Introduction to Adaptive Optics

Olivier Guyon (guyon@naoj.org)

Center for Astronomical Adaptive Optics, University of Arizona
Subaru Telescope, National Astronomical Observatory of Japan

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Why Adaptive Optics?

Imaging through ideal telescope (reminder)
Imaging through atmospheric turbulence
  Wave propagation (diffraction, geometrical)
  Atmospheric turbulence: Fried Parameter and seeing
  Modeling atmospheric turbulence and its effect on image quality

How does adaptive optics work?

Adaptive optics principle
  What is an Adaptive Optics loop?
  Identification of main components in AO system

Deformable mirrors

Wavefront sensing
  Types of wavefront sensors: SH, Curvature, Pyramid
  Laser guide stars

Making it all work together: control algorithms

Types of AO systems for Astronomy
  Wide field systems: GLAO, MCAO
  Extreme-AO for high contrast imaging

Useful references:
Adaptive Optics in Astronomy (2004), by Francois Roddier (Editor), Cambridge University Press
Adaptive Optics for Astronomical Telescopes (1998), by John W. Hardy, Oxford University Press
Imaging through an ideal telescope (reminder)

Diffraction by an aperture – telescope diffraction limit

Fresnel diffraction integral:

\[ E(x, y, z) = \frac{e^{ikz}}{i\lambda z} \int \int E(x', y', 0) e^{\frac{ik}{z}[(x-x')^2+(y-y')^2]} \, dx' \, dy' \]

In imaging telescope, focal plane is conjugated to infinity (far field)
Fraunhofer is far field approximation of the Fresnel diffraction integral – and can easily be computed as a Fourier transform.

For circular aperture without obstruction : Airy pattern
First dark ring is at \(~1.22 \, \lambda/D\)
Full width at half maximum \(~1 \, \lambda/D\)
The “Diffraction limit” term = \(1 \, \lambda/D\)

D=10m, \(\lambda=0.55 \, \mu m\) \(\rightarrow \lambda/D = 0.011\) arcsec

On large telescopes, image angular resolution is limited by atmospheric turbulence on the ground, at about 1 arcsecond
\(\rightarrow\) Adaptive optics required for < arcsecond imaging
Imaging through Atmospheric turbulence

Diffraction by an aperture $\rightarrow$ Images should get sharper as telescope size increases (angular resolution $\sim \lambda/D$)

Below: Images of a point source at infinity at $\lambda = 0.6 \, \mu m$

![Images of a point source at infinity at $\lambda = 0.6 \, \mu m$](image)
Imaging through Atmospheric turbulence

Unfortunately, atmospheric turbulence dominates angular resolution of telescope for $D > \sim 0.2\text{m}$ in the visible (unless AO is used)

Below: Images of a point source at infinity at $\lambda = 0.6\ \mu\text{m}$ with turbulence
What is Atmospheric Turbulence?

Spatial variations in refractive index → poor image quality

Turbulence is energy dissipation effect:
Large motions → breaks down into smaller turbulence cells → friction (heat dissipation) at inner scale
Refractive index spatial structure function (3D):
\[ D_N(\rho) = <|n(r)-n(r+\rho)|^2> = C_N^2 \rho^{2/3} \]  
(equ 1)
Equation is valid between inner scale (~mm) and outer scale (few m)

Taylor approximation: turbulence is a frozen wavefront pushed by the wind (frozen flow)
Between inner and outer scale, turbulence is well described by this power law.

Refractive index temporal structure function under Taylor approximation:
\[ D_N(\tau) = <|n(r,t)-n(r,t+\tau)|^2> = C_N^2 |\nu\tau|^{2/3} \]
Wavefront phase spatial structure function (2D):

\[ D_{\phi_a}(\rho) = \left\langle |\phi_a(r) - \phi_a(r + \rho)|^2 \right\rangle_r \]

Can be obtained by integrating equ 1 over light path:

\[ D_{\phi_a}(\rho) = 6.88 \left( \frac{|\rho|}{r_0} \right)^{5/3} \]  
(equ 2)

With \( r_0 = \) Fried Parameter [unit = m]

\[ r_0 = \left( 16.7 \lambda^{-2} (\cos \gamma)^{-1} \int_0^\infty dh C_N^2(h) \right)^{-3/5} \]

Wavelength

Elevation (=0 for Zenith)
Fried Parameter & Seeing

Wavefront phase error over a circular aperture of diameter $d$:

$$\sigma^2 = 1.0299 \left( \frac{d}{r_0} \right)^{5/3}$$

$r_0 = \text{Fried Parameter} \ [\text{unit} = \text{m}] = \text{diameter of telescope for which atmospheric wavefront} \sim 1 \ \text{rad}^2$

Seeing $= \frac{\lambda}{r_0} = \text{angular size of PSF in long exposure}$

In this “collapsed” treatment of turbulence (what is the wavefront in a single direction in the sky), turbulence is fully described by $r_0$ and wind speed $v$

If variation of wavefront over small angles is important, the turbulence profile becomes important
Atmospheric turbulence, wavefront variance, Image quality

D = telescope diameter
\[ \sigma^2 = 1.03 \left( \frac{D}{r_0} \right)^{5/3} \]

Seeing = \( \frac{\lambda}{r_0} \)

Number of speckles = \( \left( \frac{D}{r_0} \right)^2 \)

D = 8 m, \( r_0 = 0.8 \) m
(0.2 m in visible = 0.8 m at 1.6 \( \mu \)m)

Wavefront error \( \sigma \) is in radian in all equations.

Wavefront variance \( \sigma^2 \) is additive (no correlation between different sources), and the wavefront error budget is built by adding \( \sigma^2 \) terms.

Wavefront error (m) = \( \lambda \times \frac{\sigma}{2\pi} \)

Strehl ratio \( \sim e^{-\sigma^2} \)
(Marechal approximation, valid for Strehl ratio higher than \( \sim 0.3 \))
Seeing (or its equivalent $r_0$) is the most used metric to quantify atmospheric turbulence

WITHOUT AO (and with long exposures), this is the only relevant quantity to describe atmospheric turbulence
With AO, **isoplanatic angle** and **coherence time** become important.

How quickly does the wavefront change with location on the sky is quantified by **isoplanatic angle**
- field of view of corrected image
- how far from science target can the guide star be

Speed at which wavefront changes is quantified by **coherence time**
- how fast should the AO system run?
- how faint a guide star can be used?
Example: Mauna Kea observatory forecast

Dry and Stable
$C_n^2$ profile
Differential Image Motion Monitor (DIMM)

Concept: measure differential motion, for a single star, between images formed by different subapertures of a single telescope

RoboDIMM for Isaac Newton group of Telescope (LaPalma, Canary islands, Spain)
What gains are offered by Astronomical Adaptive Optics Systems?

**Angular resolution:**
Resolve small features on Sun, Moon, planets, disks, galaxies

**Improved sensitivity for faint objects:**
Detection of faint objects is a background-limited problem. By making the image smaller, the AO system limits amount of background mixed with image, and improves sensitivity. Efficiency with AO goes as $D^4$ instead of $D^2$ without AO. This is especially important in infrared, as sky glows, and AO work well.

**Astrometry:**
Measuring the position of a source. For example: measuring the mass of the black hole in the center of our galaxy.

**Confusion limit:**
Astronomical imaging of sources is often confusion limited. Better angular resolution helps! For example: studying stellar populations in nearby galaxies.
Why Adaptive Optics?

Double star, separation=0.276"  
Seeing=0.7" @ 0.5mic  
Magnitude=10.7  
Strehl Ratio=30%

Uncorrected  
Closed Loop  
Closed Loop + Deconvolution

Io (Keck)

Neptune imaged by Keck AO

without AO  
with AO
What is Adaptive Optics?

Main components of an AO system:

**Guide star(s):** provides light to measure wavefront aberrations, can be natural (star in the sky) or laser (spot created by laser)

**Deformable mirror(s) (+ tip-tilt mirror):** corrects aberrations

**Wavefront sensor(s):** measures aberrations

**Computer, algorithms:** converts wavefront sensor measurements into deformable mirror commands
What does an Adaptive Optics system look like?

Altair Optics bench (for Gemini)
Deformable Mirror

Requirements and issues

**Stroke**: how much can the DM surface move (need few μm)
**Number of actuators**: 100s, 1000s
**Speed**: How fast does the DM respond (need ~kHz), vibrations
**Stability**: Does the surface drift with time? are the actuator responses stable in time? sensitivity to temperature, humidity, pressure

**Hysteresis** should be low (<30%, ideally less)
**Backlash** should be low
**Wavefront quality**
- Shape when not driven
- Non-correctable surface errors

**Heat output**
**Reliablility**
**Piezoelectric effect**

Coupling between electric field and mechanical strain

Applied electric field ↔ dimension

Relation is approximately linear, but:
- Hysteresis (~10%)
- Small drifts (temperature, excitation history)

Requires high voltage (typically > 100V)

Bipolar (voltage can be positive or negative)

**Electrostrictive materials**

Quadratic relationship between electric field and displacement

Smaller hysteresis, but more temperature dependance than piezoelectric materials.

Higher capacitive load → requires higher currents
Piezo stack DM

Displacement is proportional to electric field
Large displacement = high electric field over long length of material

To avoid unreasonably high voltages, stack of piezo layers is used
Voltage is applied across each layer

Piezo actuated mirror (Cilas)
Bimorph DMs

Curvature DM made by IfA, University of Hawaii
The 4356-actuator deformable mirror for PALM-3000 (Xinetics Inc.).
Magnetic force

Adaptive secondary Mirror

Thermal IR instruments need low thermal background -> fewer warm optics

adaptive secondary mirror (MMT, LBT, Magellan)
Magnetic force

Small magnetic DMs
Key advantage is large stroke

> 20 micron stroke
(high speed DM97, Alpao)

241 actuators magnetic DM
(Alpao)
Electrostatic DMs
large number of actuators in a small space

Small electrostatic MEMS mirror
(Boston Micromachines, 1024 act)
WFS: Role & Requirements

Problem: Detectors measure light intensity, not phase → an optical trick is required to convert wavefront phase into intensity.

Wavefront sensor must measure wavefront to allow correction with Deformable mirror. Wavefront measurement is done by Wavefront Sensor (hardware) + wavefront reconstructor (Software, translates WFS signal into DM language)

Requirements (need to be balanced in AO system design):
- **Accuracy**
- **Spatial resolution** (number of modes measured - ideally as many as can be corrected by DM)
- **Efficiency** (good use of photons)
- **Speed** (coupled with accuracy and efficiency)
- **Linearity** (faster reconstruction → helps with speed)
- **Range** (ability to measure large wavefront errors)
- **Robustness** (chromaticity, ability to work on extended sources, etc ...)
- **Match with DM** (WFS must see what DM can correct)
Shack-Hartmann WFS

Measures wavefront slope in front of each subaperture

courtesy: Boston Micromachines
Pyramid WFS

Separates focal plane into 4 quadrants, each quadrant re-imaged in pupil plane

Geometrical optics explanation: parts of the pupil with a given slope correspond to light in the corresponding focal plane quadrant
Curvature WFS

Light propagation turns phase into amplitude (similar to scintillation)

Lenslet array used to inject light into a series of fibers, which are connected to photon-counting Avalanche PhotoDiodes (APDs)
Laser Guide stars can create a spot in the atmosphere for the WFS to measure the WF

**Allows quasi-complete sky coverage**

**Cone effect** due to finite altitude of LGS (90km sodium, few km for Rayleigh)

- can be solved by using several lasers and tomography

**Tip/Tilt & Focus sensing**

Upstream & downstream paths are the same: tip/tilt not seen

Sodium layer altitude not fixed: LGS focus info is incomplete (can be used to sense fast focus)

- Still need NGS(s) for tip/tilt & Focus
AO control

How should the AO system drive the DM from WFS measurements?

“standard” solution (fast, linear):

- Measure/model how WFS measures DM commands
- If relationship is linear, this is stored as a “response matrix”
  “response matrix” is inverted -> “control matrix” (this step usually includes some filtering – see next slide)
- WFS measurements x control matrix = DM commands

This could also be done by computing explicitly the wavefront:

WFS measurements -> wavefront -> DM commands

Good AO control now allows to separate WFS choice from DM choice: example: Curvature WFS could run with a MEMs DM
Linear control of AO system: response and control matrix

Wavefront sensor response to DM commands is linear:

*If DM command increased by factor \( x \), WFS signal multiplied by \( x \)*

*WFS signal to sum of 2 DM commands = sum of the 2 WFS signals*

→ Relationship can be written as matrix multiplication:

\[
A = M_{\text{resp}} B
\]

Assuming \( m \) actuators, \( n \) sensing elements

\( A_{i=0...n-1} \): WFS signal vector (for example, \( x,y \) centroids for SH)

\( B_{j=0...m-1} \): DM commands (can be voltages, displacements)

\( M_{\text{resp}} \): \( m \times n \) Response matrix (usually not a square matrix !)

**AO control problem:**

Given \( A \) (WFS measurement), and knowing \( M_{\text{resp}} \), what is the DM command \( B \) which will produce the WFS signal \(-A\) ?

How to do this in a robust way, in the presence of noise, and with \( M_{\text{resp}} \) which is generally not not irreversible?
Wavefront sensor response to DM commands is linear → DM commands to produce a given WFS signal is obtained by multiplication of $A$ (WFS signal) by the control matrix $M_{\text{contr}}$

$B = M_{\text{contr}} A$

With $M_{\text{contr}}$ the pseudo-inverse of $M_{\text{resp}} = M_{\text{resp}}^+ = (M_{\text{resp}}^\top M_{\text{resp}})^{-1} M_{\text{resp}}^\top$

If $M_{\text{resp}}$ is an invertible square matrix, $M_{\text{contr}} = M_{\text{resp}}^{-1}$

$M_{\text{contr}}$ can be computed by Singular Value Decomposition (SVD) of $M_{\text{rest}}$

**Singular Value Decomposition:**

$M = U \Sigma V^*$

- $U$: Unitary matrix
- $\Sigma$: diagonal matrix (Eigenvalues $a_i$)
- $V$: Unitary matrix, $V^*$ its conjugate transpose ($=V^\top$ if $V$ real)

**Pseudo-inverse:**

$M^+ = V \Sigma^+ U^*$

With $\Sigma^+ = 1/a$ if $|a|>0$, and 0 if $a=0$
Linear control of AO system: response and control matrix

In practice:

- Although $M_{\text{resp}}$ could be in theory computer, $M_{\text{resp}}$ is usually measured by poking DM actuators and measuring the corresponding change in the WFS signal.

- $M_{\text{resp}}$ can be measured quickly by driving simultaneously several actuators if $M_{\text{resp}}$ is a sparse matrix (each DM actuator has an effect on a small number of sensors).

- $M_{\text{contr}}$ is usually computed by SVD, and presence of noise in the measurement forces modes of $M_{\text{resp}}$ with small eigenvalues to be discarded from the control loop (their eigenvalue considered $=0$ in the pseudo-inverse computation).
System response matrix: example (simulation)

Measured response matrix includes system defects/imperfections, such as:
- alignment errors
- defective sensor(s)
- defective actuator(s)
- crosstalk

Measuring response matrix is very good system diagnostics
AO loop control: loop gain

At each step of the loop, offset \( dDM (= -M_{\text{contr}} A) \) required to cancel WFS signal is computed. Ideally, with \( k \) the loop step (= time): \( DM_k = DM_{k-1} + dDM \)

Problem: with above equation, loop would likely be unstable
   Effective time lag in the measurement is 1/sampling time
   \( \rightarrow \) some temporal frequencies are amplified
   Measurement is noisy, and several consecutive measurements should be averaged

Solution: use loop gain \(< 1\):

\( DM_k = DM_{k-1} + g \, dDM \)

With \( 0 < g < 1 \)
   Noisy WFS measurement (faint guide star) \( \rightarrow \) small \( g \)
   High quality WFS measurement (bright guide star) \( \rightarrow \) large \( g \)

Loop gain can be different for different modes (Modal control)
Predictive control can improve AO system performance
### Types of AO systems

<table>
<thead>
<tr>
<th>Field of view</th>
<th># of DMs</th>
<th># of guide stars</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single conjugate AO</td>
<td>~ 30 arcsecond</td>
<td>1 DM, usually conjugated to ground</td>
<td>1 (LGS or NGS)</td>
</tr>
<tr>
<td>ExAO (ExAO)</td>
<td>~1 arcsecond</td>
<td>1 DM, or 2 DMs in woofer/tweeter</td>
<td>1 (on-axis NGS)</td>
</tr>
<tr>
<td>Laser Tomography AO</td>
<td>~30 arcsecond</td>
<td>1 DM</td>
<td>&gt;3 (LGS or NGS)</td>
</tr>
<tr>
<td>GLAO (GLAO)</td>
<td>very wide – up to ~degree</td>
<td>1 DM, conjugated to ground</td>
<td>&gt;3</td>
</tr>
<tr>
<td>MCAO (MCAO)</td>
<td>wide, &gt; arcmin</td>
<td>2 to 3</td>
<td>&gt; 3, can be LGS or NGS</td>
</tr>
<tr>
<td>MOAO (MOAO)</td>
<td>wide but fragmented</td>
<td>1 DM per object (+ 1 DM in common path?)</td>
<td>about 1 GS per field</td>
</tr>
</tbody>
</table>
Astronomical AO system diversity:
Field of view vs. Wavefront error

- Narrow field visible AO
- Narrow field NGS in near-IR
- High contrast “Extreme-AO”
- 1 guide star OK usually also >1 LGS
- >1 guide star needed

- “Low order AO”
- Multi Conjugate AO (MCAO)
- Multi Object AO (MOAO)
- Laser Tomography AO (LTAO)
- Ground-layer AO

- 10 nm
- 100 nm

- Optics size, optical layout complexity
- # of DM actuators
- # of WFS elements
- AO loop speed
- Need more photons
- Need more photons

- 0”
- 10”
- 1’

- 1 DM OK
- >1 DM needed

- Easier
- Challenging

Field of view
Wavefront Error (nm)
ExAO example: The Gemini Planet Imager Extreme-AO system
Fundamental problem to solve: Isoplanatic Angle

If we assume perfect on-axis correction, and a single turbulent layer at altitude $h$, the variance (sq. radian) is:

$$\sigma^2 = 1.03 \left(\frac{\theta}{\theta_0}\right)^{5/3}$$

Where $\alpha$ is the angle to the optical axis, $\theta_0$ is the isoplanatic angle:

$$\theta_0 = 0.31 \left(\frac{r_0}{h}\right)$$

$D = 8 \text{ m}, r_0 = 0.8 \text{ m}, h = 5 \text{ km} \rightarrow \theta_0 = 10''$
Solution:

Wavefront measurement: Several guide stars needed

Several guide stars (Laser and/or natural) → volumetric knowledge of atmospheric turbulence, instead of simply collapsed turbulence

Wavefront correction:
Several DMs if good correction over a large FOV

Or, single DM driven to correct average wavefront error over wide FOV (Ground-layer AO, partial correction)

With single DM, there is a fundamental limit in the wavefront error vs. FOV tradeoff. Multiple DMs is the only way to break this limit.
Ground Layer Adaptive Optics (GLAO)

- Significant part of turbulence (~50% or more) is located near ground level
- Ground layer turbulence is common to sources in a wide field of view

→ With correction of ground layer, image quality is improved over a wide field of view

Problem: how to isolate ground layer turbulence from high altitude turbulence.

Solution: use several WFSs. The part of the wavefront common to all WFSs is the ground layer
Fig. 8. The core of M3 imaged at 2.2 μm in two 60 s exposures. a) The full 110 arc sec field of the IR camera in the native seeing limit of 0.7 arc sec, on a logarithmic intensity scale. b,d) Two smaller 27 arc sec regions of the same image, indicated by the boxes in a) shown on a truncated linear scale in which bright stars appear saturated but which reaches the noise floor and brings out the faintest observable stars. c,e) In a second 60 s exposure of the same two regions, taken with GLAO running at 400 Hz and shown on the same linear scale as b) and d), the stellar image width is reduced to 0.3 arc sec and has a very similar PSF morphology across the whole field of view. For reference, we highlight a star in the corrected image with K band magnitude of 16.5, detected at a SNR of 26. In the uncorrected image, stars must be 2 mag brighter to be seen at the same SNR.
Multi-Conjugate Adaptive Optics (MCAO)
Concept: Use several DMs conjugated at different altitudes to perform correction over a wide field of view

Gemini South MCAO system
NGC6369 - GeMS/GMOS-S
FWHM = 0.08"
10min exposure

i-band & z-band
FoV = 2 x 1.3arcmin
Multi Object Adaptive Optics (MOAO)

Can be visualized as several tomographic AO systems sharing the same set of wavefront sensors: 1 DM per object of interest.

MOAO runs DMs in open loop → need for good DM calibration (WFSs do not see DMs)
EAGLE (ESO E-ELT project)
MCAO system with 20 fields

1. Shutter
2. Laser Guide Star Sensing System
3. Pick-off System (Focal Plane)
4. Target Reimaging and Magnification System (including Deformable Mirror)
5. Integral Field Unit and Spectrograph System
The next generation of large telescopes combine AO with telescope design

The 39m diameter European Extremely Large Telescope (EELT) optical design includes DM as large fold mirrors (M4: 2.4m diameter, 8000 actuators).
The Giant Magellan Telescope (GMT) secondary mirrors are adaptive and serve as DMs for the AO system(s).
The Thirty Meter Telescope (TMT), just like GMT and ELT, includes adaptive optics for first generation instruments.