Characterization of habitable exoplanets with simultaneous coronagraphy and astrometry with a single aperture telescope

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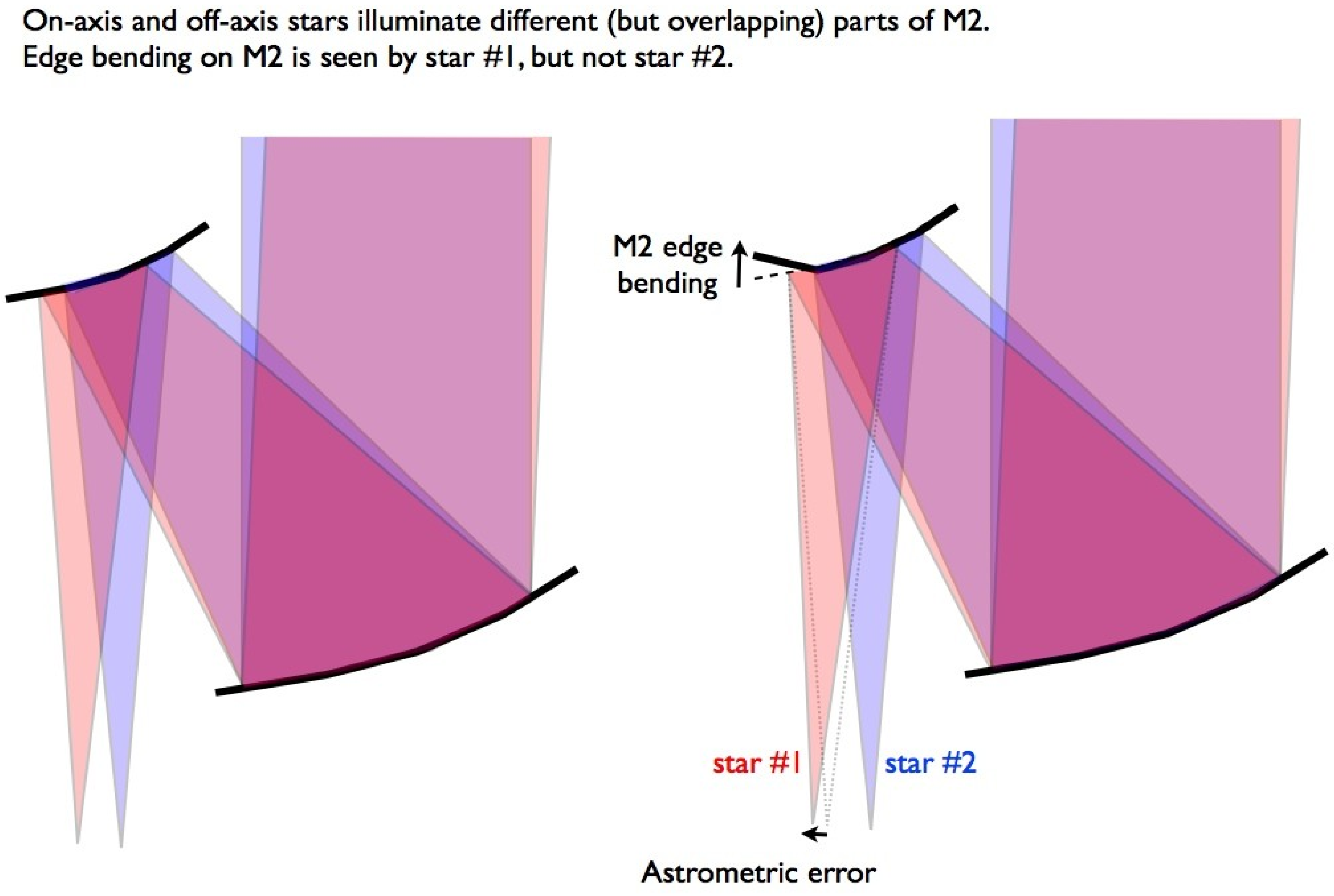
Wide field imaging technologies are now mature (optics, detectors)
Can we use wide field images for astrometry to identify and characterize exoplanets ?

Fundamental problem:
what do you do if you are looking at this field from the bottom of a swimming pool ?

Principle: use background stars around coronagraph target as an astrometric reference

With a 1.4-m telescope in the visible, 0.25 sq deg offers sufficient photons from stars at the galactic pole to provide an astrometric reference at the <50 nano-arcsec after taking into account realistic efficiency, zodi light and pixel sampling (1 day exposure)

Why is imaging astrometry difficult ?



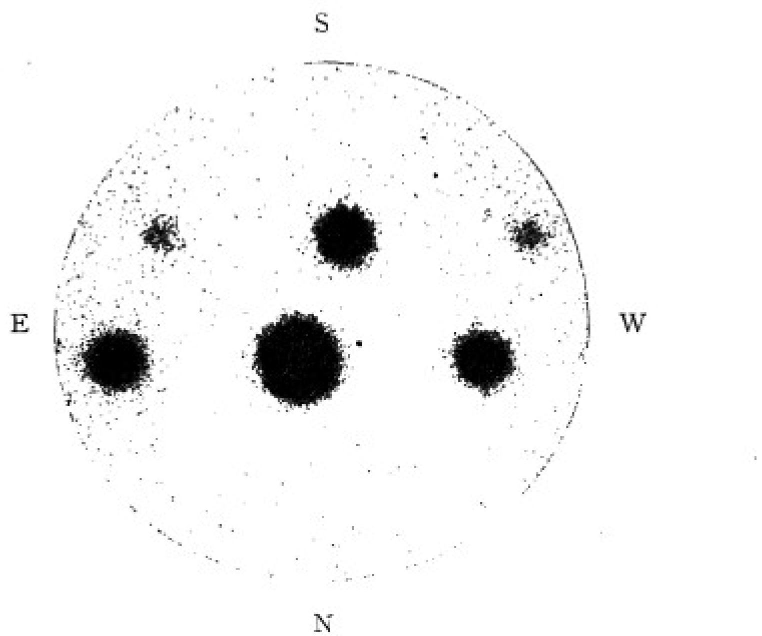
(1) Light from different stars on the sky travels different paths → small bending of optics produces field distortions

(2) The detector can move between observations (especially when using large mosaics)

(3) Pixels are not perfect and their response changes with time

+ (4) Central star is much brighter than background stars

Precursors ...



A 5-seconds exposure of Castor, enlarged 75 times. The separation of the components is 3″.74 or 0.198 mm on the plate. The first order spectra are one magnitude fainter than the central image. Taken December 1, 1939, by K. Aa. Strand, with the Sproul 24-inch refractor, aperture reduced to 13 inches, Eastman IV G emulsion, Wratten No. 12 (minus-blue) filter.

age of the fainter component, a compensation for possible magnitude error is provided by using the mean of the measured positions of the two spectral images instead of the central image. As long as the difference in intensity between the images does not exceed half a magnitude, the magnitude error is usually negligible; it is therefore sufficient to have a limited number of gratings, producing first-order spectra which are a whole number of magnitudes fainter than the central image. For example, in his work with the Sproul refractor, Strand[3] used four gratings, made of duraluminum, giving differences of one, two, three, and four magnitudes, respectively, between the central image and the first-order spectra. The bars are mounted on 10 cm-wide annular frames, cut from sheets of duraluminum, 3 mm thick. The constants of the four gratings are given below.

Constants of Sproul Objective Gratings

Grating	—width of— bar	opening	extinction for central image	first order minus central image mag. difference	distance
1	11.25 mm	11.21 mm	1.51 mag	.98 mag	.270 mm = 5″.10
2	7.12 mm	15.06 mm	.84 mag	2.05 mag	.273 mm = 5.15
3	3.98 mm	14.80 mm	.52 mag	3.01 mag	.322 mm = 6.08
4	3.20 mm	19.06 mm	.34 mag	3.95 mag	.272 mm = 5.13

“Long-focus photographic astrometry”, van de Kamp, 1951

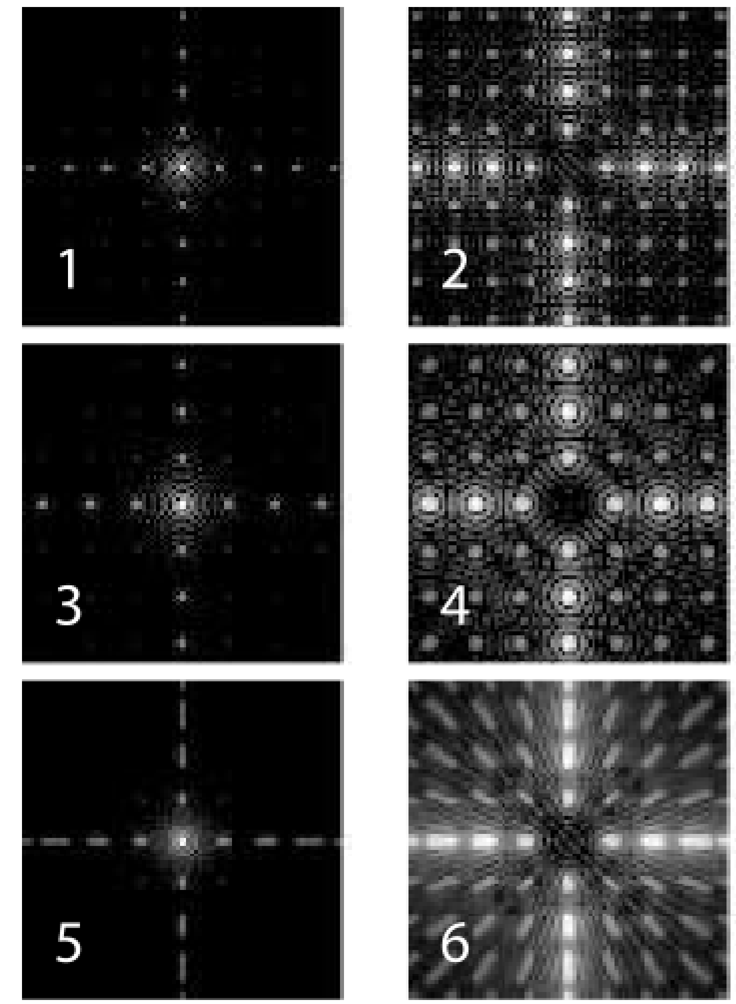


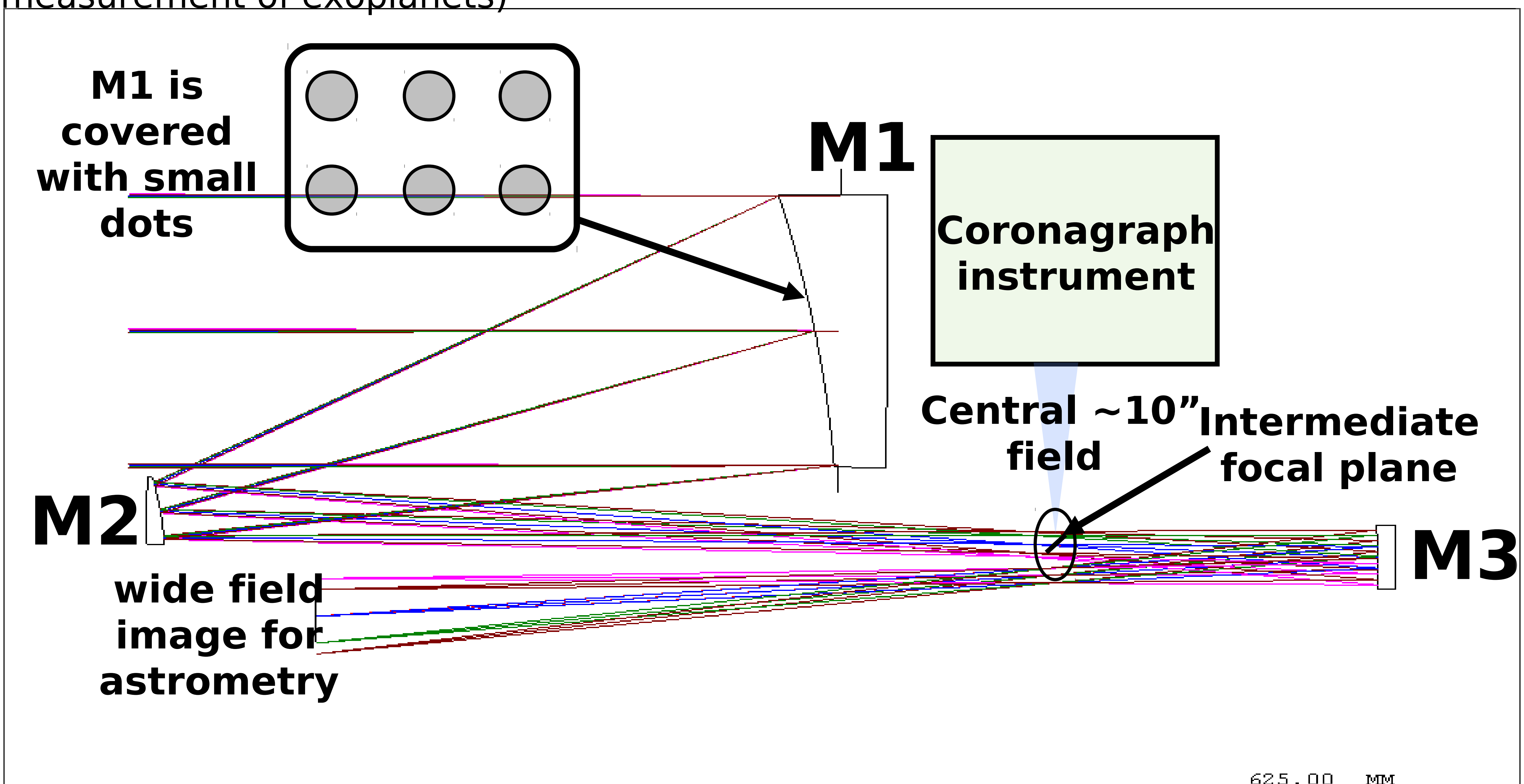
Fig. 1.—Monochromatic and broadband direct and coronagraphic PSFs with a square-geometry reticulate pupil mask. All images are on a logarithmic gray scale stretching 10 mag fainter than their peaks. The pupil is 128 pixels across, and the grid has a wire spacing of 16 pixels, with 2-pixel-wide wires. (1): Direct PSF for the shortest wavelength of a 20% bandwidth filter with uniform transmission within the bandpass, in the absence of phase errors. The satellite PSFs off the origin but along the horizontal and vertical axes are fainter than the central core of the PSF by a factor $\epsilon^2 = (g/d)^2$, where g is the wire thickness and d is the wire spacing. The satellite spots off the axes are ϵ^4 fainter than the corresponding central peak. (2): Coronagraphic PSF at the shortest wavelength of the filter. The off-axis sea of satellite spots are more visible in the coronagraphic image because the core has been suppressed. (3) and (4): Direct and coronagraphic PSFs for the longest wavelength of the filter. (5) and (6): Direct and coronagraphic PSF for the full bandpass. The length of any particular radial streak in this last pair of images (in resolution elements at the central wavelength of the bandpass) is approximately the fractional filter bandwidth multiplied by the radial distance of the spot at band center. The streaks all point toward the origin, so the smearing has no effect on astrometric precision according to Fraunhofer regime image formation theory. We suggest using the four satellite peaks closest to the core as fiducials for the position of the central occulted star in coronagraphic images.

“Astrometry and Photometry with Coronagraphs”, Sivaramakrishnan, Anand; Oppenheimer, Ben R., 2006

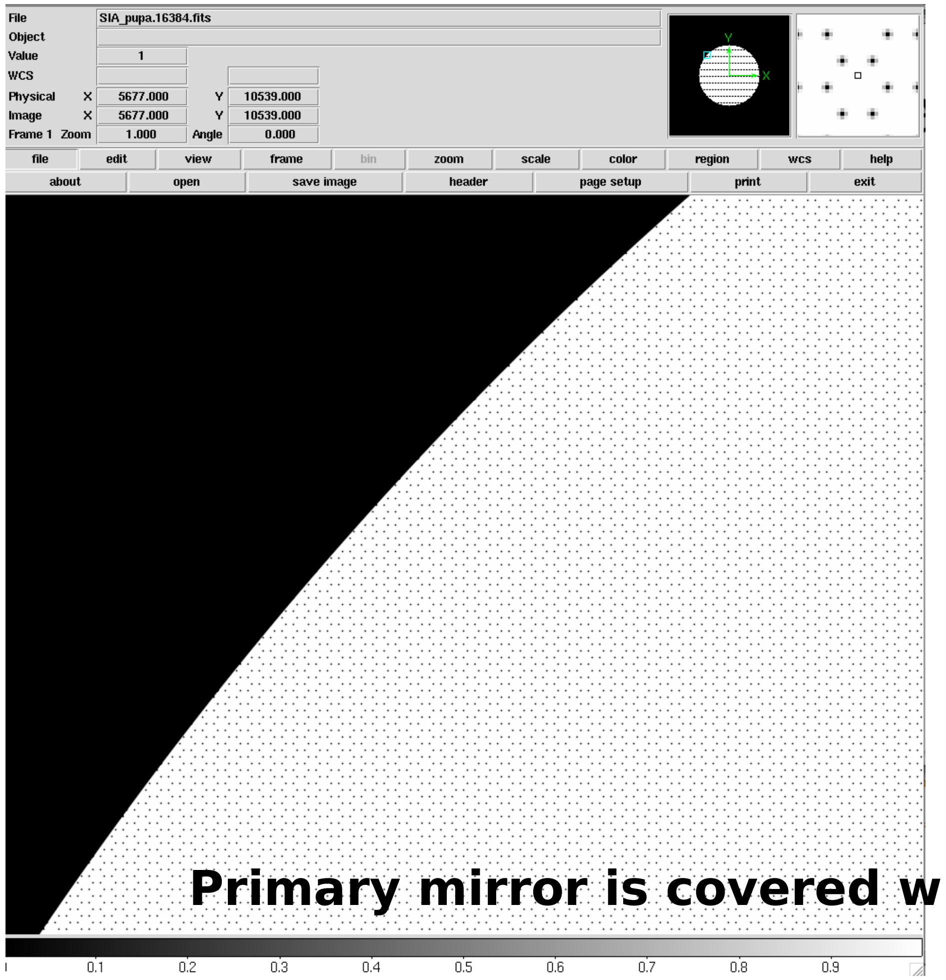
Optical Layout for simultaneous coronagraphy and astrometry

The telescope is a conventional TMA, providing a high quality diffraction-limited PSF over a 0.5 x 0.5 deg field with no refractive corrector. The design shown here was made for a 1.4m telescope (PECO).

Light is simultaneously collected by the coronagraph instrument (direct imaging and spectroscopy of exoplanet) and the wide field astrometric camera (detection and mass measurement of exoplanets)

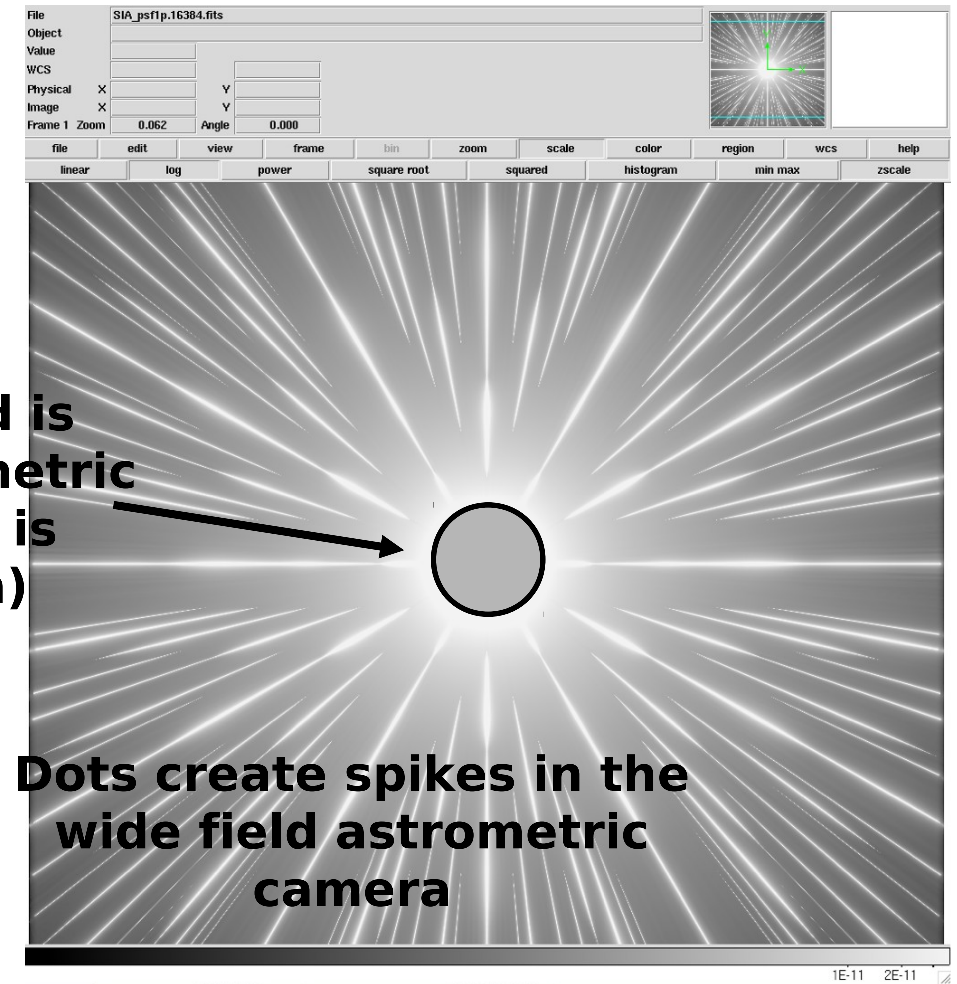


Dots on primary mirror create a series of diffraction spikes used to calibrate astrometric distortions



Primary mirror is covered with small dots

The center of the field is missing from the astrometric camera (central light is sent to coronagraph)

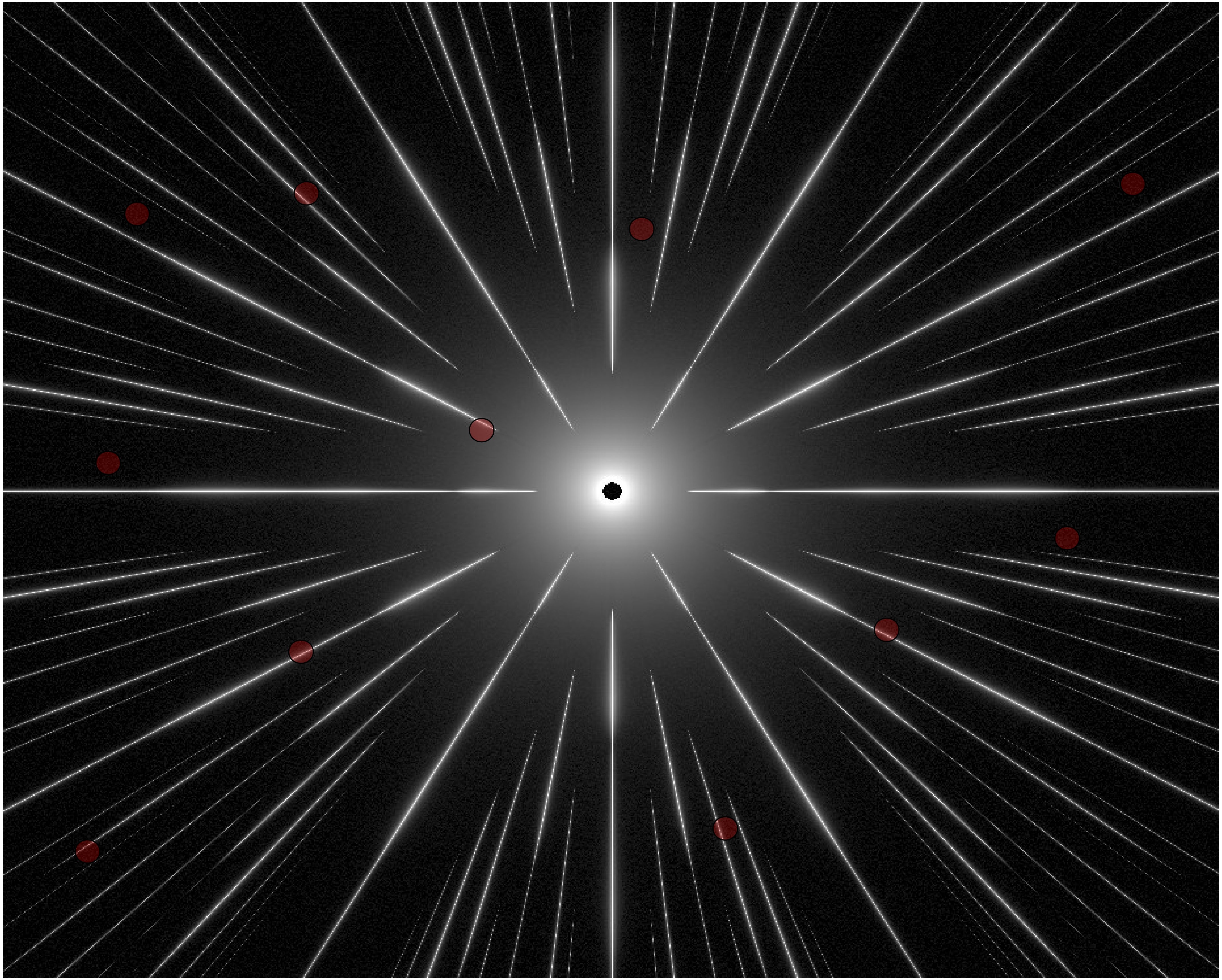


Dots create spikes in the wide field astrometric camera

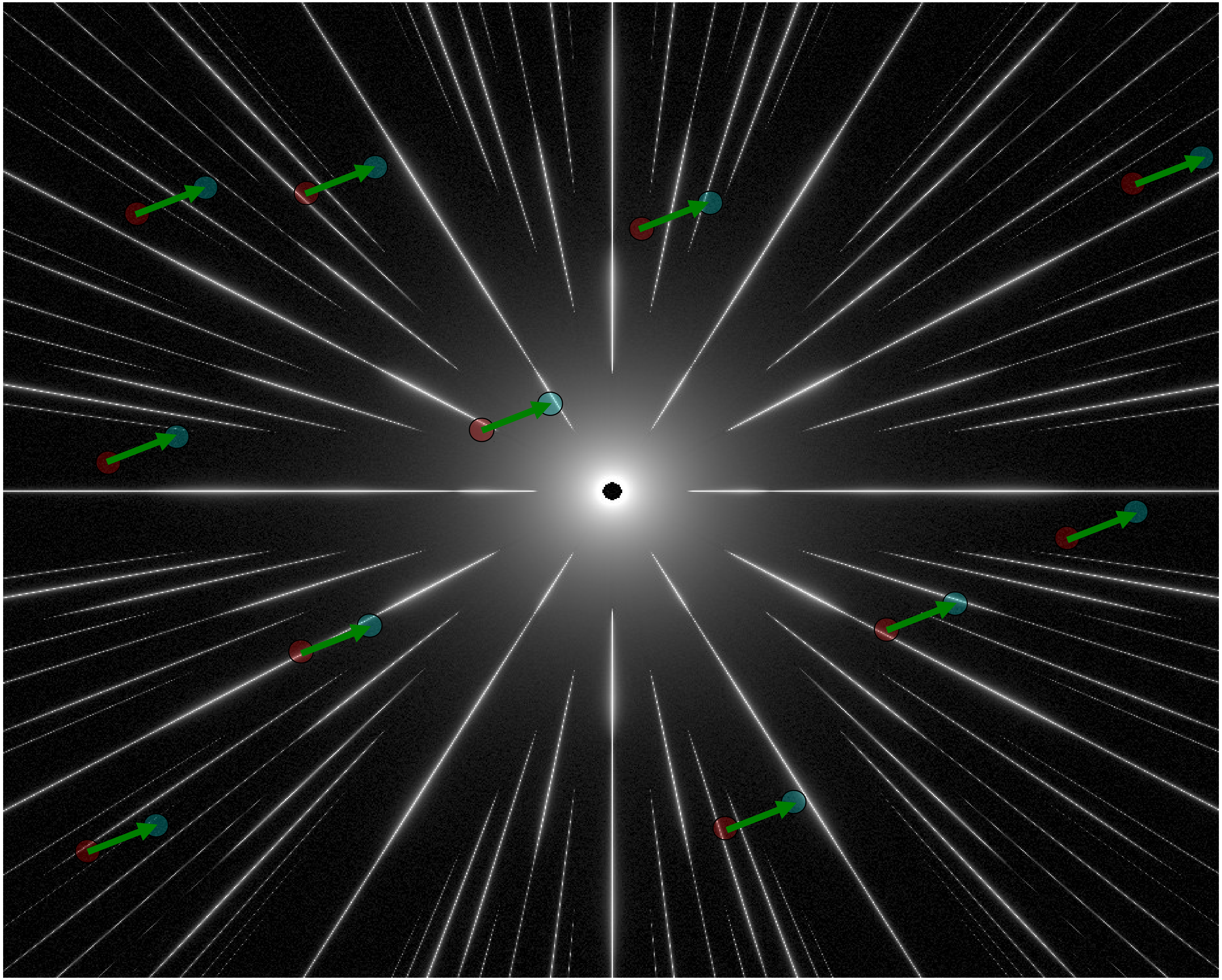
All astrometric distortions (due to change in optics shapes of M2, M3, and deformations of the focal plane array) **are common to the spikes and the background stars**. By referencing the background star positions to the spikes, the astrometric measurement is largely immune to large scale astrometric distortions.

Instead of requiring ~pm level stability on the optics over yrs, the stability requirement on M2, M3 is now at the nm-level over approximately a day on the optics surfaces, which is within expected stability of a coronagraphic space telescope. (Note: the concept does not require stability of the primary mirror).

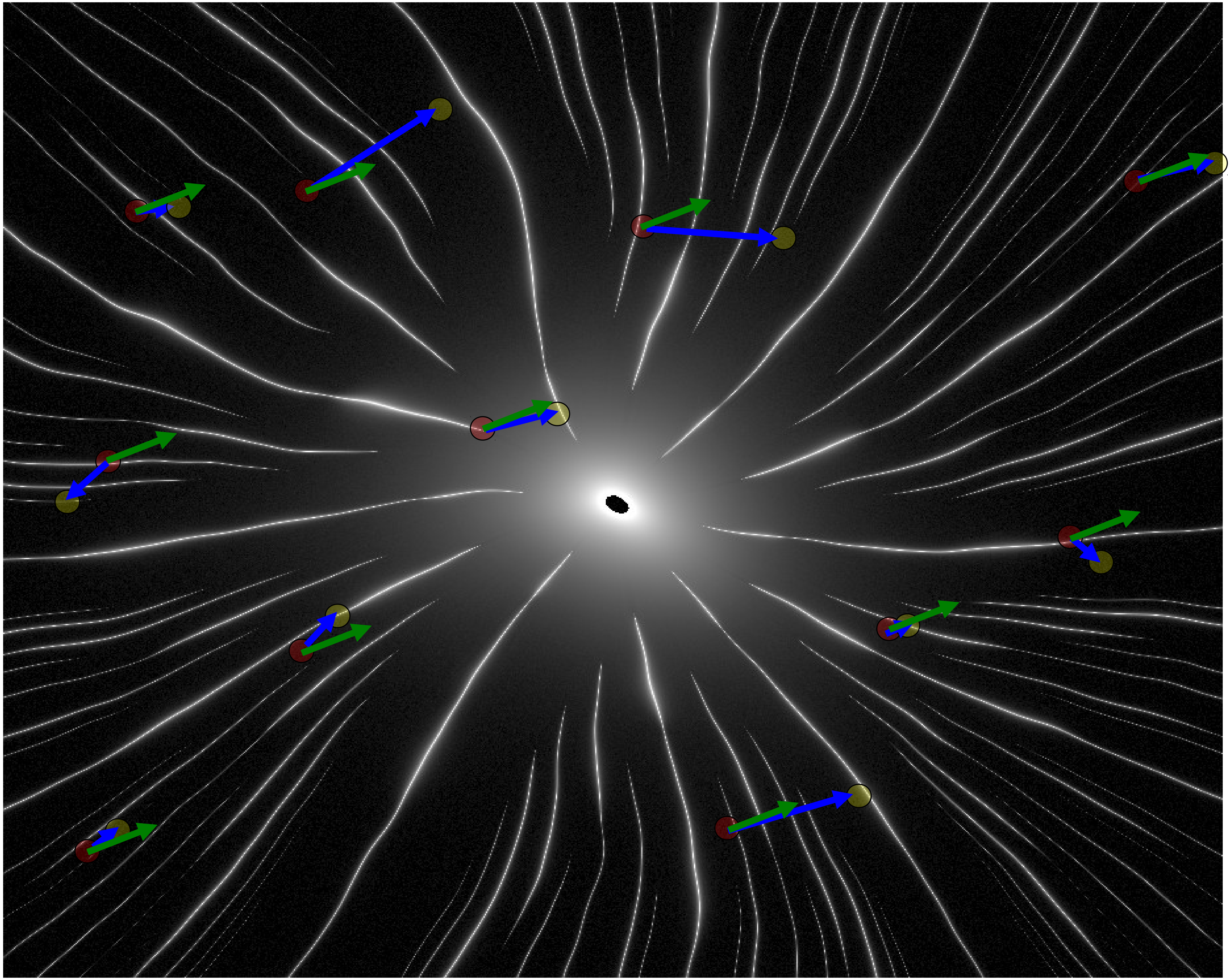
Red points show the position of background stars at epoch #1 (first observation)



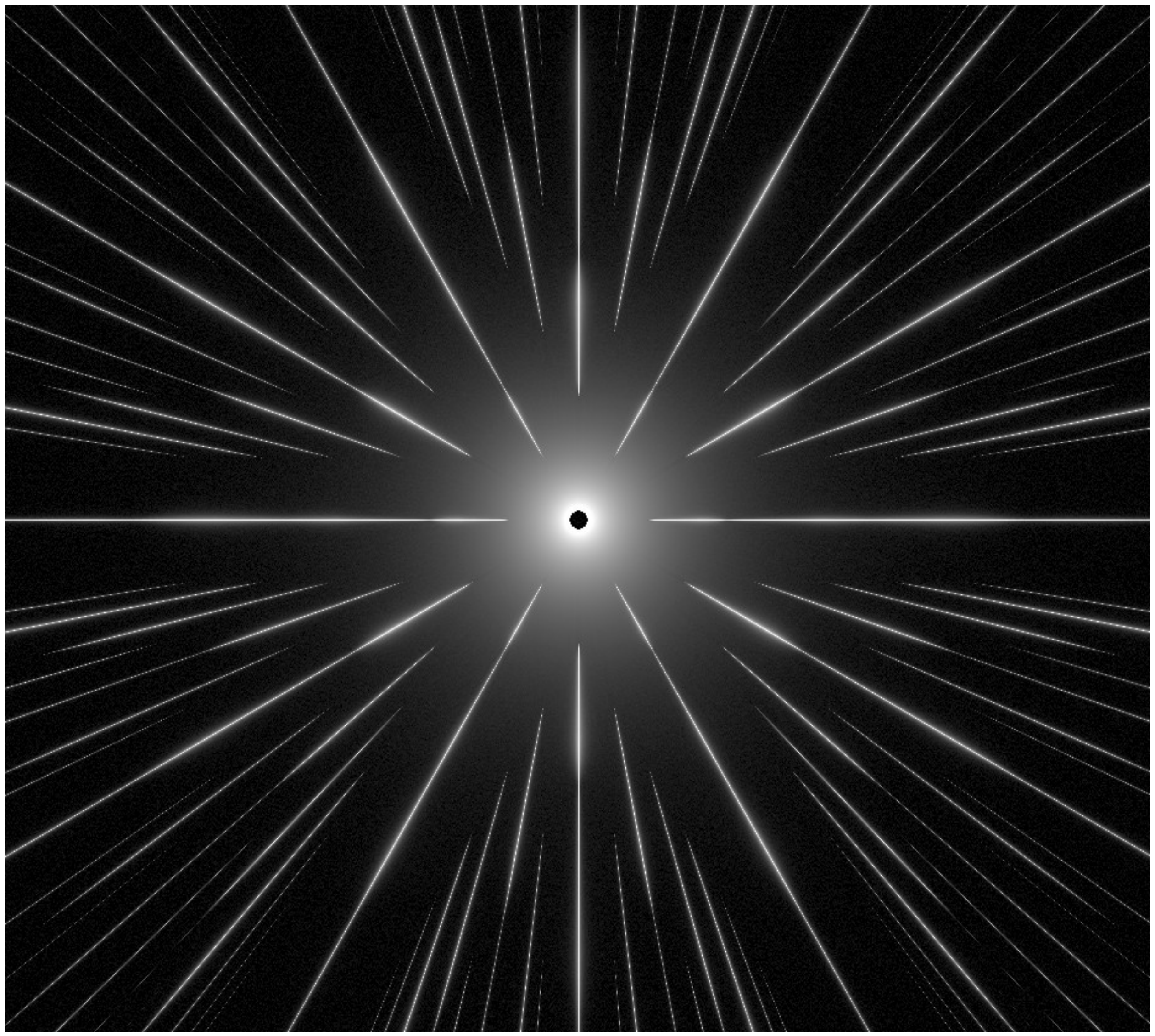
Blue points show the position of background stars at epoch #2 (second observation)
The telescope is pointed on the central star, so the spikes have not moved between the 2 observations, but the position of the background stars has moved due to the astrometric motion of the central star (green vectors).

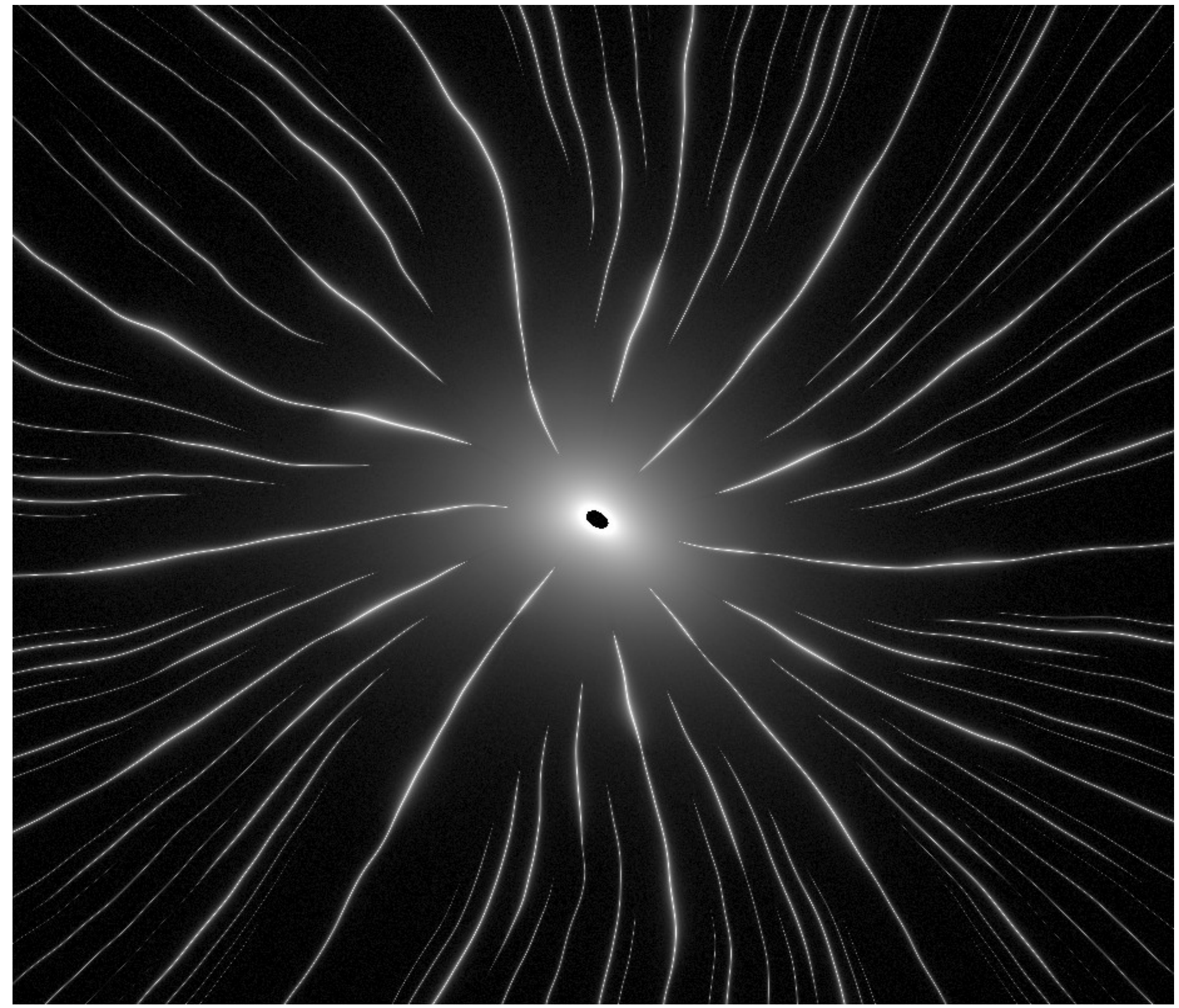


Due to astrometic distortions between the 2 observations, the actual positions measured (yellow) are different from the blue point. The error is larger than the signal induced by a planet, which makes the astrometric measurement impossible without distortion calibration.

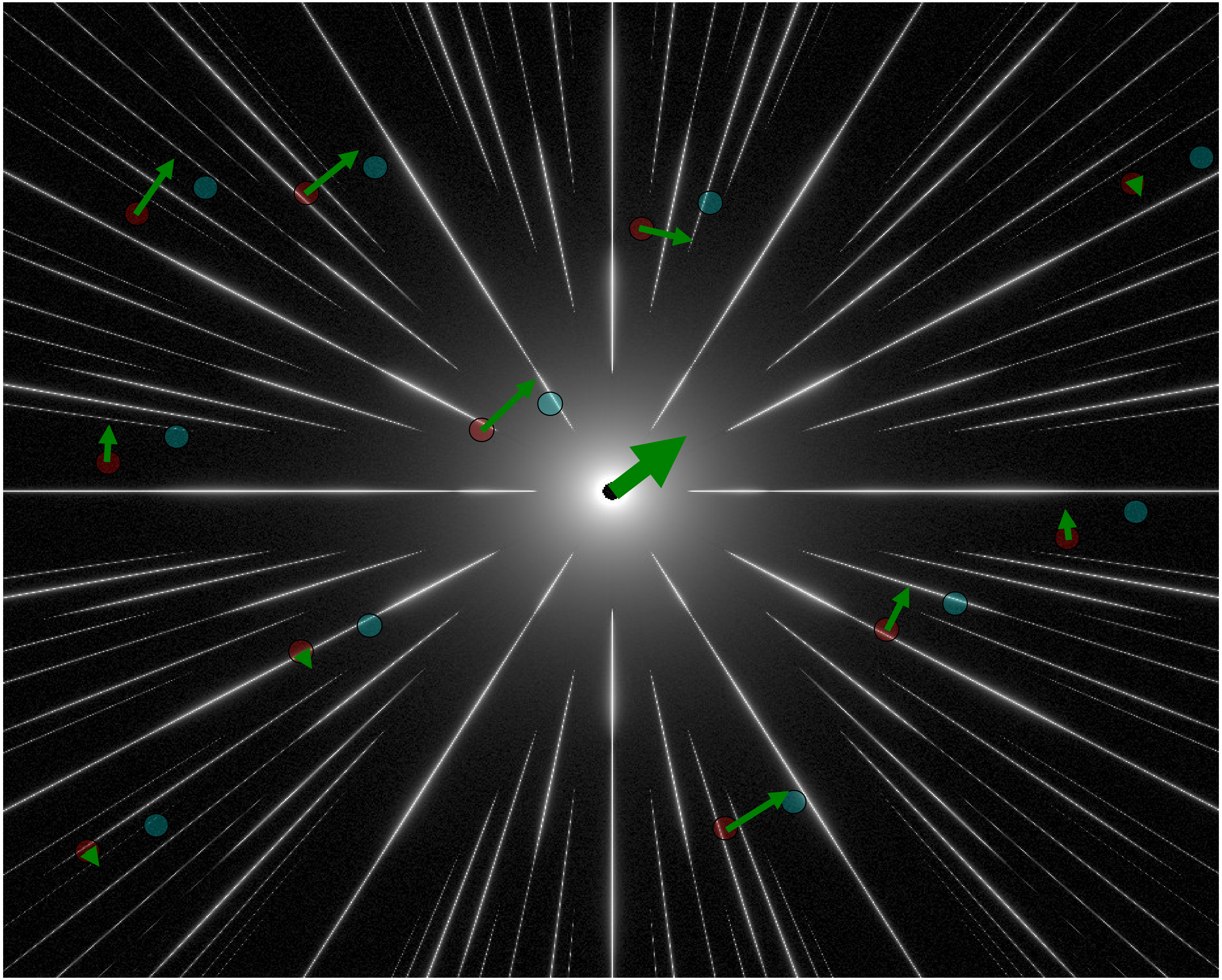


The measured astrometric motion (blue vectors in previous slide) is the sum of the true astrometric signal (green vectors) and the astrometric distortion induced by change in optics and detector between the 2 observations.
Direct comparison of the spike images between the 2 epochs is used to measure this distortion, which is then subtracted from the measurement to produce a calibrated astrometric measurement.





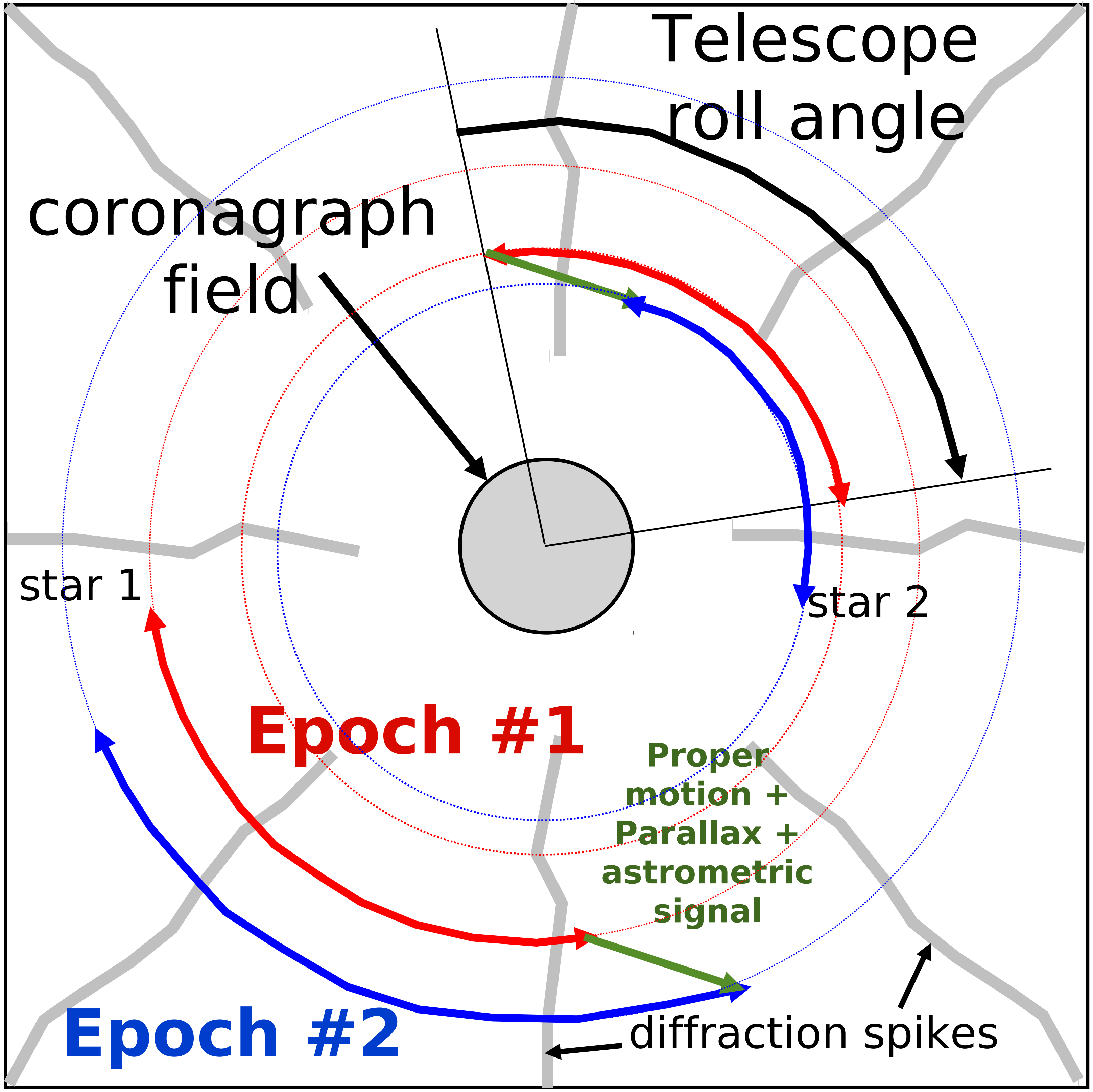
The calibration of astrometric distortions with the spikes is only accurate in the direction perpendicular to the spikes length. For a single background star, the measurement is made along this axis (1-D measurement), as shown by the green vectors. The 2-D measurement is obtained by combining all 1-D measurements (large green vector).



Telescope roll

A slow telescope roll is used to average out small scale distortions, which are due to non-uniformity in the pixel size, (spectral) response, and geometry

The green vector is what should be measured



Simulation input: Assuming optics + detector not specifically designed or calibrated for astrometry

Table 2. System parameters for the baseline design and the numerical simulation

	Baseline design	numerically simulated	Rationale
Telescope diameter	1.4 m		Assumed (cost constrained)
Detector pixel size	44 mas		Nyquist sampled at 600 nm
Field of view (FOV)	0.29 deg^2 (0.6 deg diam)	0.03 deg^2 (0.2 deg diam)	Set to meet astrometric accuracy requirement Optical design allows diffraction-limited imaging over FOV
Single measurement duration	48 hr		Typical single coronagraphic observation duration
Dot coverage on PM (area)	1 %	0.12 %	Keeps throughput loss moderate
Dot size / pitch (μm)	120 / 932 (black dots)	360 / 2800 (gray dots)	dot diameter imposed by FOV
Flat field error, static	1.02 % RMS, 6% peak		Conservative estimate for modern detectors
Flat field error, dynamic	0.1% RMS per pixel spatially uncorrelated		Undetected cosmic ray damage on detector
Detector distortion, static	0.2% of pixel size		
Telescope roll (full range)	± 10 deg	± 28 deg	Manageable PSF elongation at edge of FOV
M2/M3 surface change	40 pm		Level of surface deformation experienced in optical labs
M2/M3 surface error	1.5 nm		Surface error and PSD from existing optics

Target & observation parameters

Table 1. Observation model

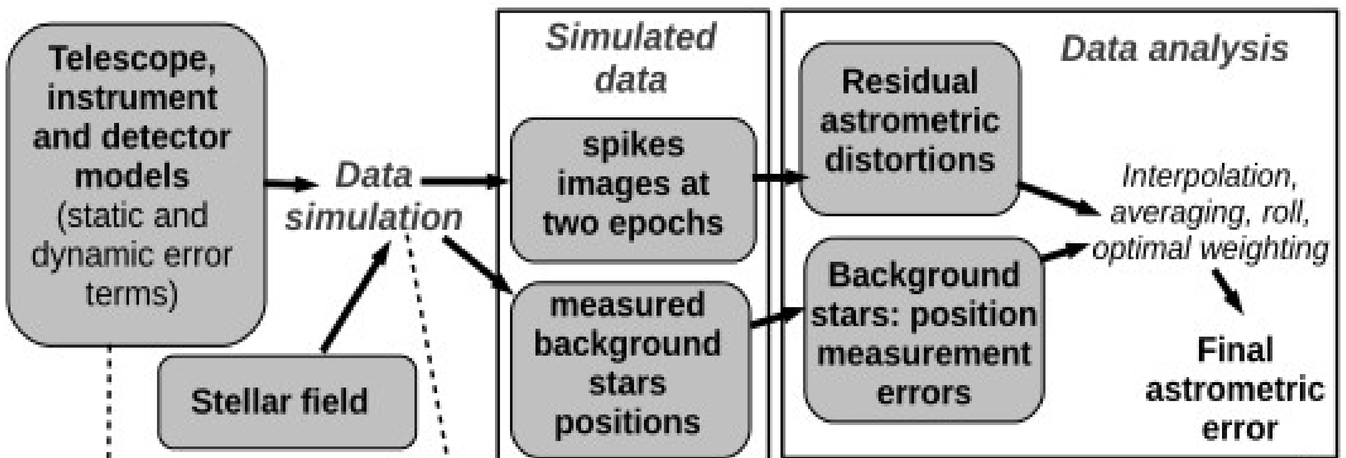
Parameter	value
Planetary system characteristics	
Star	Sun analog
Distance	6 pc
Location	Ecliptic pole
Orbit semi-major axis	1.2 AU
Orbital Period	1.3 yr
Planet Mass	1 Earth Mass
Orbit inclination	1.3 rad
Orbit Eccentricity	0.2
Astrometric signal amplitude	0.5 μas
Orbit apparent semi-major axis	200 mas
Measurements	
Number of observations	32 (regularly spaced every 57 days)
Coronagraphic image: planet position precision	2.5 mas per axis (=3.6 mas in 2D) [a]
Coronagraphic image: inner working angle (IWA)	130 mas
Astrometry: single measurement precision	Variable (driven by science requirement)

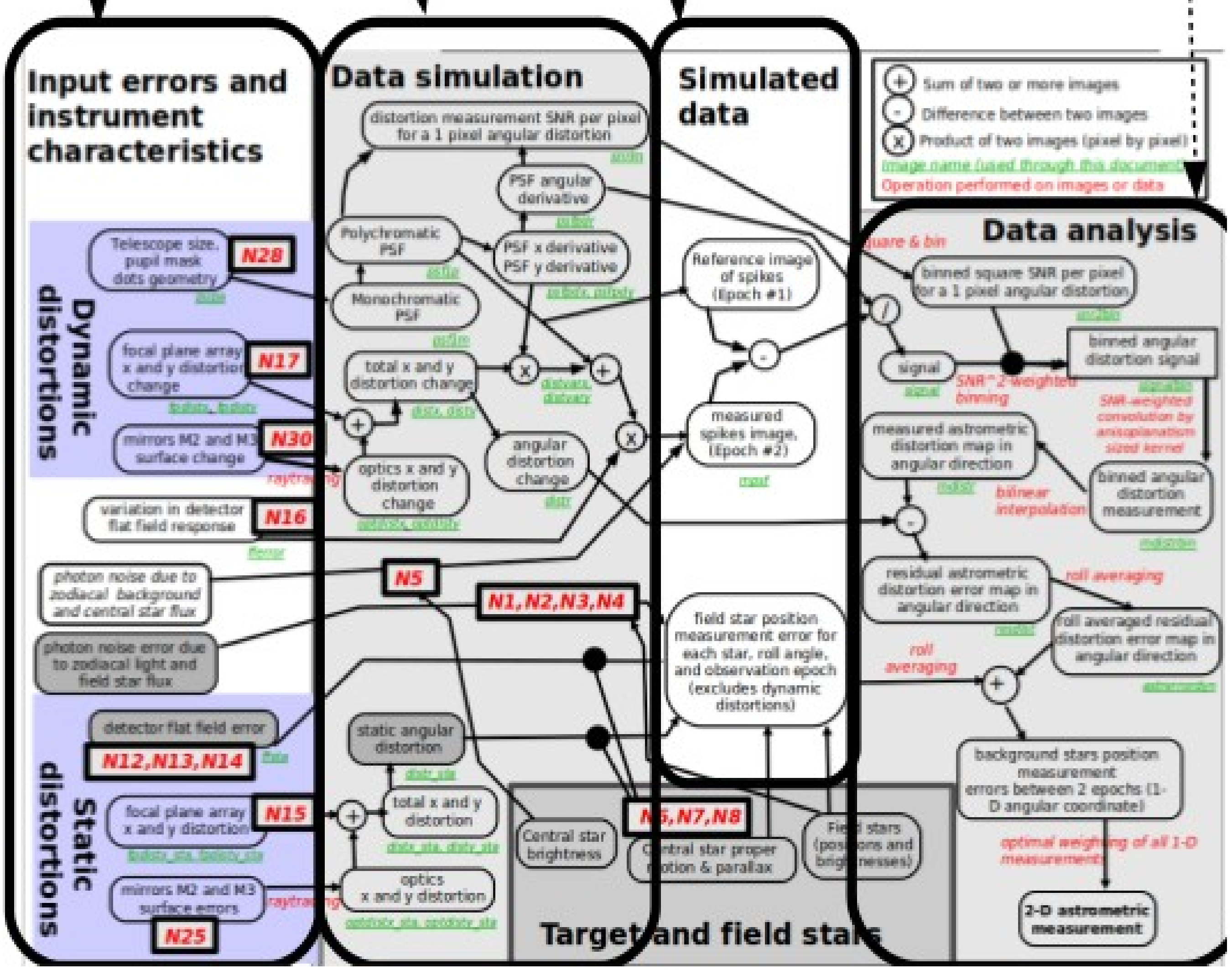
[a]Corresponds to a SNR=10 detection with $\lambda/D = 80mas$ (single axis astrometric precision is theoretically equal to $(\lambda/D)/(\pi\sqrt{Nph})$). For a photon noise limited measurement with no background, this would be achieved with Nph =100 photon at 550nm for a 1.4-m telescope.

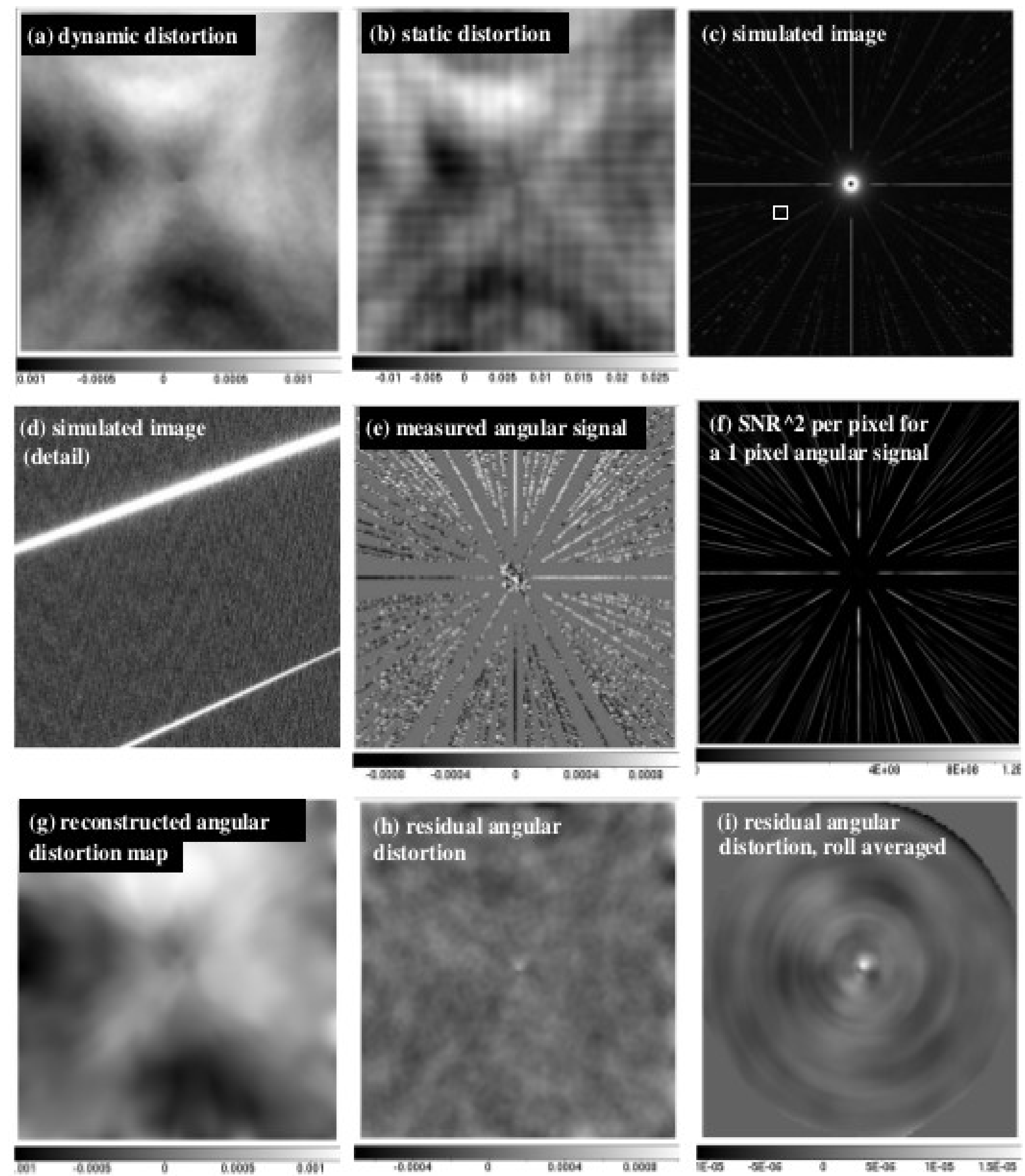
Simulation description

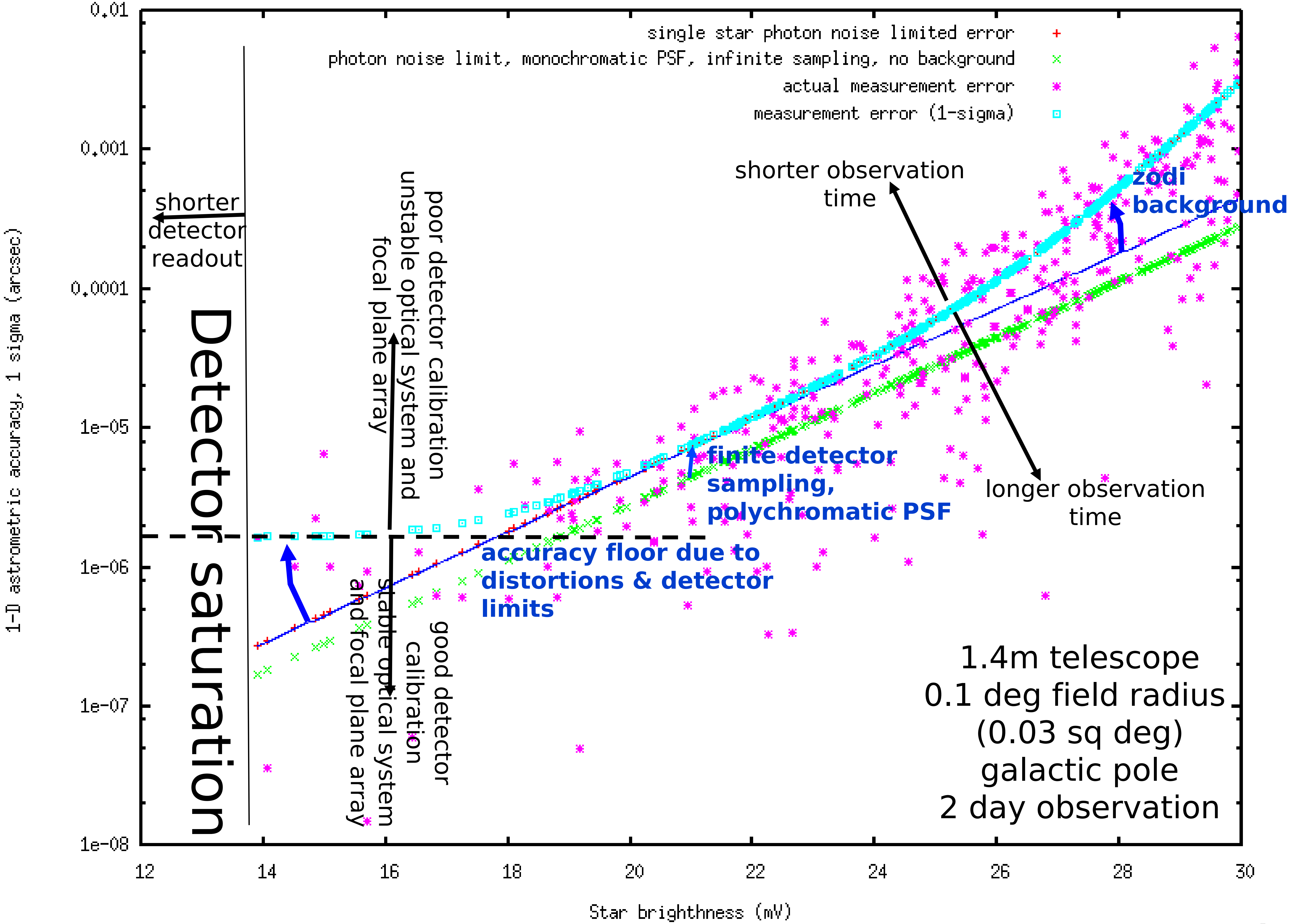
Simulation details available on: www.naoj.org/staff/guyon/

(60 slides describing this chart + C source code)









single star photon noise limited error
photon noise limit, monochromatic PSF, infinite sampling, no background
actual measurement error
measurement error (1-sigma)
shorter detector readout
Detector saturation
poor detector calibration
unstable optical system and focal plane array
good detector calibration
stable optical system and focal plane array
accuracy floor due to distortions & detector limits
finite detector sampling, polychromatic PSF
shorter observation time
longer observation time
zodi background
1.4m telescope
0.1 deg field radius
(0.03 sq deg)
galactic pole
2 day observation
1-D astrometric accuracy, 1 sigma (arcsec)
Star brightness (mV)
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
12
14
16
18
20
22
24
26
28
30

8 % area coverage on PM
$m_V = 3.7$ target
Galactic pole observation
2 day per observation

Larger telescope diameter :
- more light in spikes (D^2), finer spikes (1/D)
→ spike calibration accuracy goes as D^{-2}
- more light in background stars (D^2), and smaller PSF (1/D)
→ position measurement goes as D^{-2}

Astrometric accuracy goes as D^{-2} $FOV^{-0.5}$
Number of pixels goes as D^{-2} FOV
At fixed number of pixels, larger D is better

But: mean surface brightness of spikes gets fainter as FOV increases

Table 3. Error budget for 0.29 sq deg field, galactic pole

Error Term	Value	Notes
photon noise		
Field stars centroid error	0.128 μas	N1+N2+N3+N4
Photon noise on spikes	0.048 μas	N5
static calibration errors		
Detector flat field	0.033 μas	N12+N13+N14
Optical distortion	0.083 μas	N6+N7+N8, N25
Detector distortion	0.015 μas	N15, N25
dynamic calibration errors		
Detector flat field	0.029 μas	N16
Optical distortion	0.063 μas	N30
Detector geometry	0.076 μas	N17
Total	0.200 μas	

	FOV = 0.03 sq deg	FOV = 0.1 sq deg	FOV = 0.25 sq deg	FOV = 0.5 sq deg	FOV = 1.0 sq deg
D = 1.4 m	0.58 μas	0.31 μas	0.20 μas	0.14 μas	0.11 μas
D = 2.0 m	0.28 μas	0.15 μas	0.10 μas	0.07 μas	0.05 μas
D = 3.0 m	0.13 μas	0.067 μas	0.044 μas	0.030 μas	0.024 μas
D = 4.0 m	0.071 μas	0.038 μas	0.025 μas	0.017 μas	0.013 μas

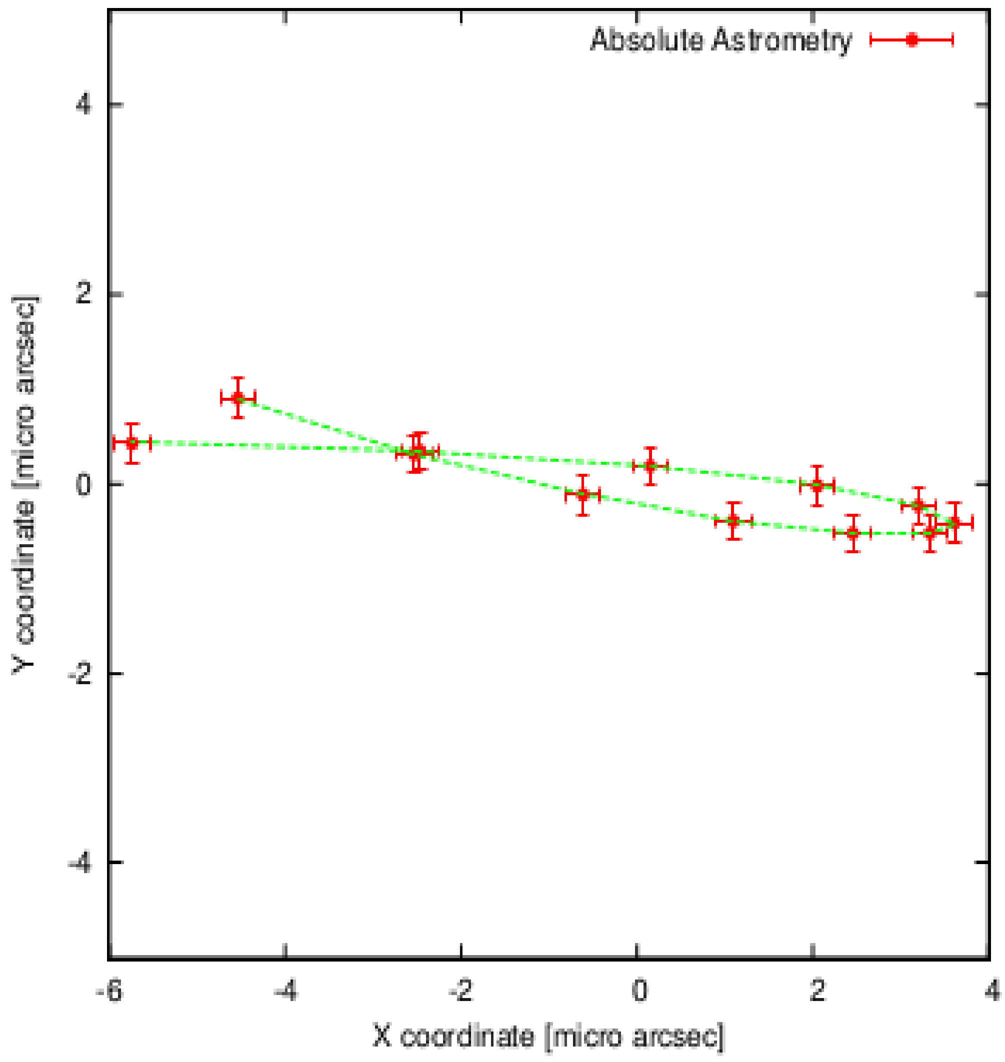
D = 4.0m, FOV = 0.1 sq deg → 0.2 uas in <2hr

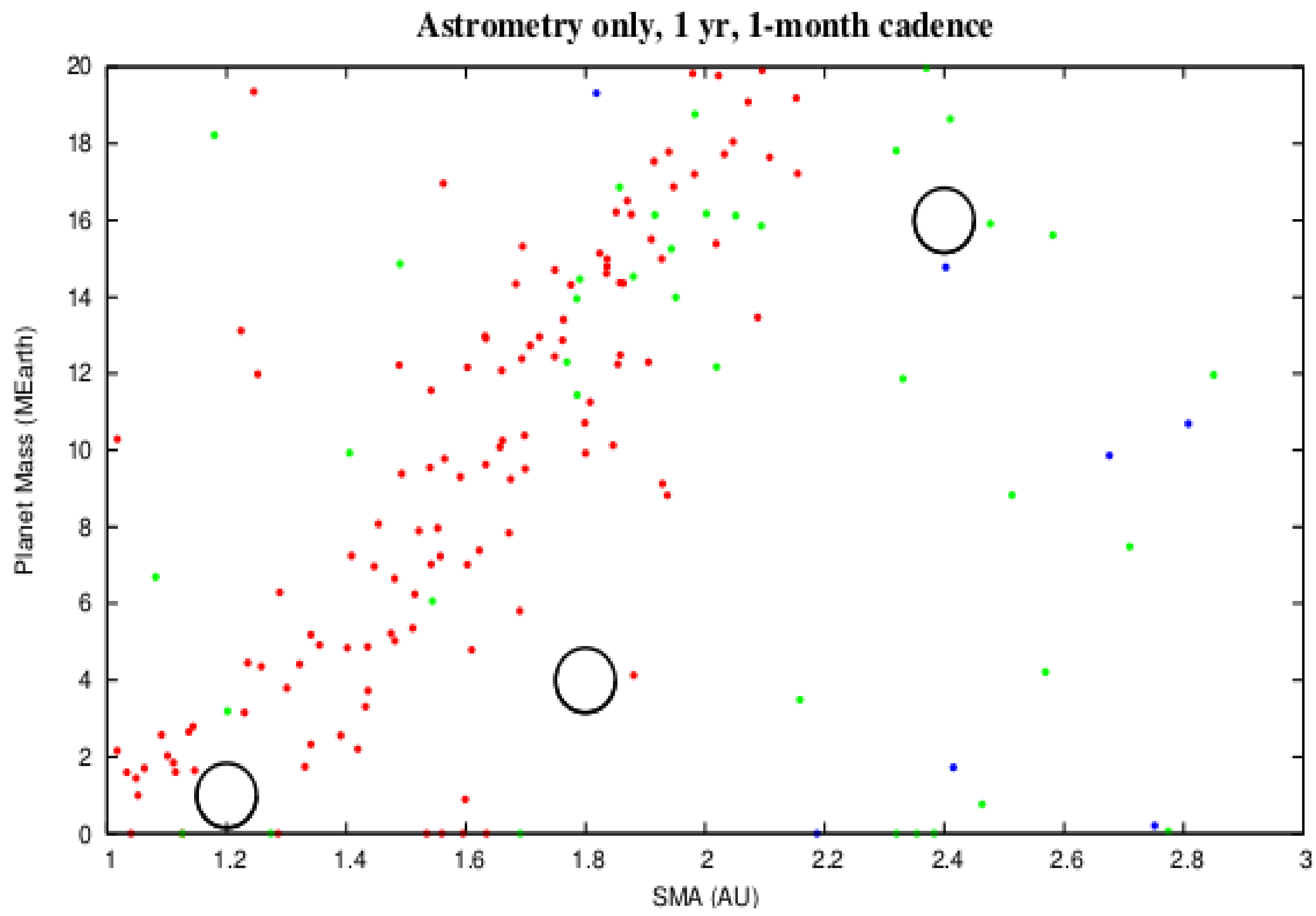
Astrometry + coronagraphy:
Very powerful combination for characterizing exoplanets

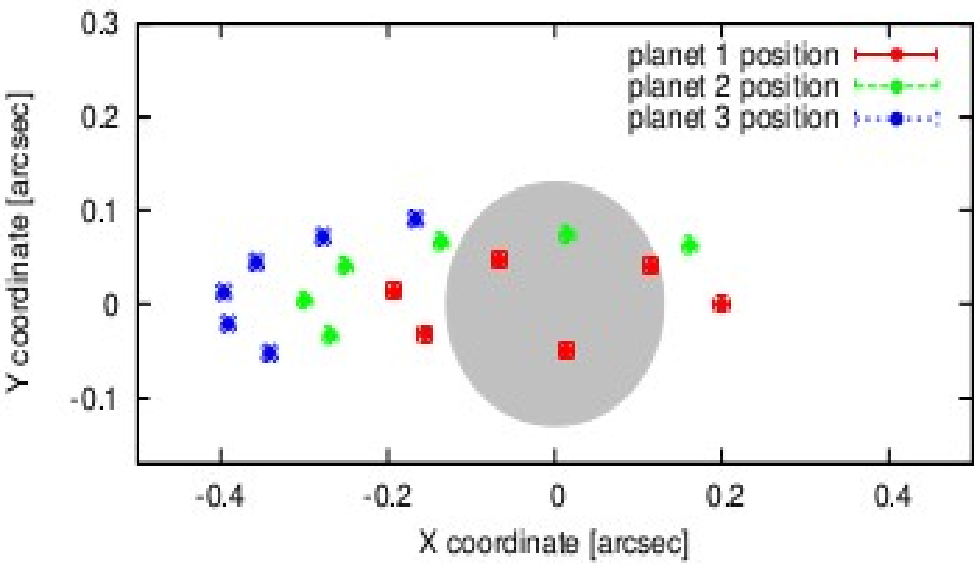
Asstrometry required for mass estimation
Coronagraphy → tight constraints on orbit parameters

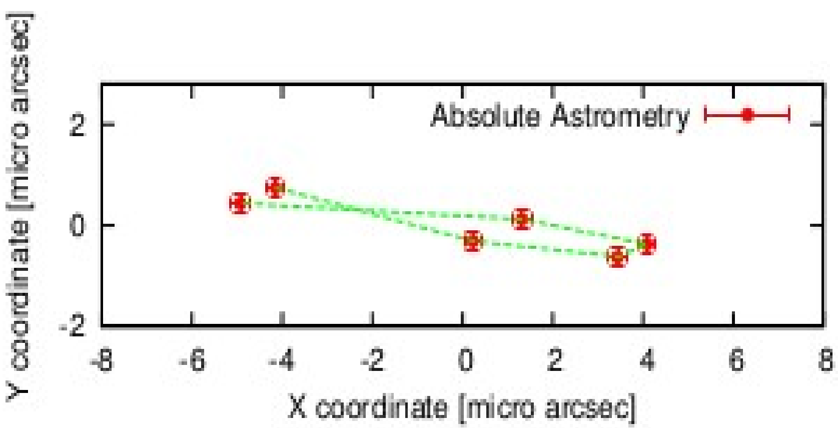
Table 3. Multiple planets system characteristics

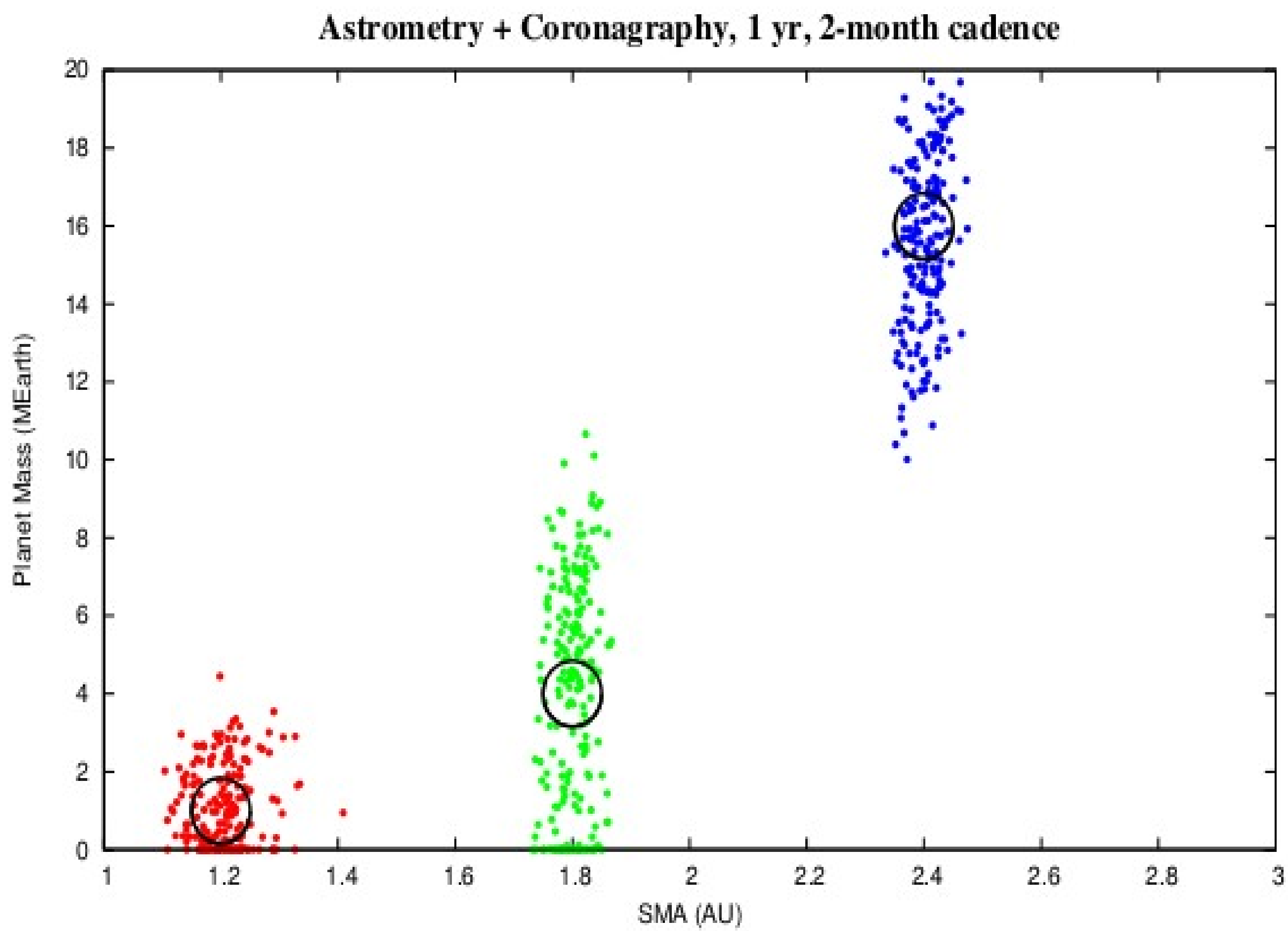
	Mass (M_{Earth})	SMA (A.U.)	Period (yr)	inclin. (sin(i))
Planet 1	1.0	1.2	1.31	0.25
Planet 2	4.0	1.8	2.41	0.25
Planet 3	16.0	2.4	3.72	0.25

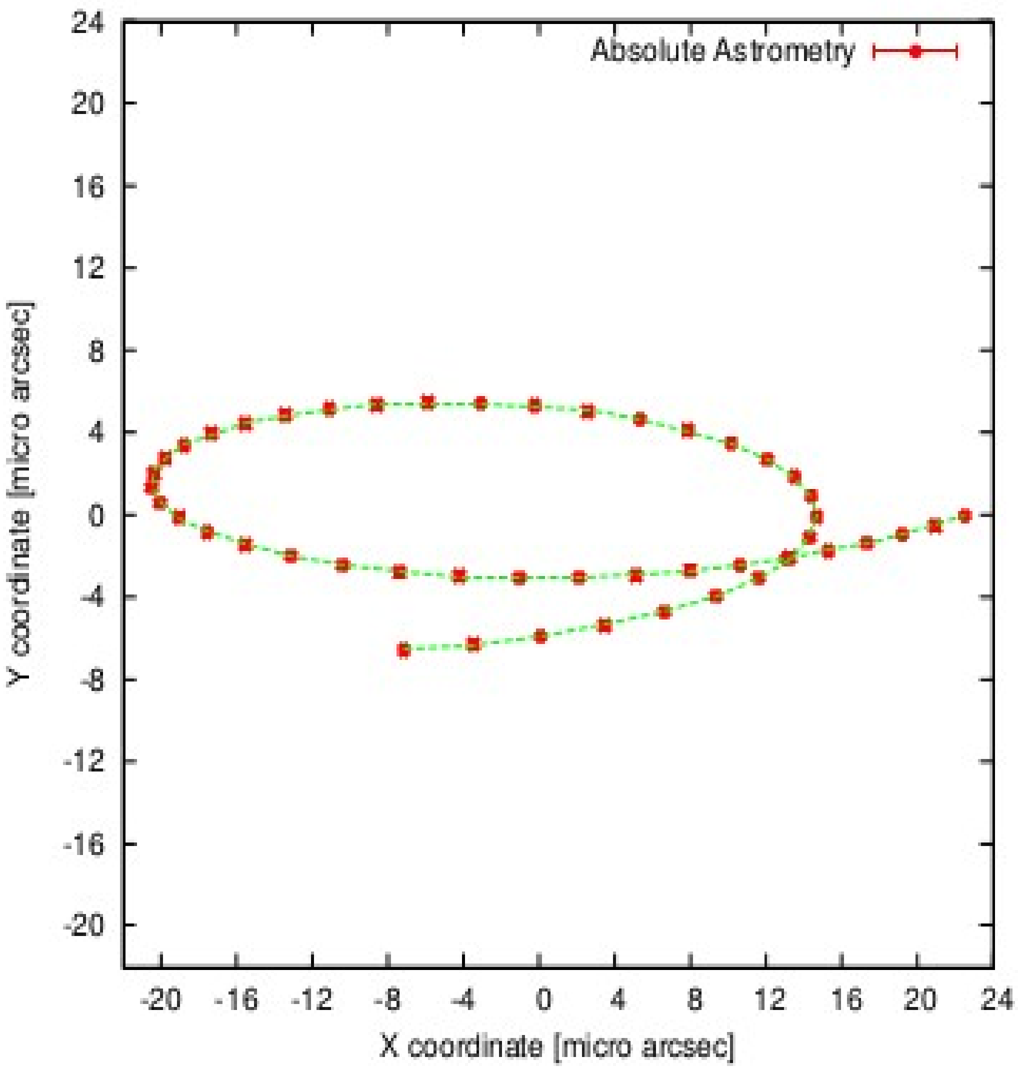

Absolute Astrometry
Y coordinate [micro arcsec]
X coordinate [micro arcsec]

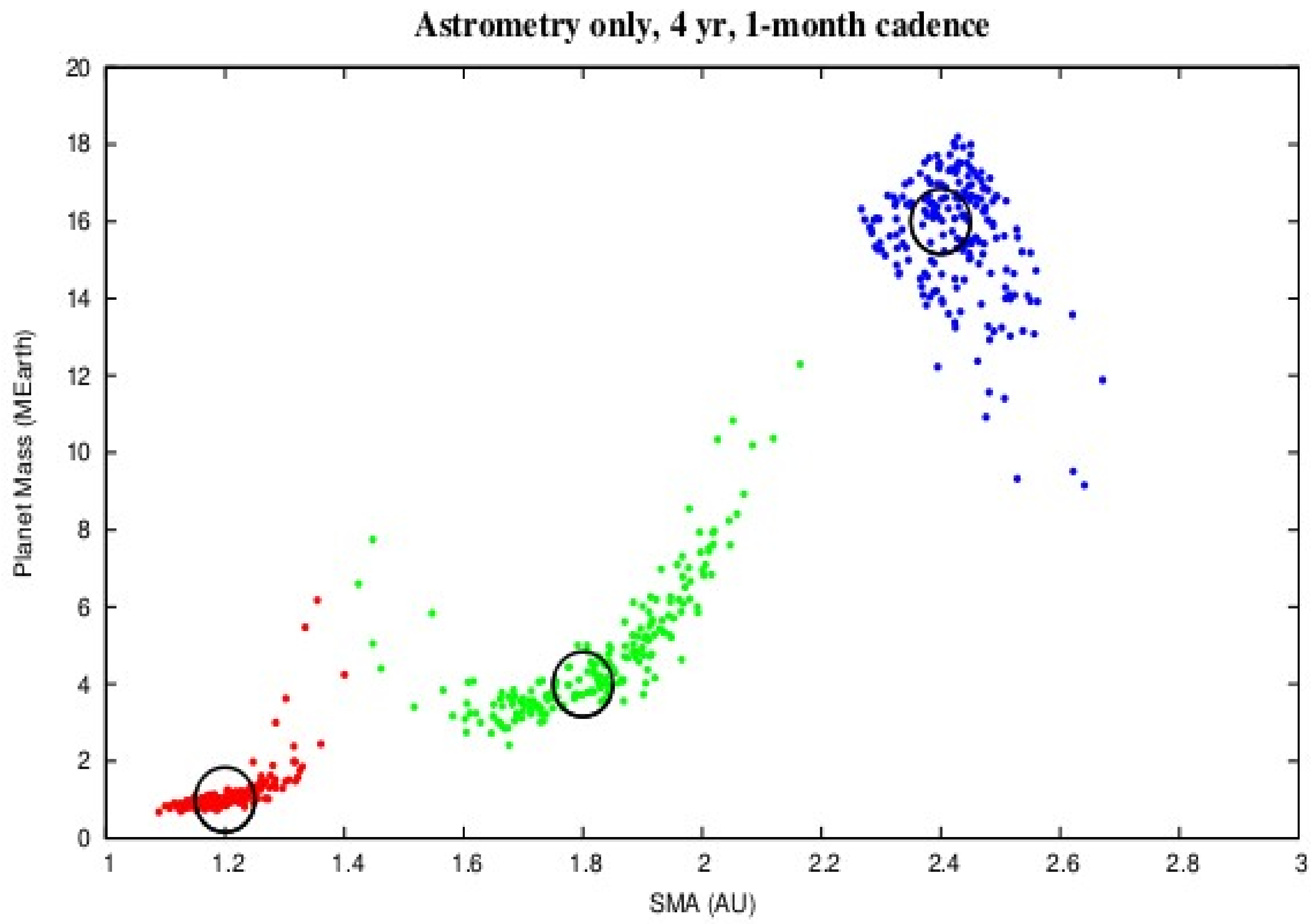

Astrometry only, 1 yr, 1-month cadence
Planet Mass (MEarth)
SMA (AU)

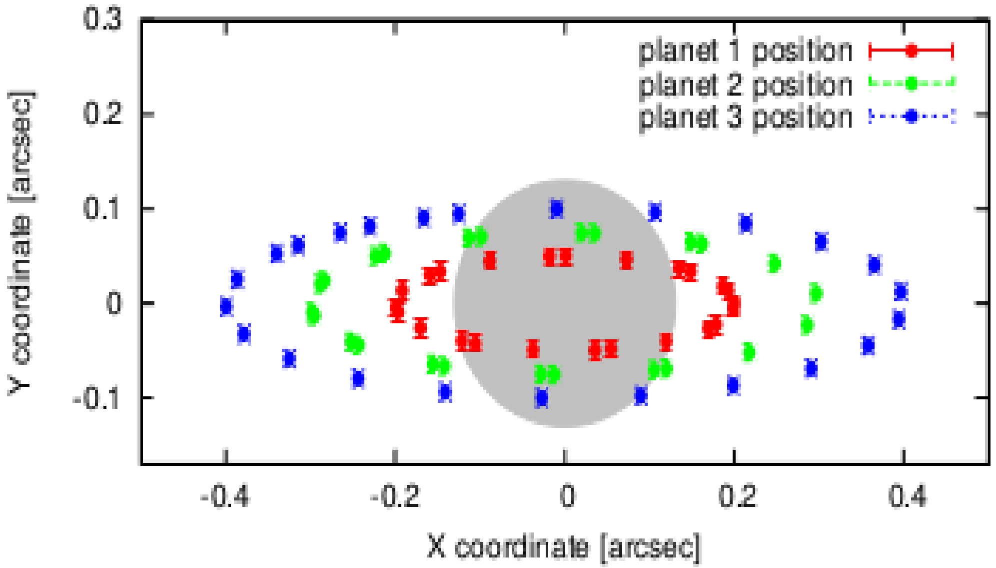

planet 1 position
planet 2 position
planet 3 position
Y coordinate [arcsec]
X coordinate [arcsec]
0.3
0.2
0.1
0
-0.1
-0.4
-0.2
0
0.2
0.4

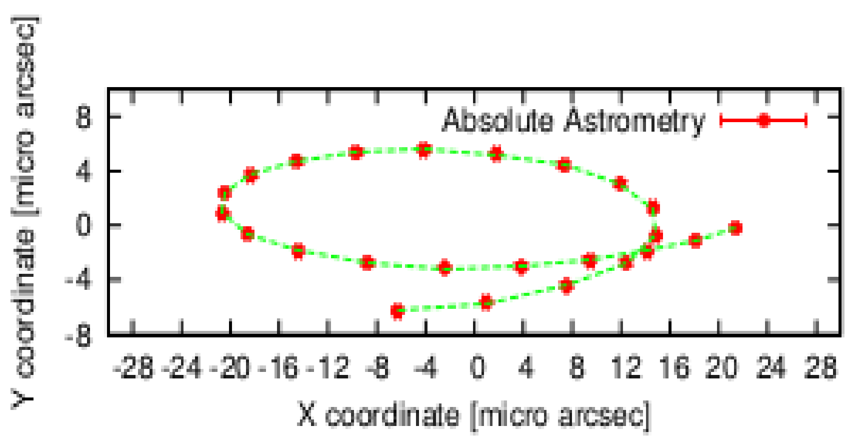

Absolute Astrometry
Y coordinate [micro arcsec]
X coordinate [micro arcsec]
2
0
-2
-8
-6
-4
-2
0
2
4
6
8

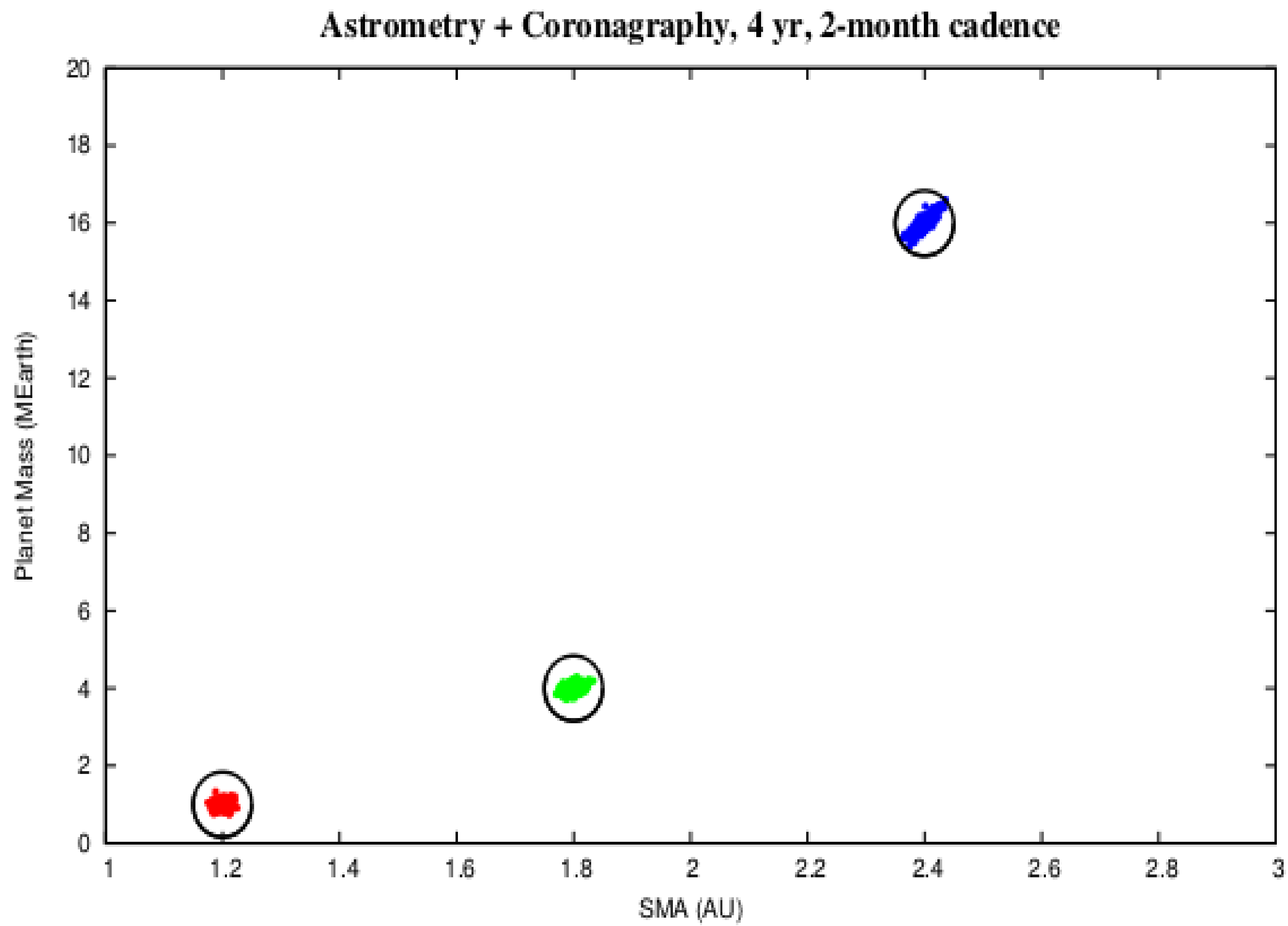

Astrometry + Coronagraphy, 1 yr, 2-month cadence
Planet Mass (MEarth)
SMA (AU)
20
18
16
14
12
10
8
6
4
2
0
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3


Absolute Astrometry
Y coordinate [micro arcsec]
X coordinate [micro arcsec]

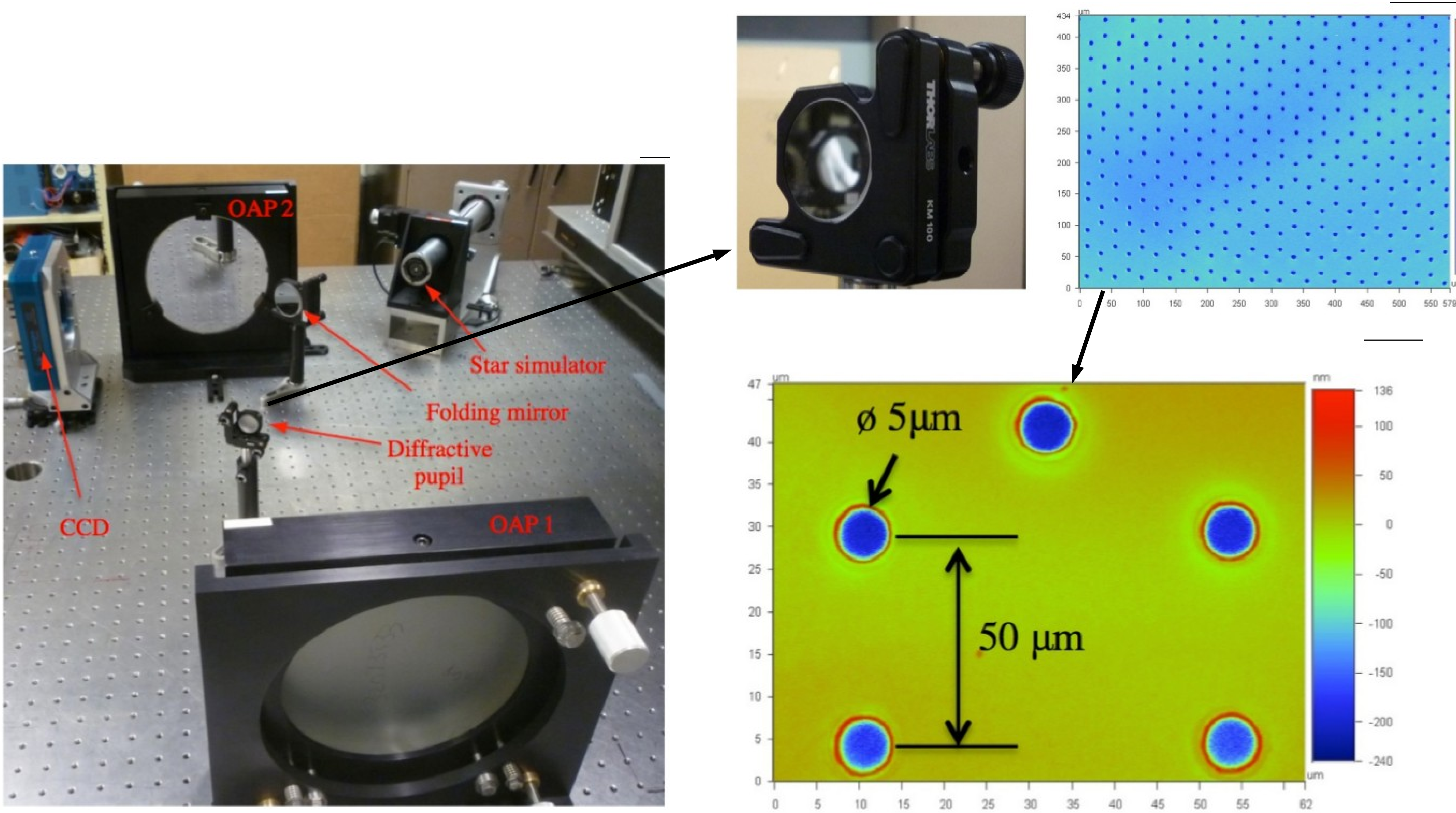

Astrometry only, 4 yr, 1-month cadence
Planet Mass (MEarth)
SMA (AU)

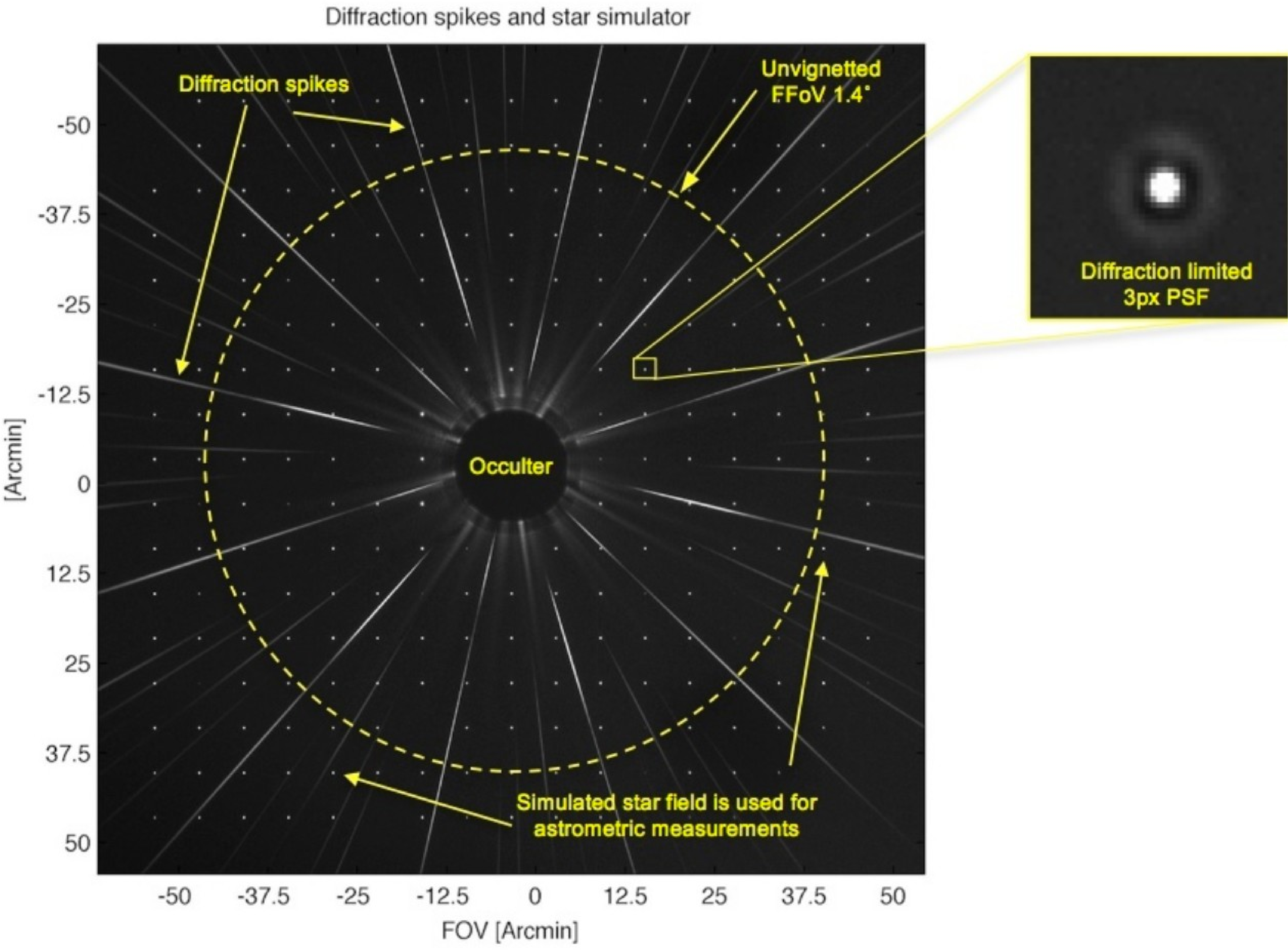

planet 1 position
planet 2 position
planet 3 position
Y coordinate [arcsec]
X coordinate [arcsec]


Absolute Astrometry
Y coordinate [micro arcsec]
X coordinate [micro arcsec]

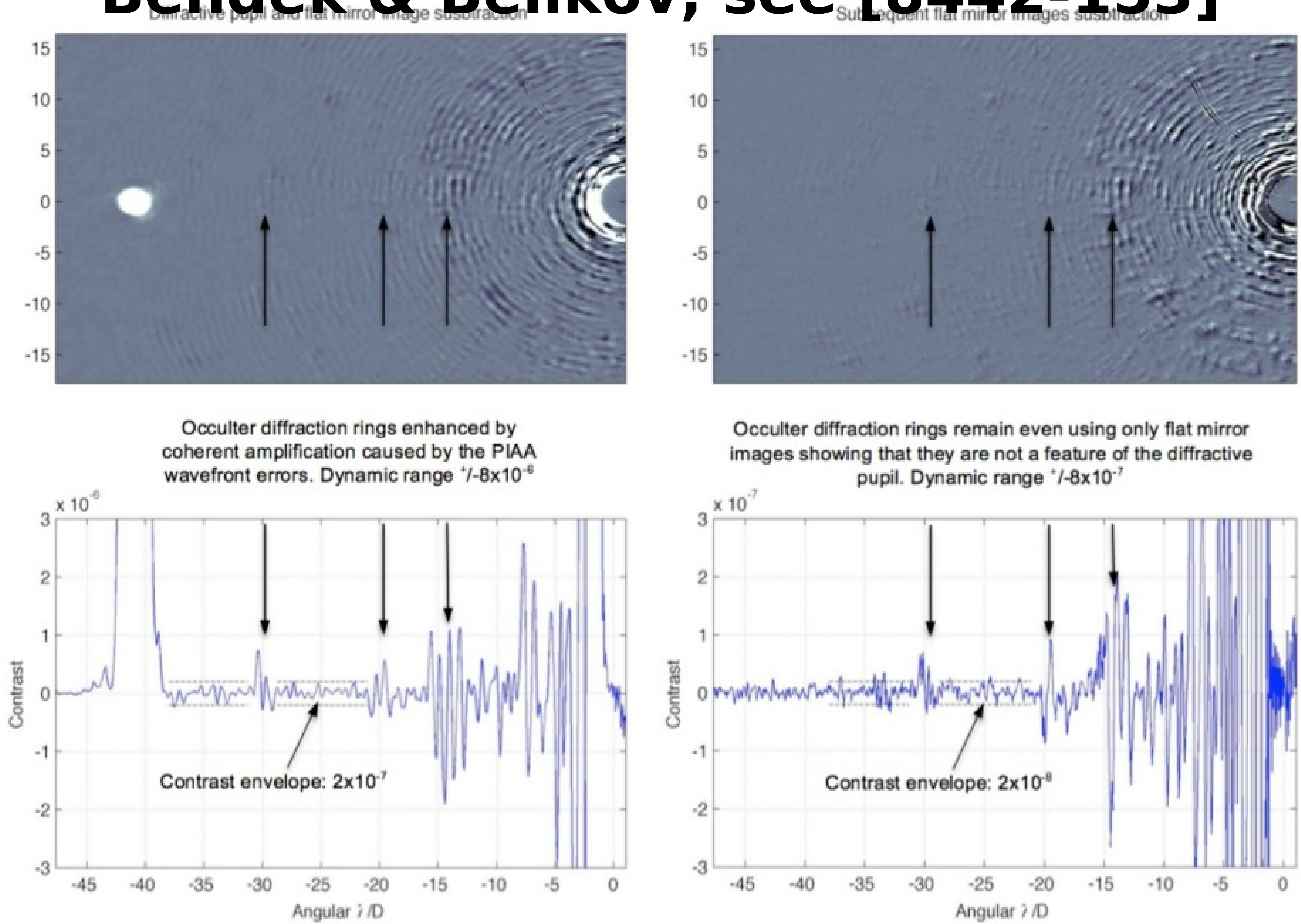

Astrometry + Coronagraphy, 4 yr, 2-month cadence
Planet Mass (MEarth)
SMA (AU)

Astrometry testbed at UofA
E. Bendek et al. [8442-153]




Diffraction spikes and star simulator
Diffraction spikes
Unvignetted
FFoV 1.4˚
Occulter
Diffraction limited
3px PSF
Simulated star field is used for
astrometric measurements
[Arcmin]
FOV [Arcmin]
-50
-37.5
-25
-12.5
0
12.5
25
37.5
50

Diffractive pupil + coronagraph test at NASA Ames
Bendek & Belikov, see [8442-153]



Conclusions & thoughts

Wide field imaging, coronagraphy and astrometry could be done on the same telescope, simultaneously and without significant performance loss … and without astrometry driving telescope requirements (other than putting dots on PM)

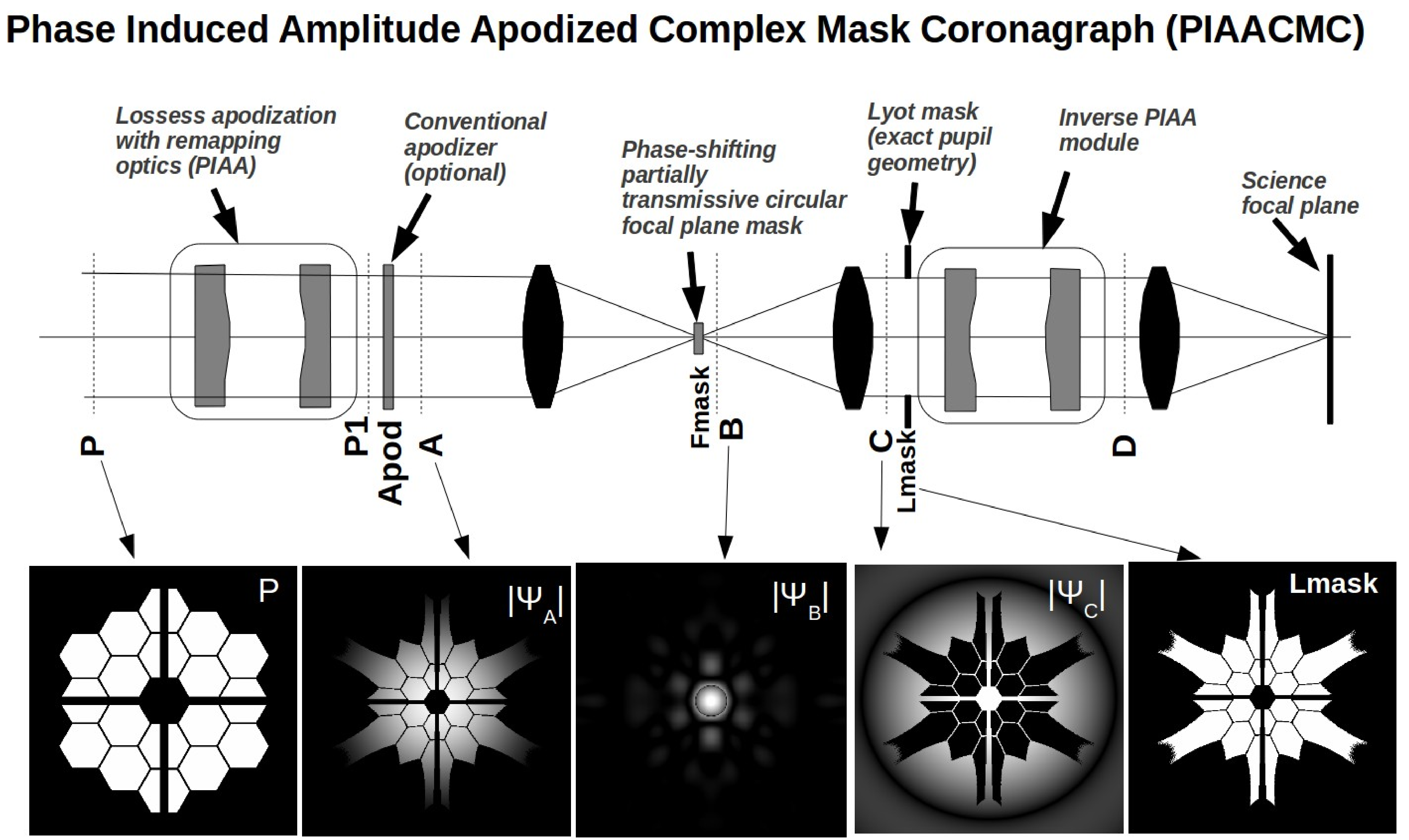
Simultaneous astrometry + coronagraphy is extremely powerful: improved detection, mass measurement

We should work hard to find a way to do exoplanet characterization on next large(ish) optical space telescope.
~Billion $ level mission for exoplanet imaging or astrometry is a hard sell if it only does exoplanets:
1B$ = <2-m telescope → very hard to do Earths
Detecting and characterizing giant planets for 1B$ in ~2030 is not compelling (competition from ground-based, transits, RV …)
Astrometry: we risk repeat of SIM history

The only credible option for characterization of Earth-like habitable planets is with ~2m telescope or larger → can be affordable if shared with other goals (wide field imaging)

Conclusions & thoughts

NOTE:
High contrast imaging may not require off-axis telescope
Example: PIAACMC offers 1e-10 contrast with 0.7 lambda/D IWA and 100% throughput



Conclusions & thoughts

Future work:
Lab demo at UofA → NASA Ames (see poster by E. Bendek)
More detailed mission concept study
Ground-based use (See talk by S. Mark Ammons in AO session 8447-25)

More details on 3 upcoming refereed papers under review:

- Principle, data reduction, error budget (Guyon et al. 2012)

- Scientific performance for exoplanet detection and characterization (Guyon et al. 2012)

- Impact on coronagraphic performance (Bendek et al. 2012)

+ PIAACMC paper (Guyon et al. 2012)

Backup slides

Combined solution derived from simultaneous coronagraphy and astrometry measurements

Known variables:

- **Star location** on the sky (effect of parallax is known except for star distance, aberration of light perfectly known)
- **observing epochs**
- **Stellar mass** (assumed to be known at the 5% accuracy level)
- **measurement noise levels** for astrometry (~µas), coronagraphy planet position (few mas) and star mass (~5%)

Measurements

Astrometry:
star position
(nb of variables = 2x #observations)

Coronagraphy:
planet position
(nb of variables = 2x #observations)

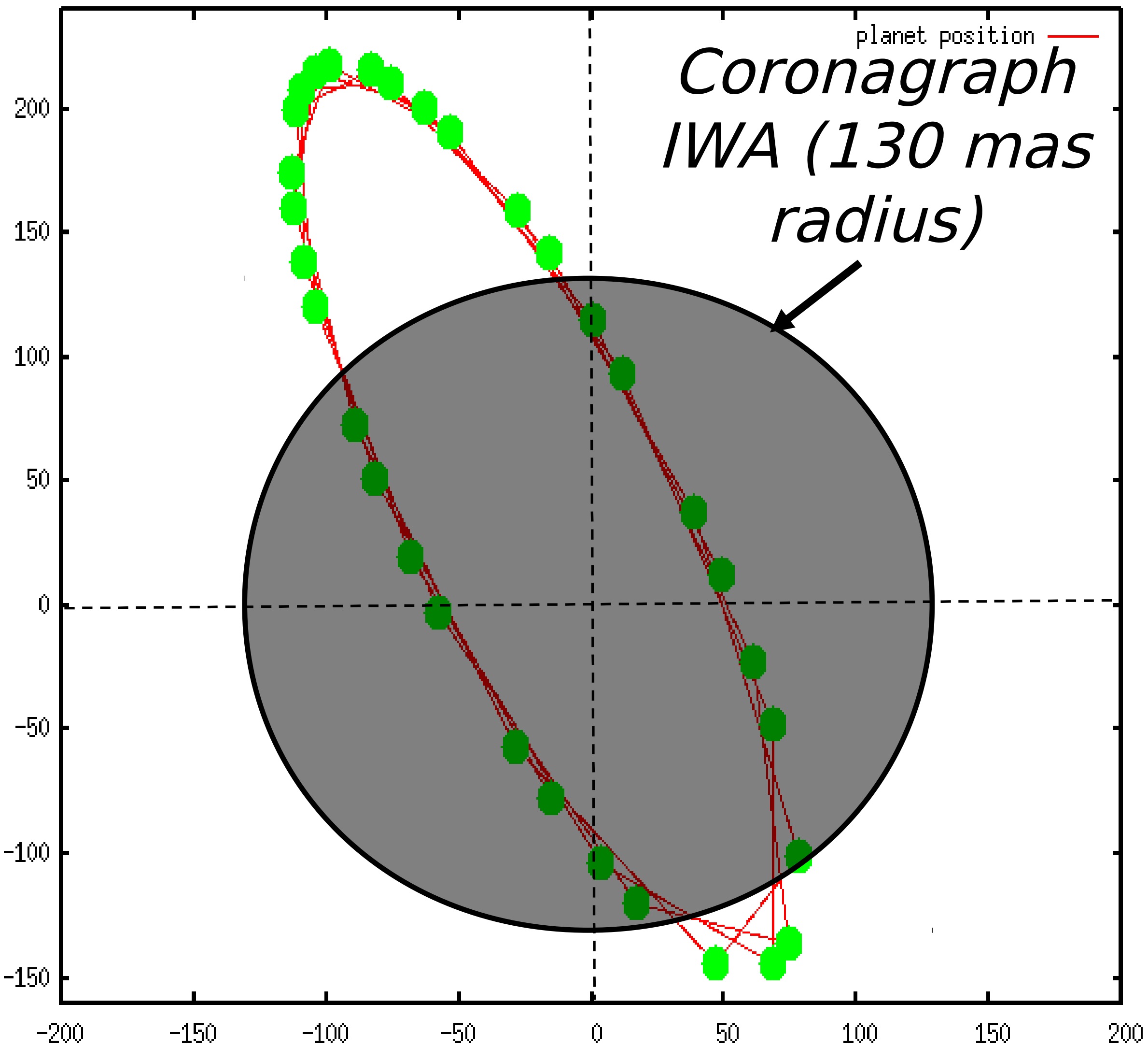
Solution

Maximum likelihood solution for 11 free parameters to be solved for:

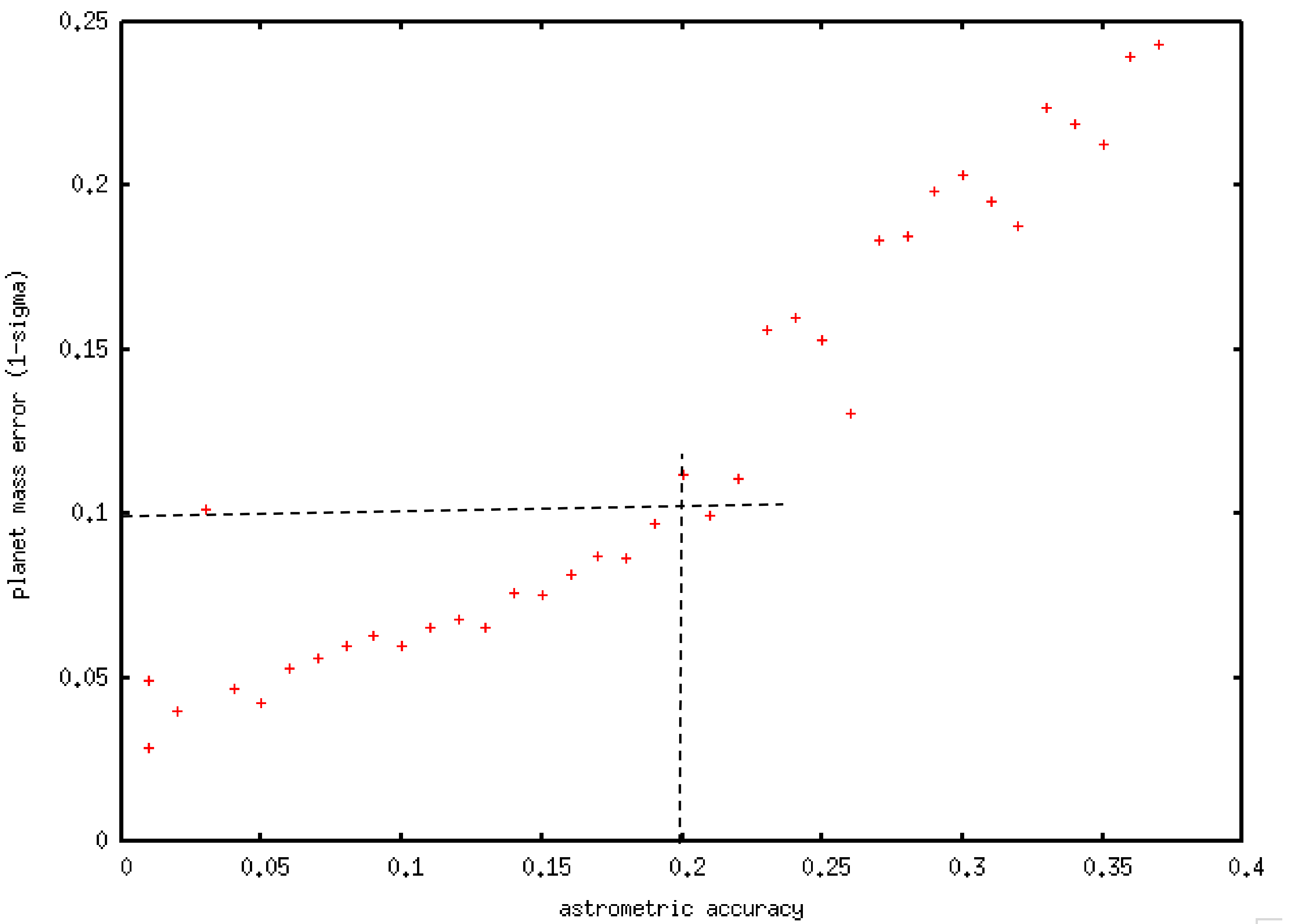
- star parallax (1 variable)
- proper motion (2 variables)
- star mass (1 variable)
- planet mass (1 variable)
- orbital parameters (6 variables)

Combined solution for simultaneous coronagraphy + astrometry

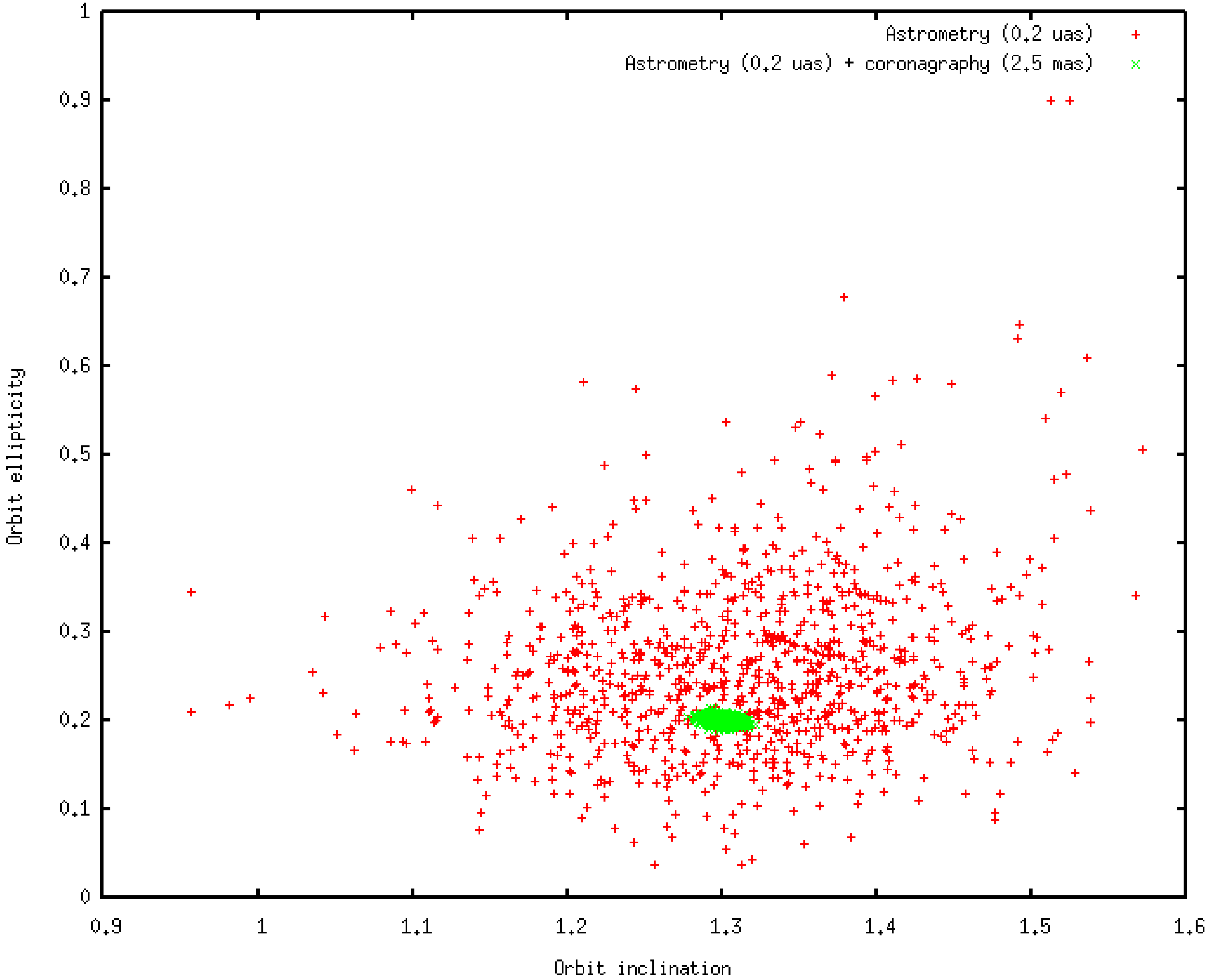
Planet on a 1.2 AU orbit (1.3 yr period), e=0.2
orbit orientation on sky: planet outside the coronagraph IWA for 17 out of the 32 observations.



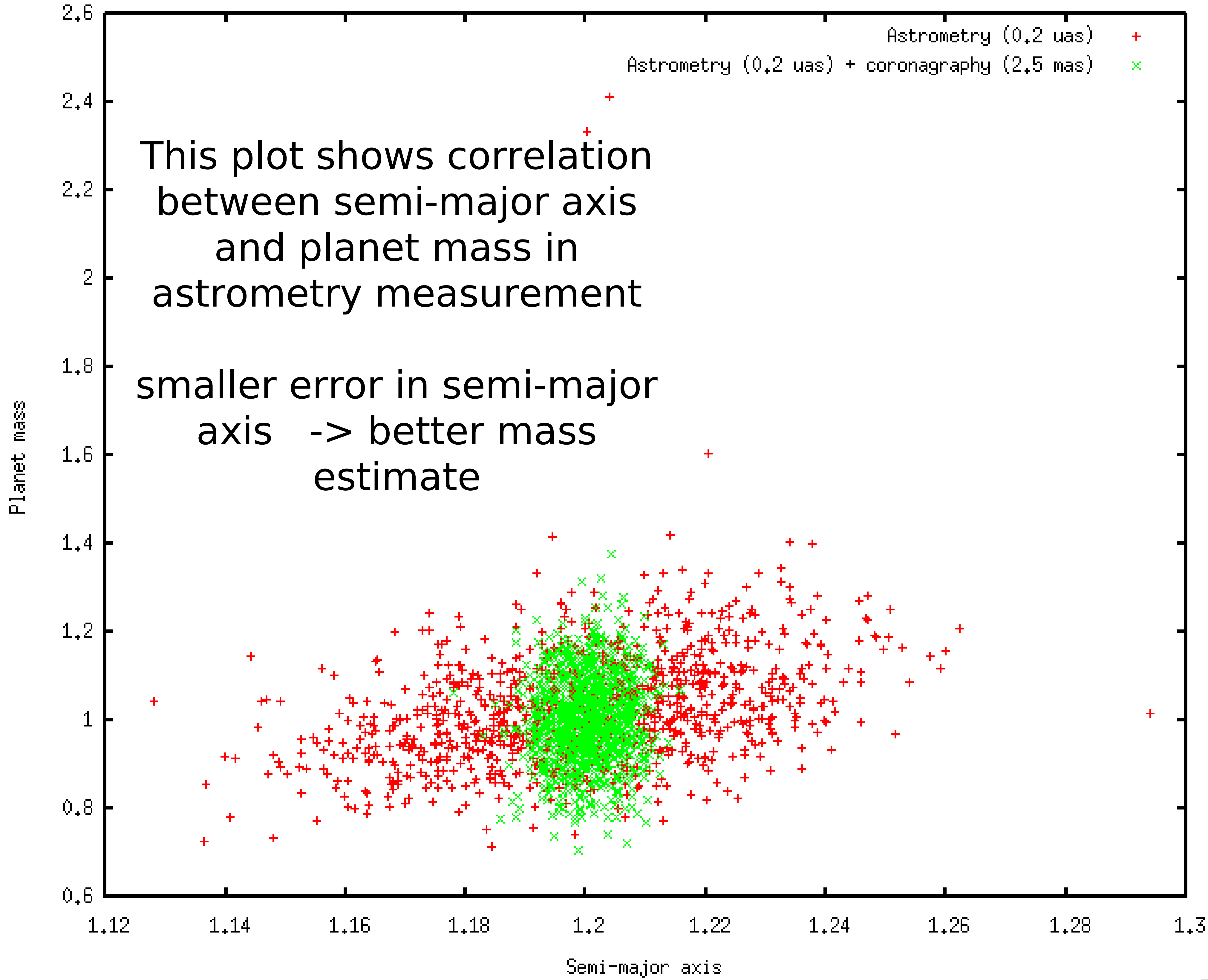
Required single measurement astrometric accuracy = 0.2 µas (1-sigma, 1D)



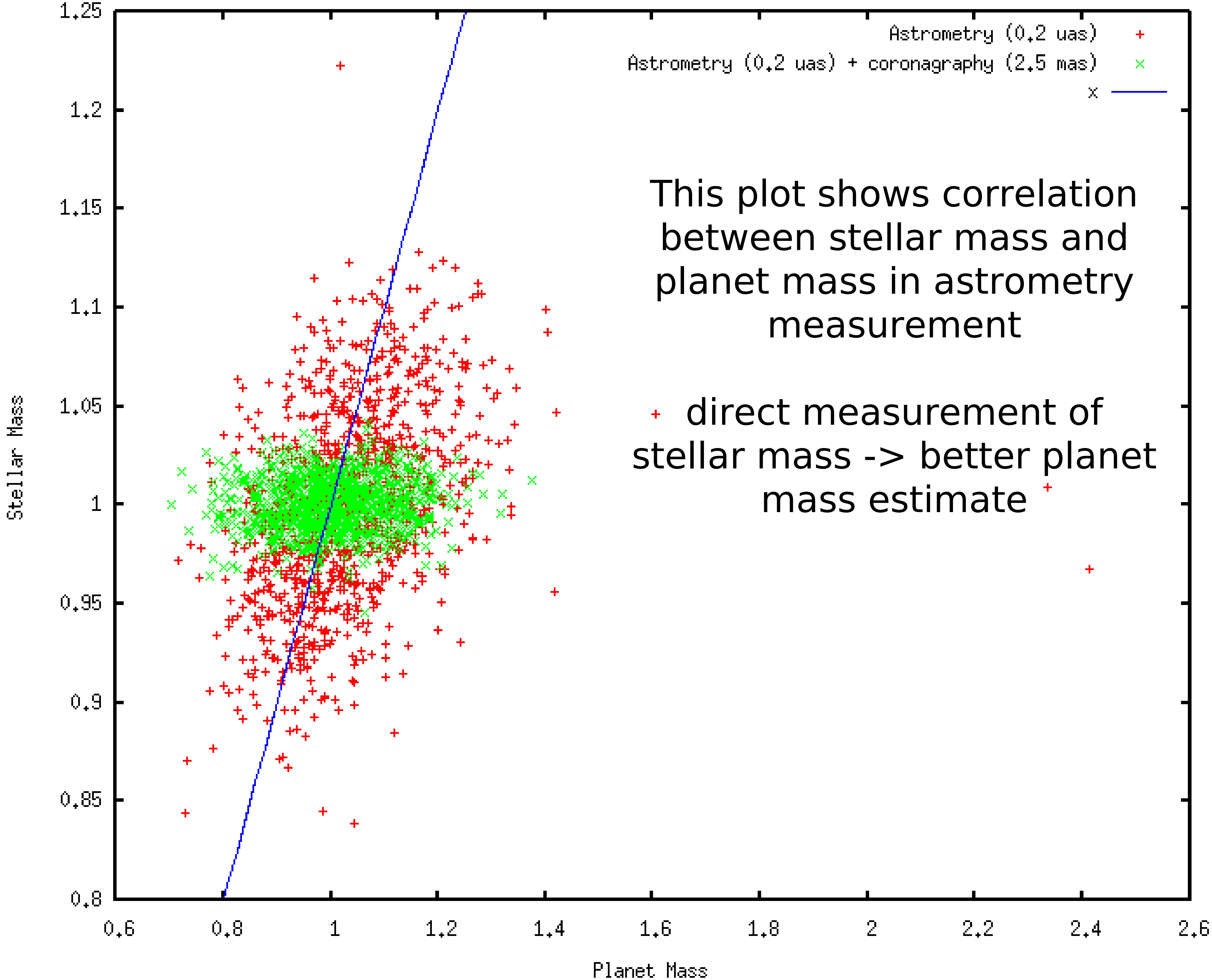
Combined solution for simultaneous coronagraphy + astrometry is very accurate for orbital parameters measurement



Better estimate of orbital parameters -> better planet mass estimate



Better estimate of stellar mass -> better planet mass estimate



Combined solution for simultaneous coronagraphy + astrometry

	Standard deviation	
	Astrometry only	**Astrometry + coronagraphy**
parallax	0.037 µas	0.035 µas
x proper motion	0.017 µas/yr	0.012 µas/yr
y proper motion	0.020 µas/yr	0.013 µas/yr
Planet mass	0.132 ME	0.098 ME
Semi-major axis	0.0228 AU	0.0052 AU
orbital phase	0.653 rad	0.039 rad
orbit inclination	0.0968 rad	0.0065 rad
sma projected PA on sky	0.1110 rad	0.0040 rad
orbit ellipticity	0.098	0.0035

~10x better estimate on orbital parameters

Direct stellar mass measurement

Benefits of simultaneous coronagraphy + astrometry

Solving for planet orbit and mass using the combined astrometry + coronagraphy measurements is scientifically very powerful:

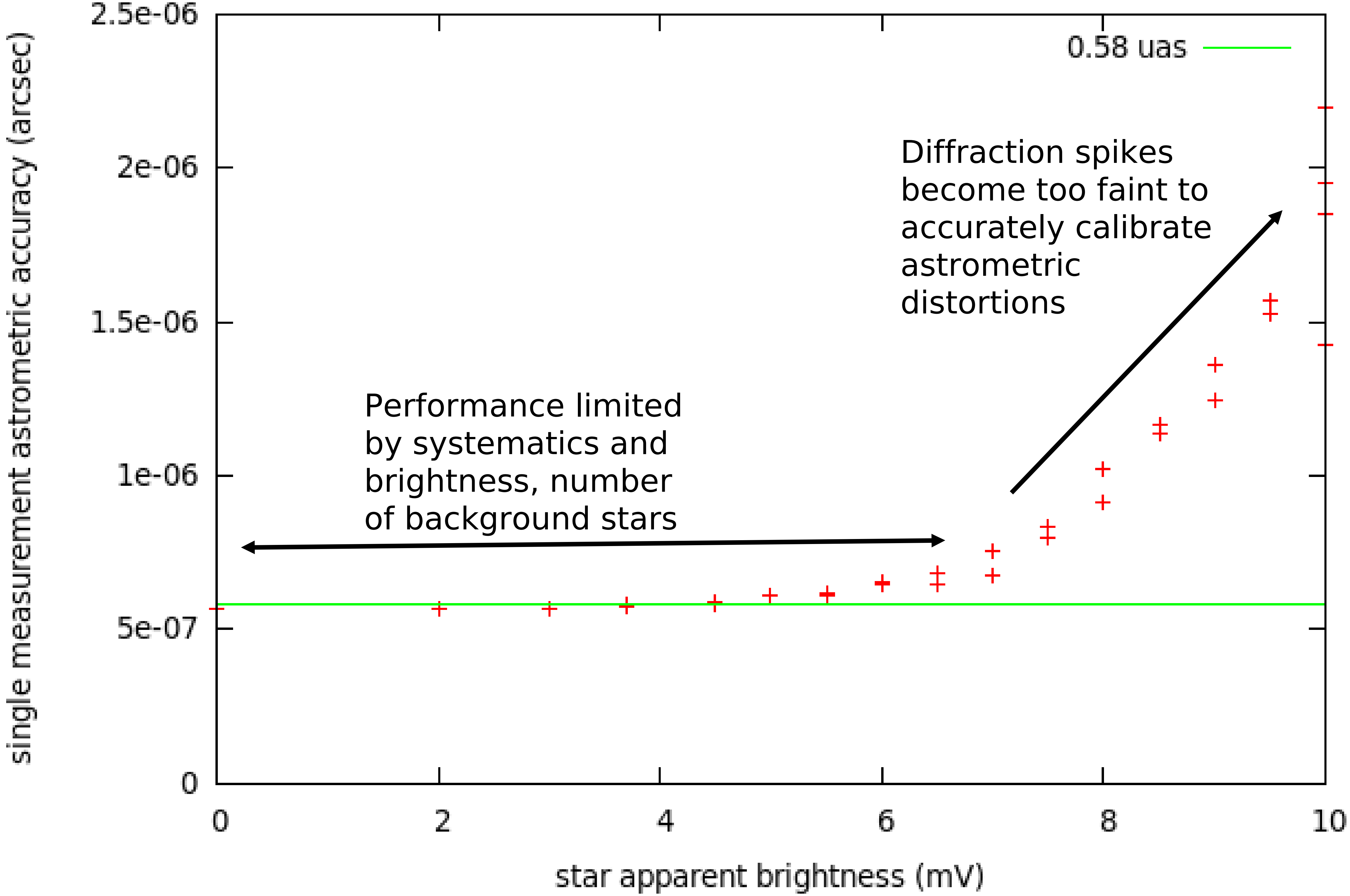
- **Reduces confusion with multiple planets**. Outer massive planets (curve in the astrometric measurement) will be seen by the coronagraph.

- Astrometry will **separate planets from exozodi clumps**.

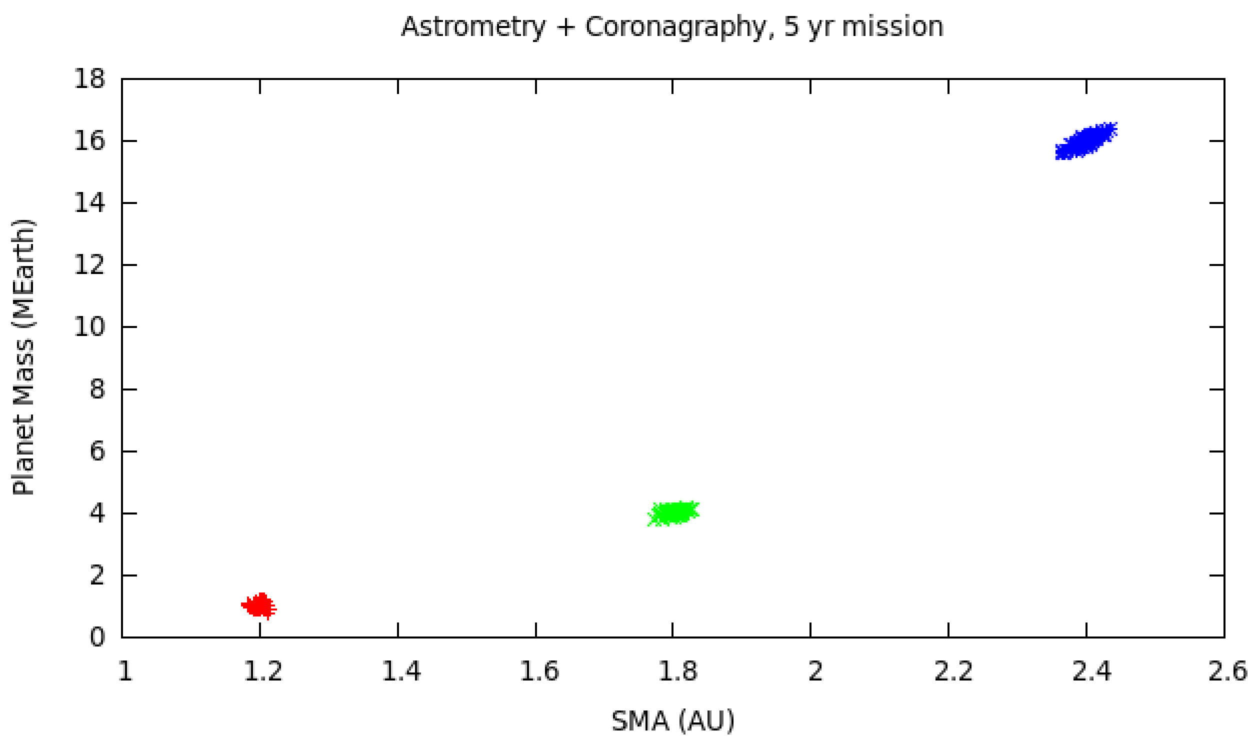
- Astrometric knowledge allows to **extract fainter planets from the images, especially close to IWA**, where the coronagraph detections are marginal.

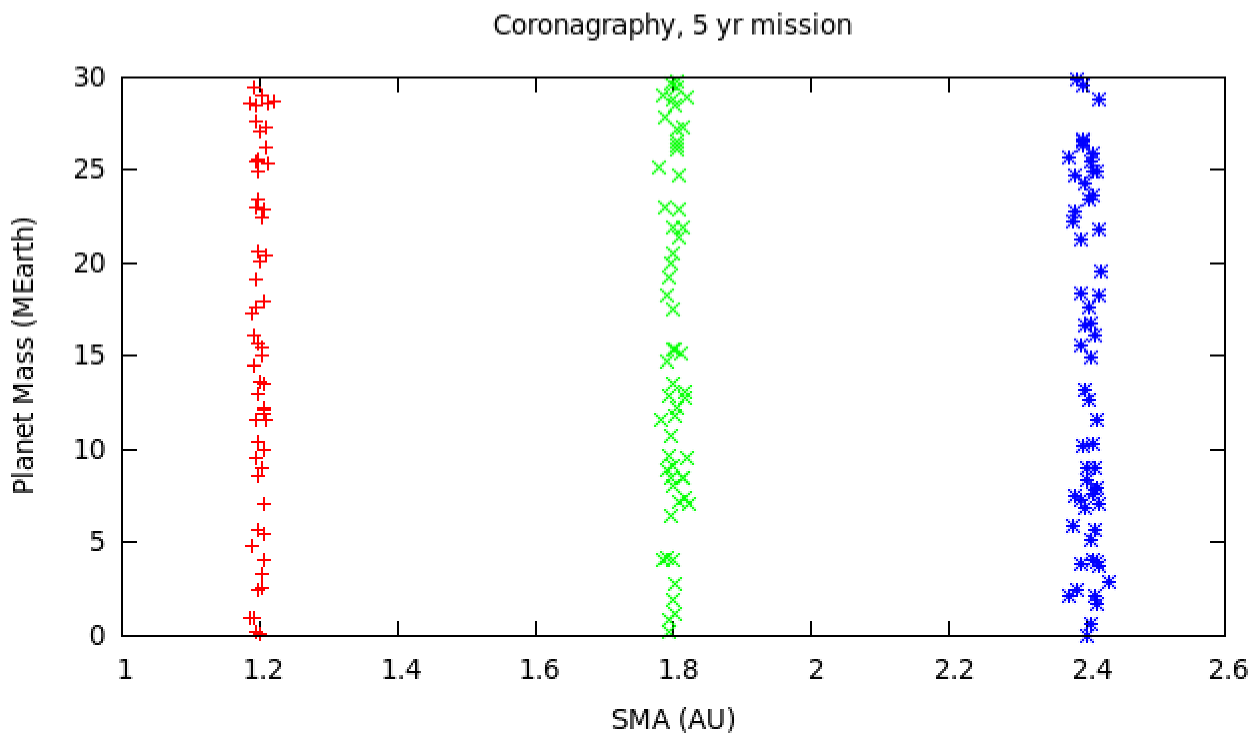
- Mitigates the **1yr period problem** for astrometry

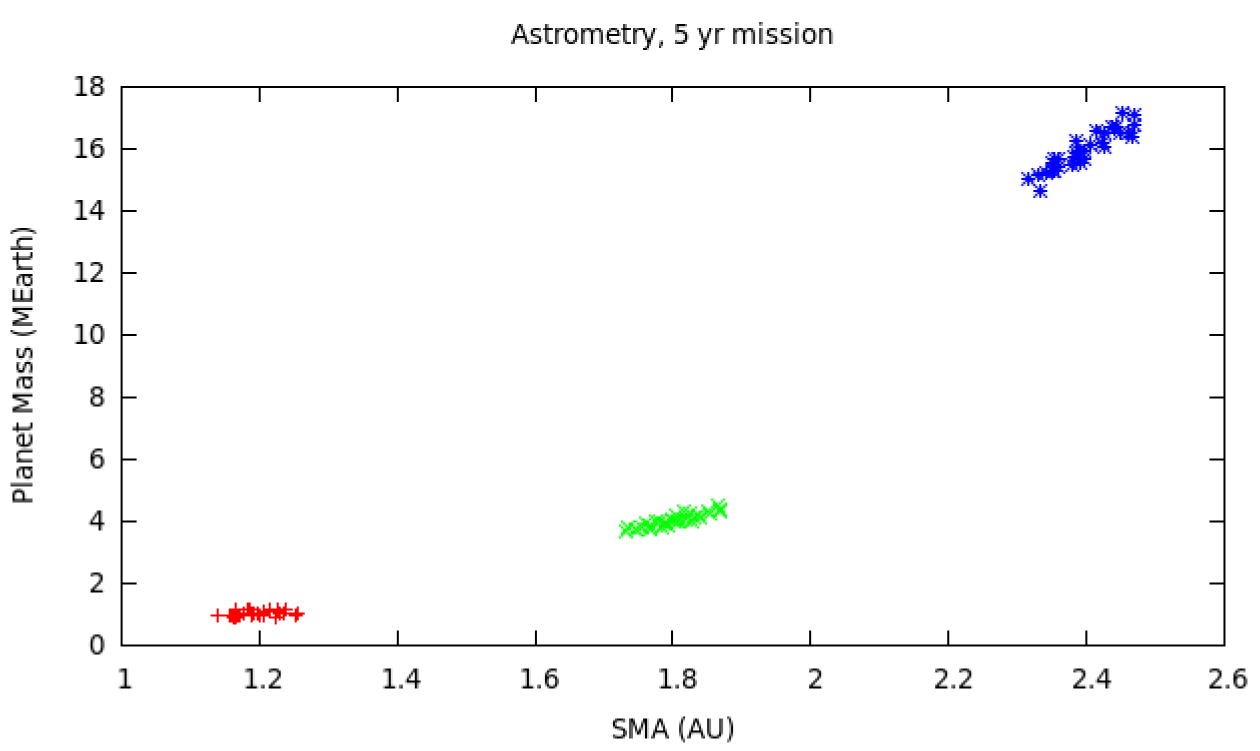
Performance as a function of star brightness (FOV = 0.03 sq deg)



Astrometry + coronagraphy can precisely measure mass and orbital parameters of multiple planets systems



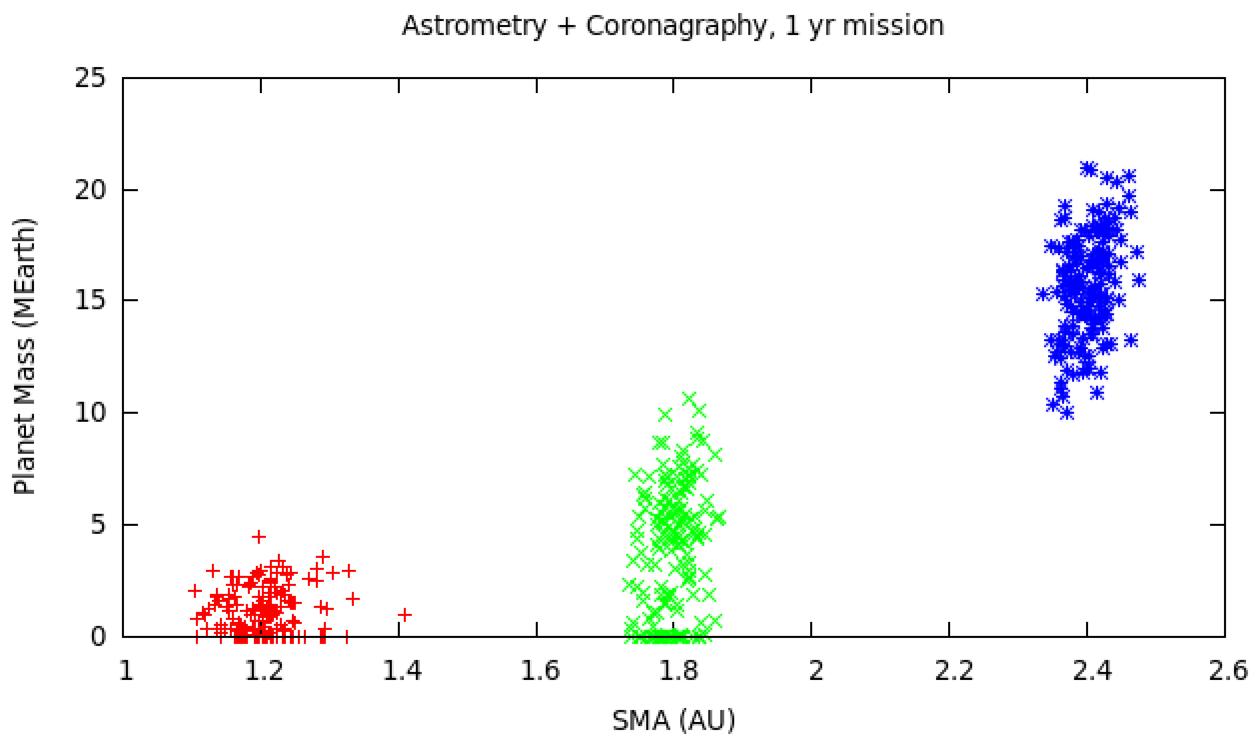


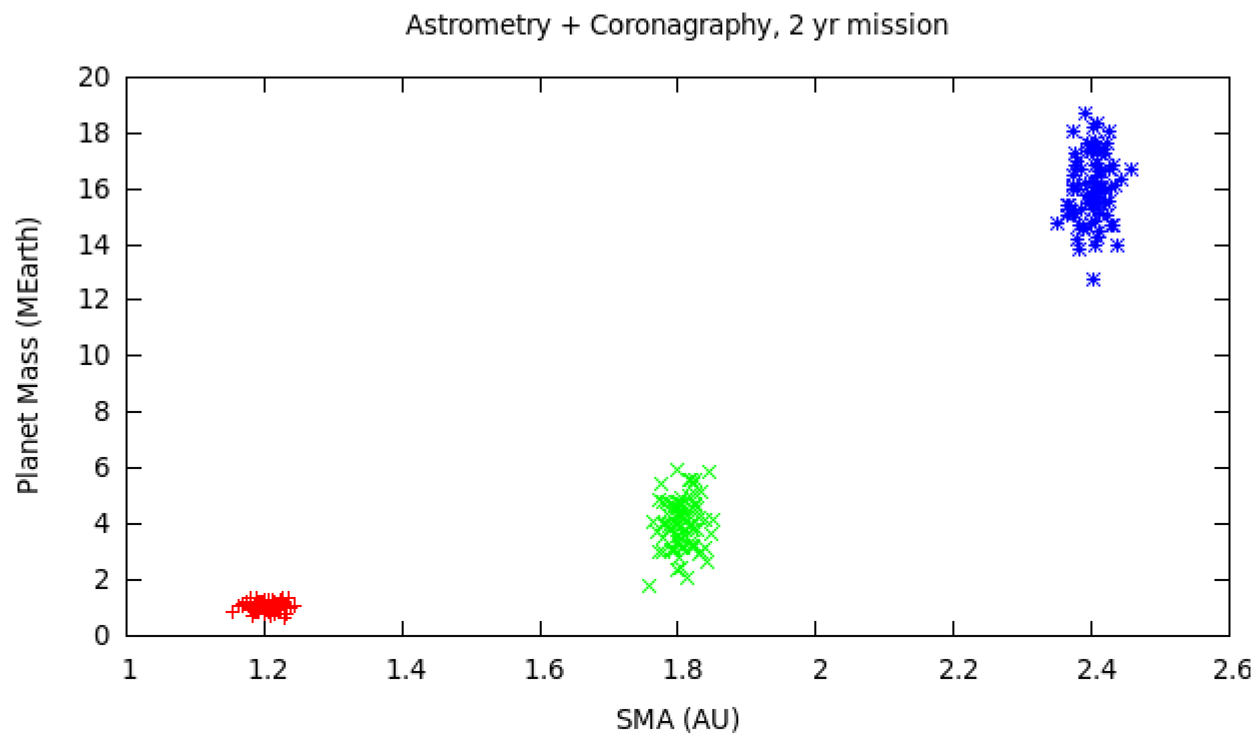


Astrometry+Coronagraphy,
Coronagraphy only :
1 observation every 2 month

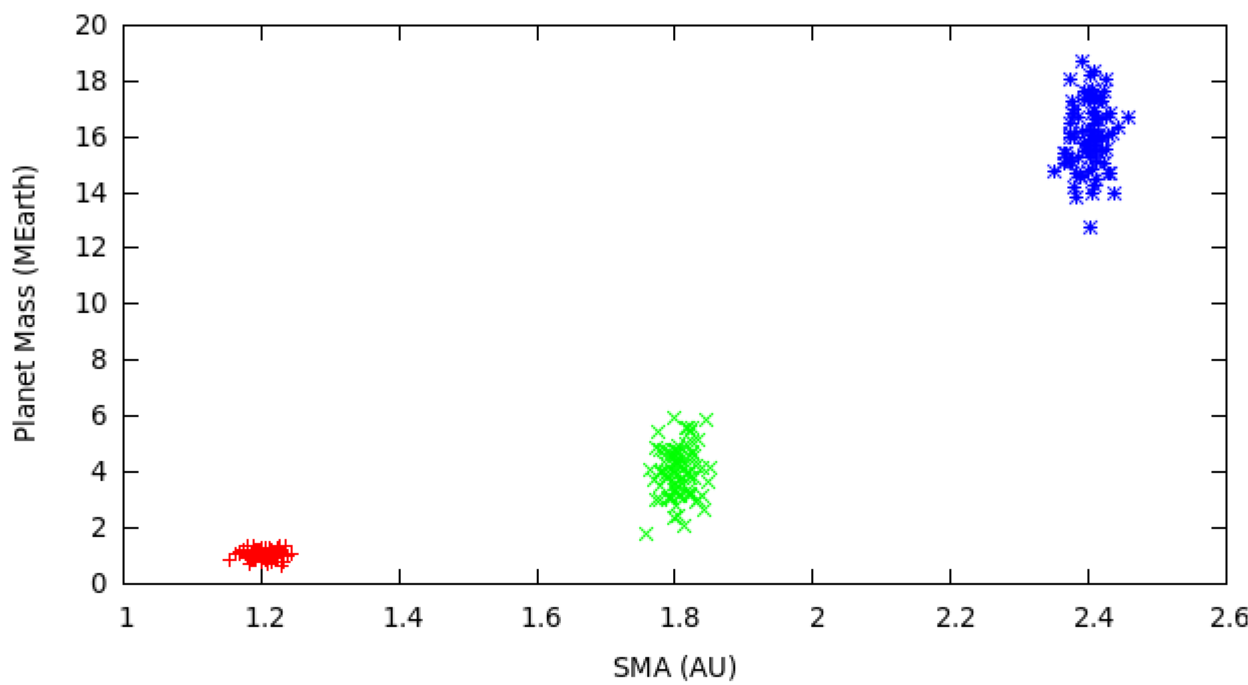
Astrometry only:
1 observation per month

Mass can be constrained quickly
→ firm identification of Earth-like planets within 1 to 2 yrs

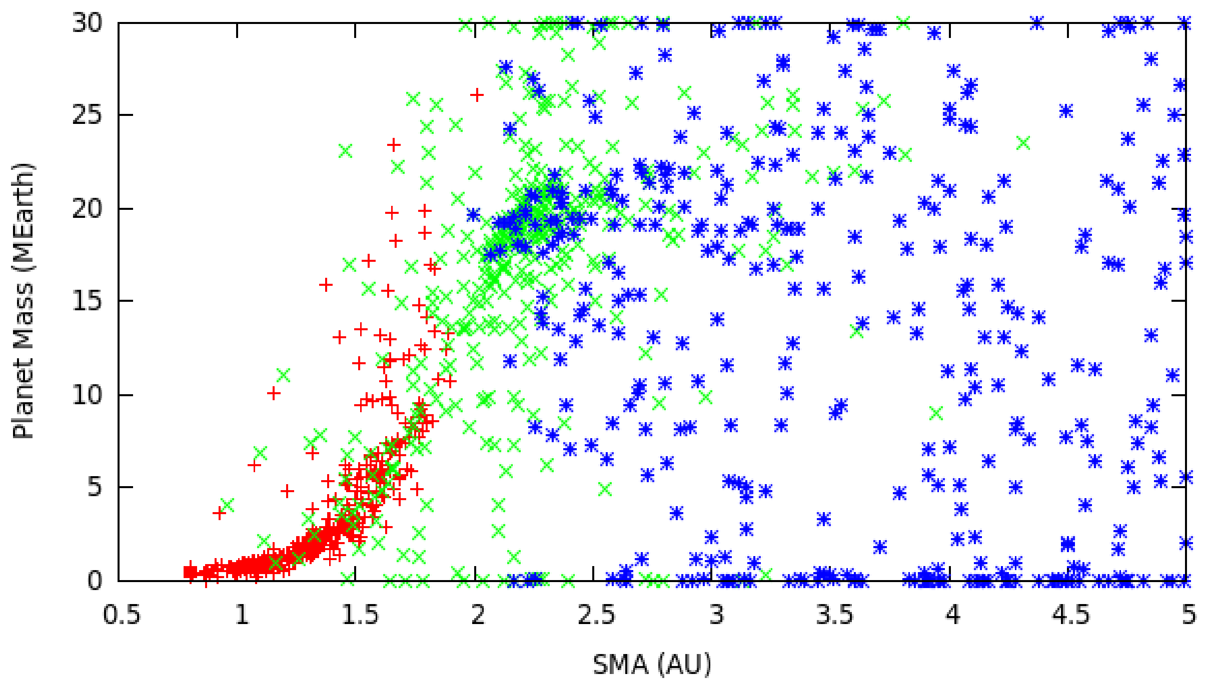




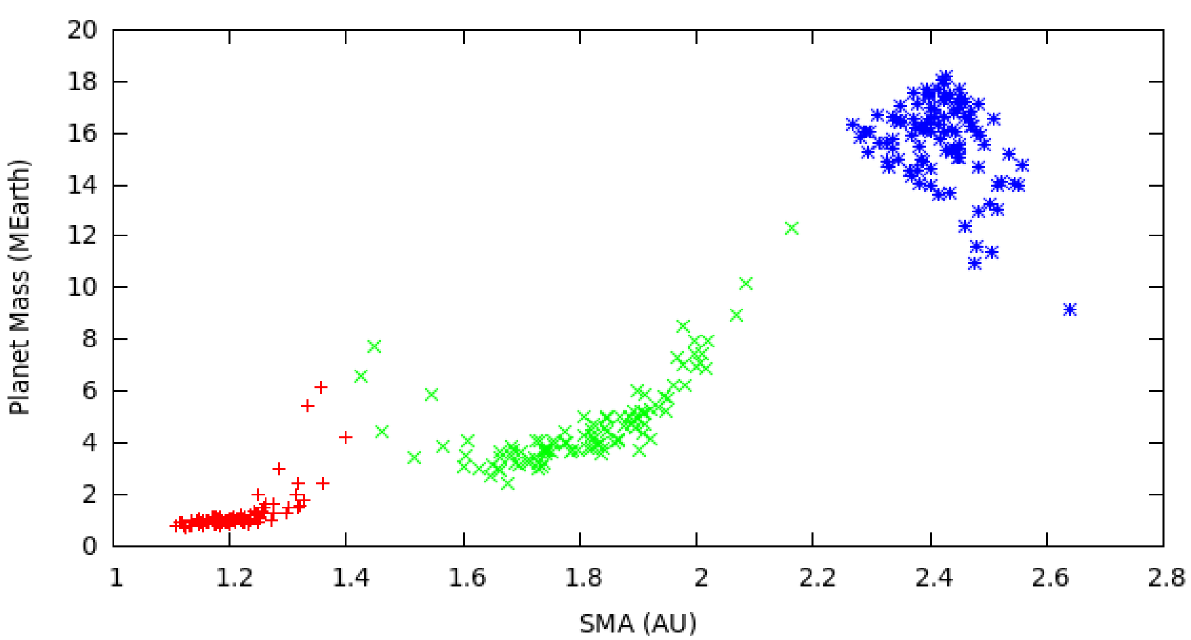
Astrometry + Coronagraphy, 2 yr mission



Astrometry only, 2 yr mission



Astrometry only, 4 yr mission



Astrometry only, 5 yr mission

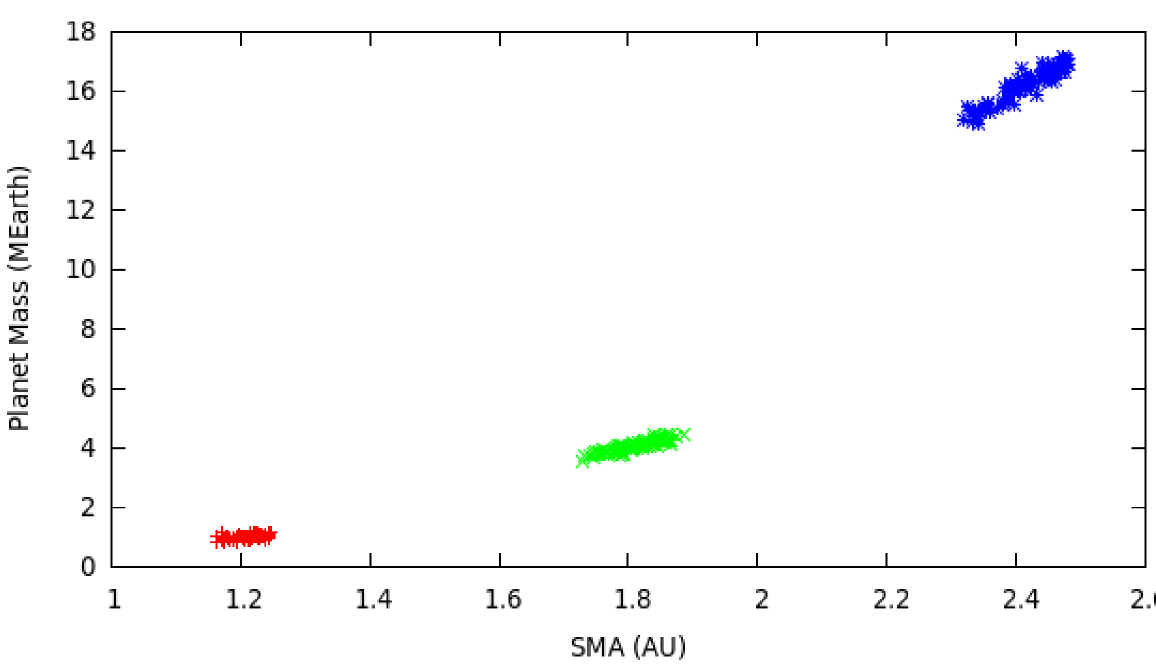


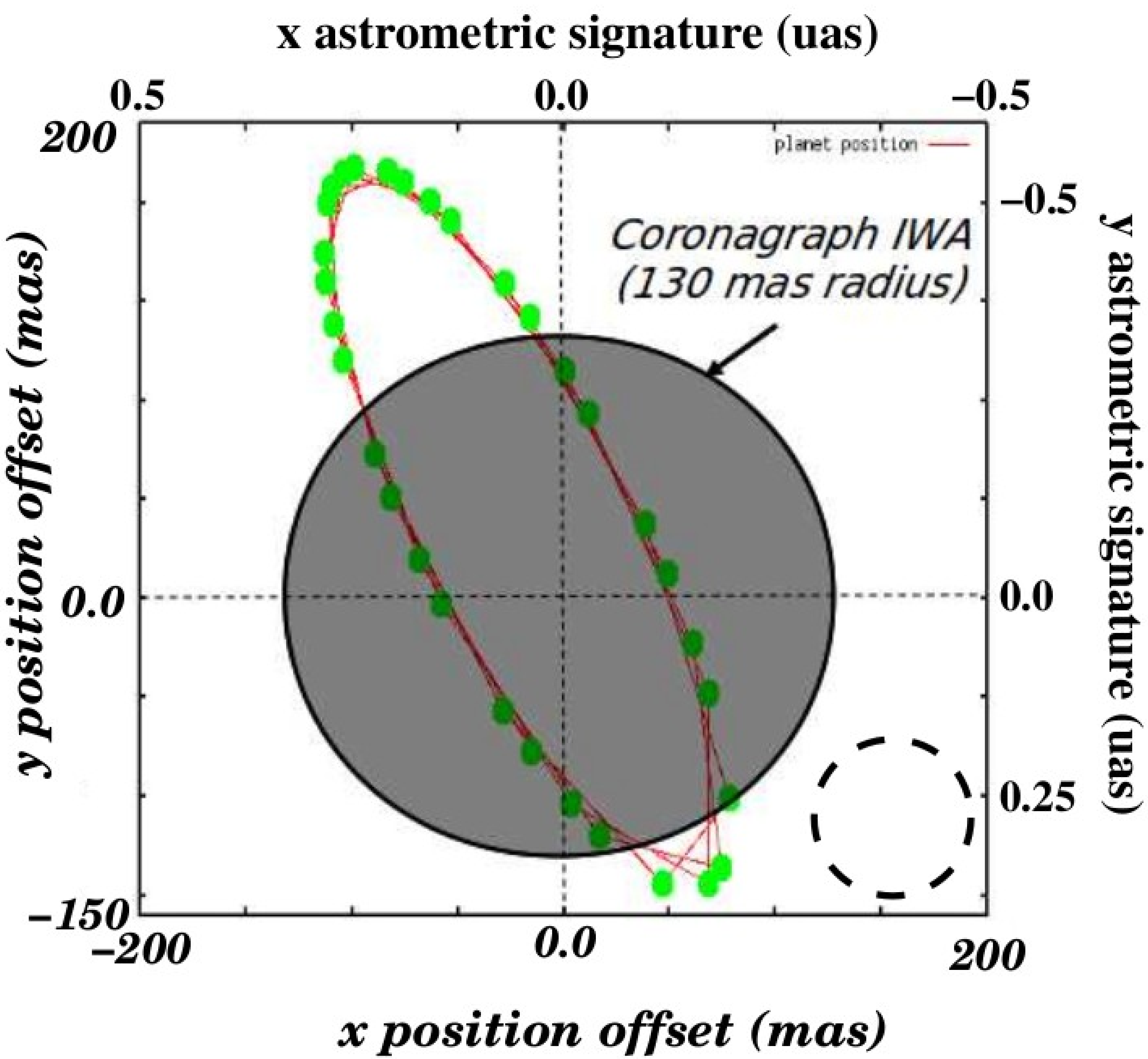
Table 2. Uncertainties in the combined (astrometry and coronagraphy) and separate (astrometry or coronagraphy) solutions[a]

Parameter	**1-σ uncertainty**		
	Astrometry only	Astrometry +coronagraphy	Coronagraphy only[b]
parallax	0.037 μas	0.035 μas	2.949 μas
x proper motion	0.017 μas/yr	0.012 μas/yr	1.304 μas/yr
y proper motion	0.020 μas/yr	0.013 μas/yr	1.288 μas/yr
Planet mass	0.132 M_{Earth}	0.098 M_{Earth}	5.355 M_{Earth}
Semi-major axis (SMA)	0.0228 AU	0.0052 AU	0.0047 AU
Orbital phase	0.653 rad	0.039 rad	0.039 rad
Orbit inclination	0.0968 rad	0.0065 rad	0.0060 rad
Position angle of SMA on sky	0.111 rad	0.0040 rad	0.0039 rad
Orbit ellipticity	0.098	0.0035	0.0034
Position angle of perihelion	0.648 rad	0.0034 rad	0.0033 rad
Stellar mass [c]	0.05 M_{Sun}	0.013 M_{Sun}	0.012 M_{Sun}

[a]Assumed measurement precisions: 0.2 μas per axis per measurement for absolute astrometry, 2.5 mas per axis per measurement for relative astrometry derived from coronagraphic images.

[b]Assumes that astrometry is available at the 20μas per axis per observation from another mission to constrain parallax. Astrometric measurement at his precision level (>> μas) only affects estimates of parallax and proper motion, and has no significan effect on other measured parameters.

[c]Assumed to be known to 5% accuracy independently of astrometry and coronagraphy measurements



x astrometric signature (uas)
0.5
0.0
−0.5
200
planet position
−0.5
y astrometric signature (uas)
Coronagraph IWA
(130 mas radius)
y position offset (mas)
0.0
0.0
0.25
−150
−200
0.0
200
x position offset (mas)

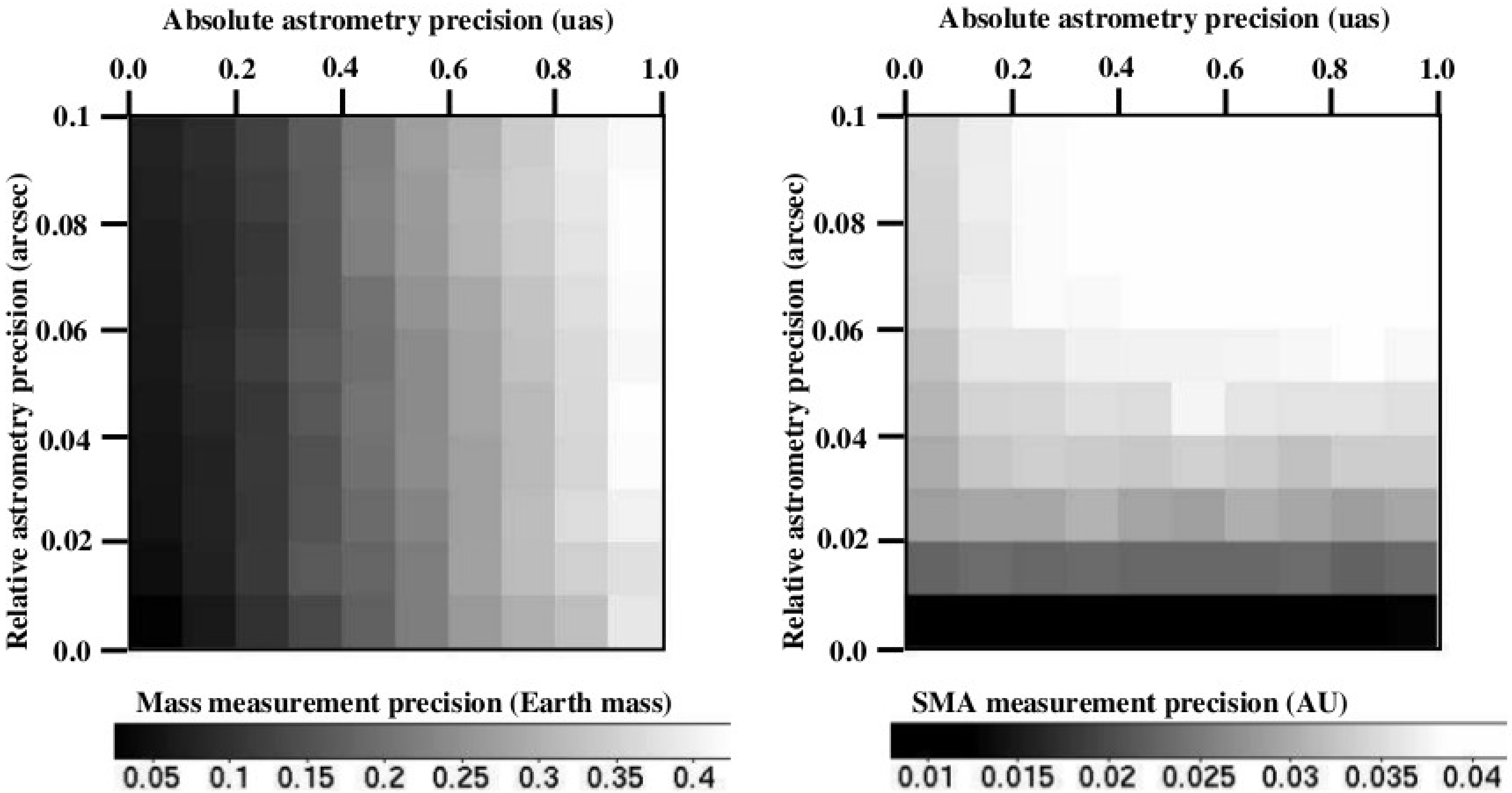

Absolute astrometry precision (uas)
0.0
0.2
0.4
0.6
0.8
1.0
Relative astrometry precision (arcsec)
0.1
0.08
0.06
0.04
0.02
0.0
Mass measurement precision (Earth mass)
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
Absolute astrometry precision (uas)
0.0
0.2
0.4
0.6
0.8
1.0
Relative astrometry precision (arcsec)
0.1
0.08
0.06
0.04
0.02
0.0
SMA measurement precision (AU)
0.01
0.015
0.02
0.025
0.03
0.035
0.04

Table 4. Scattered light due to primary mirror dots

Field percentile[a]	Flux[$ph.sec^{-1}.pix^{-1}$][b]	$Flux/Flux_{zodi}$ [c]
1 %	8.28e-6	7.66e-5
5 %	1.08e-5	9.96e-5
10 %	1.28e-5	1.19e-4
20 %	1.68e-5	1.55e-4
50 %	3.23e-5	2.99e-4
80 %	9,27e-5	8.56e-5
90 %	3.06e-4	2.82e-3
95 %	1.19e-3	1.10e-2
99 %	4.35e-2	4.02e-1

[a]All fluxes are measured in a 3x3 arcmin box centered 6 arcmin from the optical axis. Scattered light becomes smaller at larger separations due to the Airy pattern contribution and the diffraction envelope of the spikes (equal to the diffraction pattern of a single dot)

[b]All values in this table assume an unfiltered exposure (zero point = $5.5e10\,ph.sec^{-1}$), and a $m_V = 3.7$ central star

[c]Assuming $m_V = 22.5 magn.arcsec^{-2}$ zodiacal light background level

Table 1. Observation model

Parameter	value
Planetary system characteristics	
Star	Sun analog
Distance	6 pc
Location	Ecliptic pole
Orbit semi-major axis	1.2 AU
Orbital Period	1.3 yr
Planet Mass	1 Earth Mass
Orbit inclination	1.3 rad
Orbit Eccentricity	0.2
Astrometric signal amplitude	0.5 μas
Orbit apparent semi-major axis	200 mas
Measurements	
Number of observations	32 (regularly spaced every 57 days)
Coronagraphic image: planet position precision	2.5 mas per axis (=3.6 mas in 2D) [a]
Coronagraphic image: inner working angle (IWA)	130 mas
Astrometry: single measurement precision	Variable (driven by science requirement)

[a]Corresponds to a SNR=10 detection with $\lambda/D = 80mas$ (single axis astrometric precision is theoretically equal to $(\lambda/D)/(\pi\sqrt{Nph})$). For a photon noise limited measurement with no background, this would be achieved with Nph =100 photon at 550nm for a 1.4-m telescope.

Table 3. Multiple planets system characteristics

	Mass (M_{Earth})	SMA (A.U.)	Period (yr)	inclin. (sin(i))
Planet 1	1.0	1.2	1.31	0.25
Planet 2	4.0	1.8	2.41	0.25
Planet 3	16.0	2.4	3.72	0.25