Specifications
Number of sources	100
Image dimension	11×11 pixels
ePSF model	variable, symmetric along x - and y -axes
Color parameter	sampled from $(-1, 1)$ uniform distribution
Pixel phases (δ_x, δ_y)	sampled from $(-0.5, 0.5)$ uniform distribution
Source intensity	sampled from $\mathcal{N}(10^{12}, 10^{10})$
Background noise	sampled from $\mathcal{N}(0.0, 1.0)$

Experiments

HePSF model with $M=4$ was applied, where 12 basis functions were included.

Considering the xy symmetry, only the even terms were set color-dependent.

The *HePSF* model contained 22 shape parameters.

2 scale parameters (σ_x, σ_y), 12 color-independent coefficients, 8 color-dependent coefficients.

The model parameters were simultaneously optimized with source intensities and positions using the least-squares method. The colors were treated as known parameters.

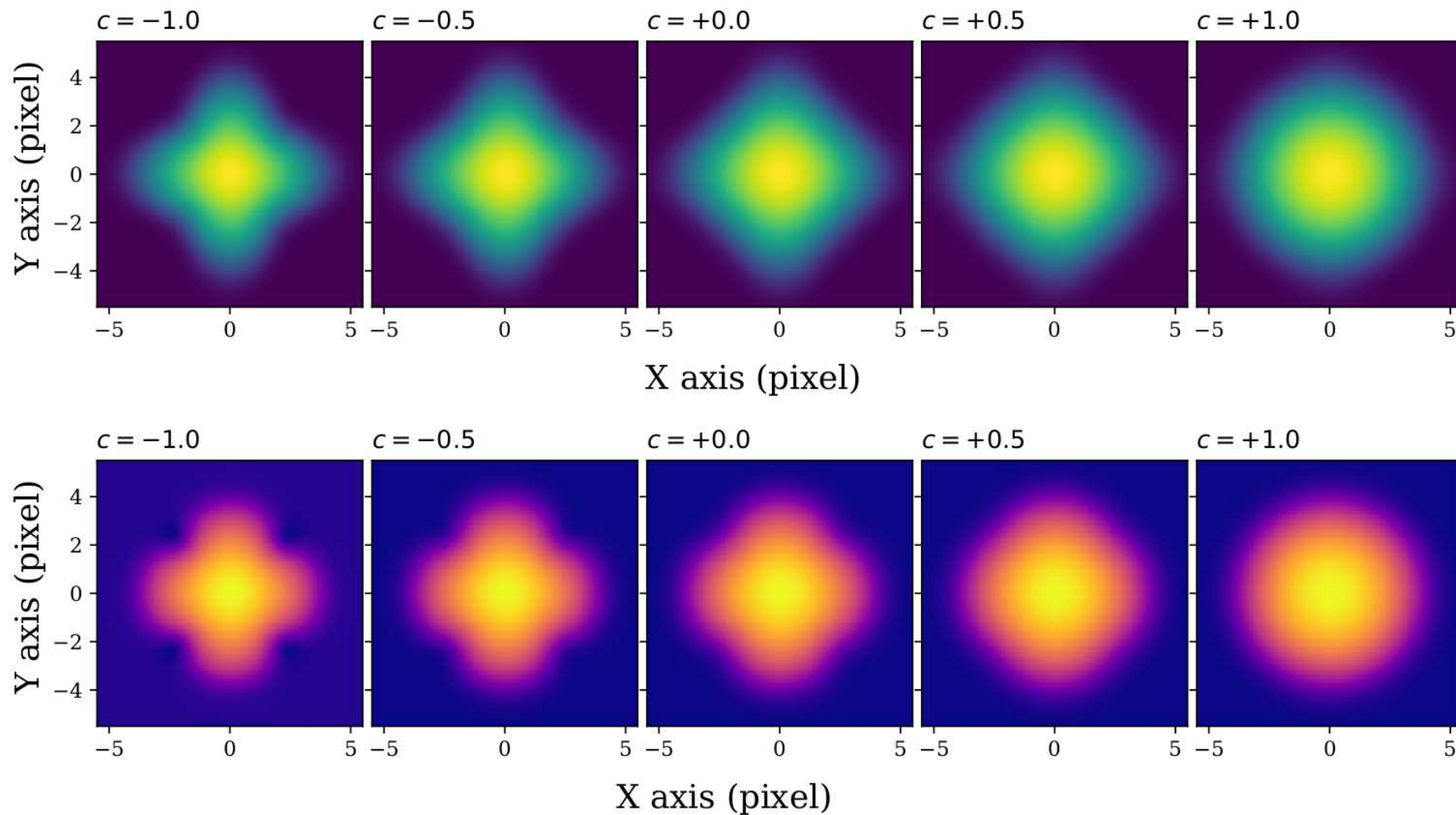
$M=2$ terms	$x^2 - 1, xy, y^2 - 1$
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$M=3$ terms	$x^3 - 3x, x^2y - y, xy^2 - x, y^3 - 3y$
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$M=4$ terms	$x^4 - 6x^2 + 3, x^3y - 3xy,$ $x^2y^2 - x^2 - y^2 + 1, xy^3 - 3xy, y^4 - 6y + 3$
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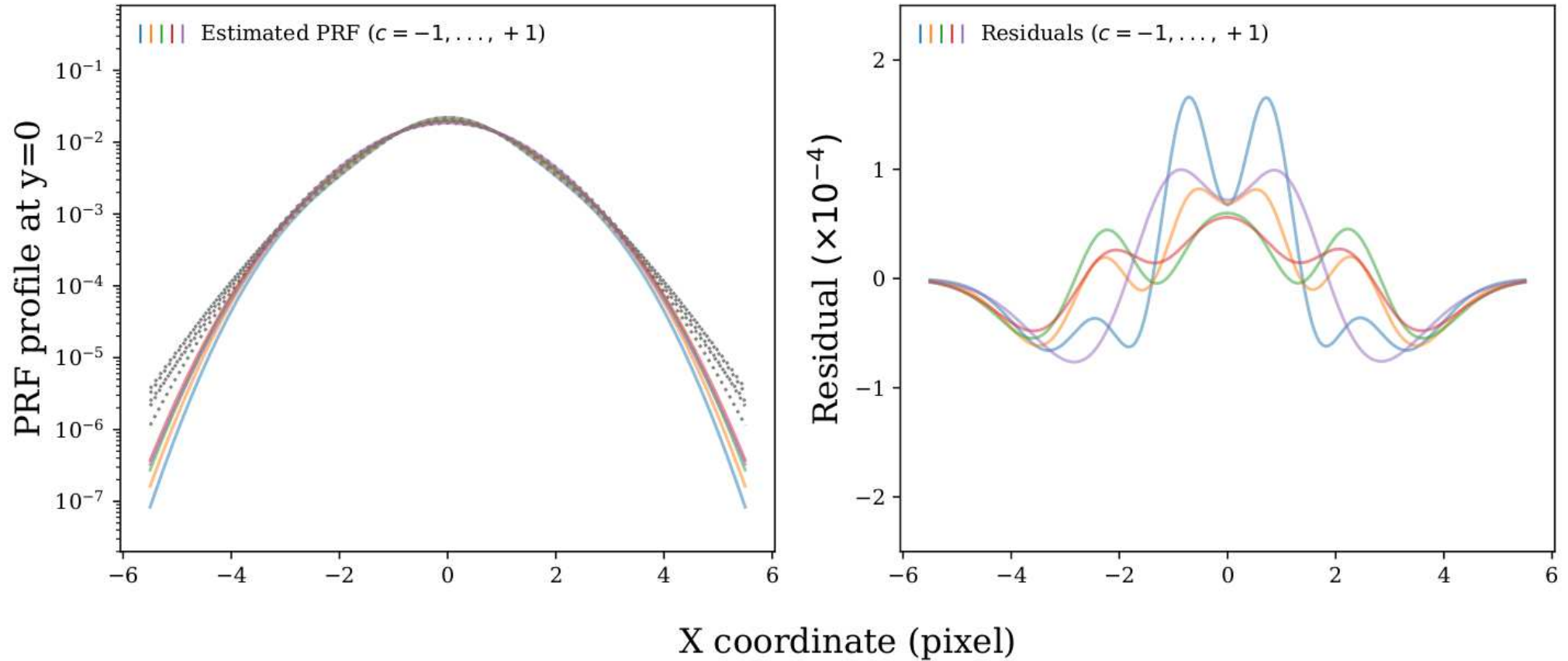
Results

Original (top) and reconstructed (bottom) ePSF

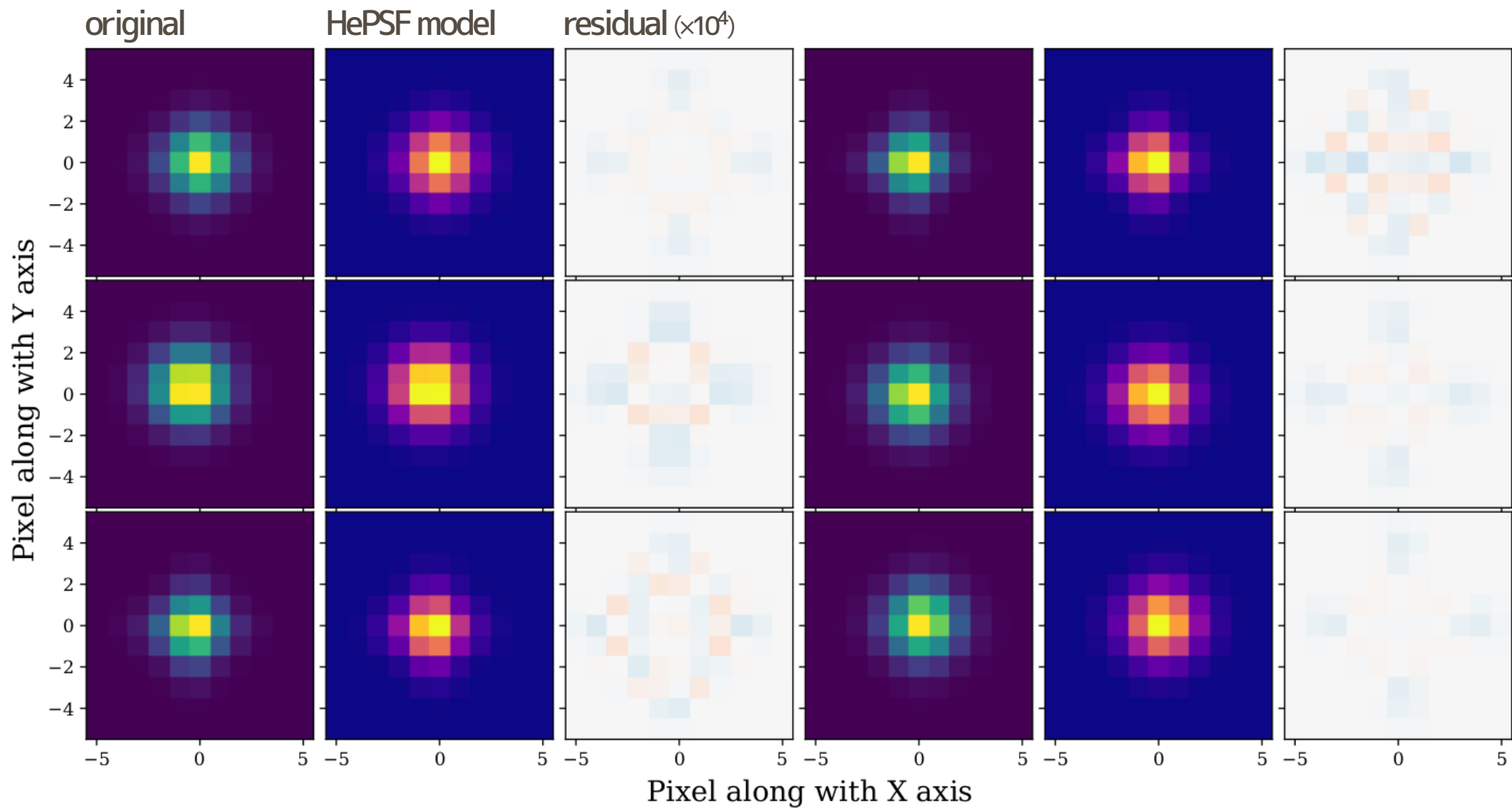


Results

Profiles of the reconstructed ePSF at $y=0$ and residuals



Results

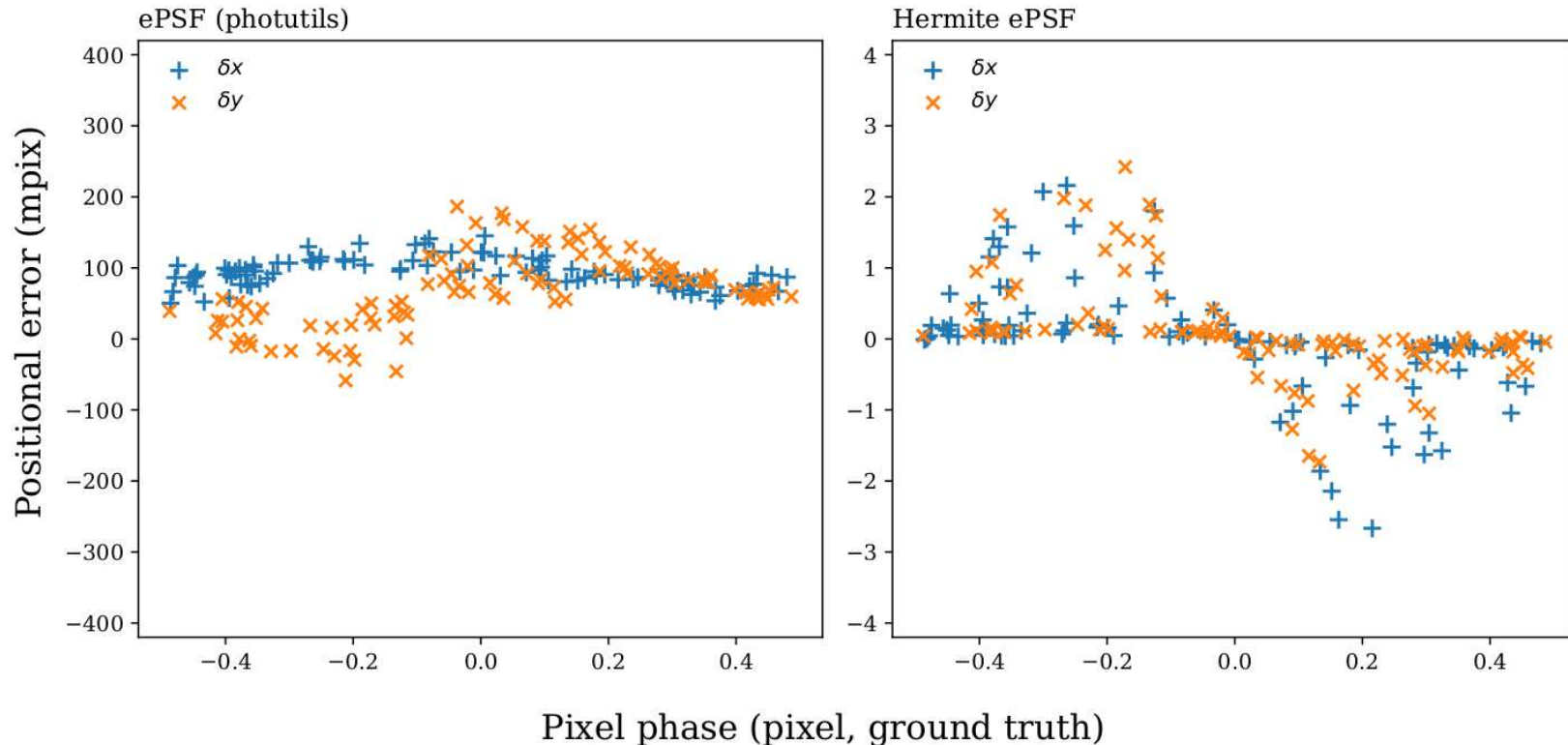


Results

Astrometric performance comparison with a single ePSF (photutils).

The *HePSF* method achieved much better accuracies.

Outliers are sources of $c \approx -1$, suggesting that higher terms are required to fit fine structures.



Discussion

We confirmed that the *HePSF* successfully represents a variable ePSF.

- ▶ High astrometric accuracies are achieved with small degrees of freedom.
- ▶ The model complexity is manually adjustable.
- ▶ Additional variabilities are easily implemented.

More realistic and practical experiments remain to be done.

- ▶ The ePSF has a single color parameter...
- ▶ The ePSF shape is relatively simple...
- ▶ Color parameter of each source is precisely known...
- ▶ Source SNRs are extremely high ($\approx 10^6$)...

Summary

We propose an approach to model a variable ePSF.

An ePSF is expanded using Hermite polynomials.

Expansion coefficients are defined as functions with additional parameters.

$$\Psi(x, y) \simeq \frac{1}{Z} \left[1 + \sum_{2 \leq k+l \leq M} \nu_{k,l} \mathcal{H}_k\left(\frac{x}{\sigma_x}\right) \mathcal{H}_l\left(\frac{y}{\sigma_y}\right) \right] \exp\left[-\frac{1}{2} \left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right)\right]$$

The *HePSF* model successfully worked with a variable ePSF.

100 stellar images were generated using an artificial variable ePSF.

The color dependence of the ePSF was successfully reconstructed.

The *HePSF* model provided accurate positional measurements (\sim mpix).

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