# Combined astrometric imaging and coronagraphy

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  - Improvements to original concept, error budget, exoplanet science
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  - Error budget, mission architecture
- Robert Woodruff (LMC)  
  - Optical design for wide field telescope compatible with coronagraphy
- Bijan Nemati (NASA JPL)  
  - Numerical simulations, modeling approach
- Mark Ammons (UofA)  
  - Lab demo design & operation
- **Eduardo Bendek (UofA)**  
  - Lab demo optical design & operation
- Marie Levine (NASA JPL)  
  - System engineering, mission architecture
- Joe Pitman (Expl. Sci.)  
  - System engineering

- Tom Milster (UofA)  
  - Mask manufacturing, scaling of mask manufacturing to full scale PM
- Jim Burge (UofA)  
  - Mask manufacturing, scaling of mask manufacturing to full scale PM
- Neville Woolf (UofA)  
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  - Exoplanet science, concept definition
- Josh Eisner (UofA)  
  - Exoplanet and star formation/evolution science
- Ruslan Belikov (NASA Ames)  
  - Compatibility with coronagraphy
- Daniel Eisenstein (UofA)  
  - Extragalactic science enabled with wide field camera
- Ann Zabludoff (UofA)  
  - Extragalactic science with wide field camera
- Dennis Zaritsky (UofA)  
  - Extragalactic & galactic science with wide field camera
- Jay Daniel (L3/Tinsley)  
  - Optics manufacturing
epoch #1 (first observation)
epoch #2 (first observation)
Blue points show the position of background stars at epoch #2 (second observation). The telescope is pointed on the central star, so the spikes have not moved between the 2 observations, but the position of the background stars has moved due to the astrometric motion of the central star (green vectors).
Why is imaging astrometry difficult?

On-axis and off-axis stars illuminate different (but overlapping) parts of M2. Edge bending on M2 is seen by star #1, but not star #2.

(1) Light from different stars on the sky travels different paths → small bending of optics produces field distortions

(2) The detector can move between observations (especially when using large mosaics)

(3) Pixels are not perfect and their response changes with time

+ (4) Central star is much brighter than background stars

Principle: use background stars around coronagraph target as an astrometric reference

With a 1.4-m telescope in the visible, 0.25 sq deg offers sufficient photons from stars at the galactic pole to provide an astrometric reference at the <50 nano-arcsec after taking into account realistic efficiency, zodi light and pixel sampling.
epoch #1 (first observation)
Optical Layout for simultaneous coronagraphy and astrometry

The telescope is a conventional TMA, providing a high quality diffraction-limited PSF over a 0.5 x 0.5 deg field with no refractive corrector. The design shown here was made for a 1.4m telescope (PECO).

Light is simultaneously collected by the coronagraph instrument (direct imaging and spectroscopy of exoplanet) and the wide field astrometric camera (detection and mass measurement of exoplanets)

M1 is covered with small dots

Central field ~10” Intermediate focal plane

 Coronagraph instrument

wide field image for astrometry
Dots on primary mirror create a series of diffraction spikes used to calibrate astrometric distortions. All astrometric distortions (due to change in optics shapes of M2, M3, and deformations of the focal plane array) are common to the spikes and the background stars. By referencing the background star positions to the spikes, the astrometric measurement is largely immune to large scale astrometric distortions. Instead of requiring ~pm level stability on the optics over yrs, the stability requirement on M2, M3 is now at the nm-level over approximately a day on the optics surfaces, which is within expected stability of a coronagraphic space telescope. (Note: the concept does not require stability of the primary mirror).
Red points show the position of background stars at epoch #1 (first observation)
Blue points show the position of background stars at epoch #2 (second observation). The telescope is pointed on the central star, so the spikes have not moved between the 2 observations, but the position of the background stars has moved due to the astrometric motion of the central star (green vectors).
Due to astrometric distortions between the 2 observations, the actual positions measured (yellow) are different from the blue point. The error is larger than the signal induced by a planet, which makes the astrometric measurement impossible without distortion calibration.
The measured astrometric motion (blue vectors in previous slide) is the sum of the true astrometric signal (green vectors) and the astrometric distortion induced by change in optics and detector between the 2 observations. Direct comparison of the spike images between the 2 epochs is used to measure this distortion, which is then subtracted from the measurement to produce a calibrated astrometric measurement.
The calibration of astrometric distortions with the spikes is only accurate in the direction perpendicular to the spikes length. For a single background star, the measurement is made along this axis (1-D measurement), as shown by the green vectors. The 2-D measurement is obtained by combining all 1-D measurements (large green vector).
A slow telescope roll is used to average out small scale distortions, which are due to non-uniformity in the pixel size, (spectral) response, and geometry.

The green vector is what should be measured.
Science goals

Primary science goal: **Measure planet mass with 10% accuracy (1-σ) for an Sun/Earth analog at 6pc.** This allows mass measurement of all potentially habitable planets (Earth-like & SuperEarths) imaged by PECO.

Table 4-2: Stars with Earth-like planets in habitable zones (1 AU equiv) easily detectable with PECO

<table>
<thead>
<tr>
<th>HIP#</th>
<th>dist (pc)</th>
<th>max el (λ/D)</th>
<th>*rad (λ/D)</th>
<th>SNR (1s, tp)</th>
<th>t20% (s, tp)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>71683</td>
<td>1.3</td>
<td>11.5</td>
<td>0.06</td>
<td>0.49</td>
<td>35</td>
<td>Alf Cen A G2 V, V=0</td>
</tr>
<tr>
<td>71681</td>
<td>1.3</td>
<td>6.6</td>
<td>0.04</td>
<td>0.45</td>
<td>44</td>
<td>Alf Cen B K2 IV, V=1.3</td>
</tr>
<tr>
<td>8102</td>
<td>3.6</td>
<td>2.3</td>
<td>0.01</td>
<td>0.08</td>
<td>2750</td>
<td>Tau Cet G8.5 V, V=3.5 **</td>
</tr>
<tr>
<td>16537</td>
<td>3.2</td>
<td>2.2</td>
<td>0.01</td>
<td>0.09</td>
<td>2968</td>
<td>Eps Eri K2 V, V=3.7 **</td>
</tr>
<tr>
<td>3821</td>
<td>6.0</td>
<td>2.3</td>
<td>0.01</td>
<td>0.04</td>
<td>14329</td>
<td>Eta Cas G0 V V=3.5 ***</td>
</tr>
<tr>
<td>2021</td>
<td>7.5</td>
<td>3.1</td>
<td>0.01</td>
<td>0.03</td>
<td>14878</td>
<td>Bet Hyi G0 V, V=2.8</td>
</tr>
<tr>
<td>99240</td>
<td>6.1</td>
<td>2.2</td>
<td>0.01</td>
<td>0.03</td>
<td>19636</td>
<td>Del Pav G8 IV, V=3.6</td>
</tr>
</tbody>
</table>

Table extracted from PECO SRD (http://caao.as.arizona.edu/PECO/PECO_SRD.pdf)
Combined solution for simultaneous coronagraphy + astrometry

Planet on a 1.2 AU orbit (1.3 yr period), $e=0.2$

orbit orientation on sky: planet outside the coronagraph IWA for 17 out of the 32 observations.

Required single measurement
astrometric accuracy = 0.2 μas (1-sigma, 1D)
Combined solution for simultaneous coronagraphy + astrometry is very accurate for orbital parameters measurement.
Combined solution for simultaneous coronagraphy + astrometry

<table>
<thead>
<tr>
<th></th>
<th>Astrometry only</th>
<th>Astrometry + coronagraphy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>parallax</strong></td>
<td>0.037 μas</td>
<td>0.035 μas</td>
</tr>
<tr>
<td><strong>x proper motion</strong></td>
<td>0.017 μas/yr</td>
<td>0.012 μas/yr</td>
</tr>
<tr>
<td><strong>y proper motion</strong></td>
<td>0.020 μas/yr</td>
<td>0.013 μas/yr</td>
</tr>
<tr>
<td><strong>Planet mass</strong></td>
<td>0.132 ME</td>
<td>0.098 ME</td>
</tr>
<tr>
<td><strong>Semi-major axis</strong></td>
<td>0.0228 AU</td>
<td>0.0052 AU</td>
</tr>
<tr>
<td><strong>orbital phase</strong></td>
<td>0.653 rad</td>
<td>0.039 rad</td>
</tr>
<tr>
<td><strong>orbit inclination</strong></td>
<td>0.0968 rad</td>
<td>0.0065 rad</td>
</tr>
<tr>
<td><strong>sma projected PA on sky</strong></td>
<td>0.1110 rad</td>
<td>0.0040 rad</td>
</tr>
<tr>
<td><strong>orbit ellipticity</strong></td>
<td>0.098</td>
<td>0.0035</td>
</tr>
<tr>
<td><strong>PA of perihelion on orbit plane (w)</strong></td>
<td>0.648 rad</td>
<td>0.0034 rad</td>
</tr>
<tr>
<td><strong>stellar mass</strong></td>
<td>0.050 M&lt;sub&gt;sun&lt;/sub&gt;</td>
<td>0.013 M&lt;sub&gt;sun&lt;/sub&gt;</td>
</tr>
</tbody>
</table>
Benefits of simultaneous coronagraphy + astrometry

- **Reduces confusion with multiple planets.** Outer massive planets (curve in the astrometric measurement) will be seen by the coronagraph.

- Astrometry will **separate planets from exozodi clumps.**

- Astrometric knowledge allows to **extract fainter planets from the images, especially close to IWA,** where the coronagraph detections are marginal.

- Mitigates the **1yr period problem** for astrometry
Approach & Assumptions

Baseline: 1.4-m telescope, with 0.29 sq deg FOV (0.31 deg radius)

The FOV is chosen to reach performance goal (0.2 μas/ measurement) in a sufficiently stable system (Photon noise limited performance for this FOV is 0.044 μas single measurement at galactic pole, but actual performance is significantly lower due to distortions and detector limits)
When detailed simulations are required, a smaller FOV system is used (0.1 deg radius = 0.03 sq deg FOV) to ease computations.

Baseline assumes no special requirements on detector or optics, other than a design to support wide field imaging and dots on M1: ASTROMETRY DOES NOT DRIVE TELESCOPE OR INSTRUMENT DESIGN
- no special detector requirements (standard errors on flat field, geometry), assumes no calibration beyond what is “standard”
- no component requirement exceeds what has already been demonstrated and manufactured
- assumes no data calibration is done between observations of different stars (pessimistic)
- fraction of primary mirror covered by dots kept small (1%) to avoid loss in sensitivity for general astrophysics and coronagraphy
Numerical simulation approach

1. **Pupil Mask with Dots**
   - Monochromatic PSF
   - Polychromatic PSF
   - PSF x derivative
   - PSF y derivative
   - PSF angular derivative

2. **Focal Plane Array x and y Distortion Change**
   - Distx, Disty
   - Distvarx, Distvary

3. **Optics x and y Distortion Change**
   - Distx, Disty (measured spikes image)
   - Measured spikes image

4. **Angular Distortion Change**
   - Total x and y distortion change
   - Optical x and y distortion change
   - Angular distortion change
   - Photons noise due to zodiacal background and stellar flux

5. **Detector Flat Field Response**
   - Flat field error
   - Flat field response

6. **Dynamic Distortions**
   - Mirrors M2 and M3 surface change
   - Ray tracing
   - Variation in detector flat field response

7. **Static Distortions**
   - Focal plane array x and y distortion
   - Mirrors M2 and M3 surface errors
   - Ray tracing

8. **Background Star Position Measurement Error**
   - Static angular distortion

9. **Background Stars Position Measurement Error between 2 Epochs**
   - 1-D angular coordinate
   - Optimal weighting of all 1-D measurements

10. **2-D Astrometric Measurement**
    - Residual astrometric distortion error map in angular direction

11. **Distortion Measurement SNR per Pixel**
    - Binned square SNR per pixel for 1 pixel angular distortion

12. **SNR-Weighted Convolution by Anisoplanatism Sizetd Kernel**
    - Residual astrometric distortion error map in angular direction

13. **Bilinear Interpolation**
    - Measured astrometric distortion map in angular direction

14. **Quick Look at Residual Distortion**
    - (Useful for optimizing anisoplanatism kernel)

15. **Roll Averaging**
    - Roll averaged residual distortion error map in angular direction

16. **Sum of Two or More Images**
    - Product of two images (pixel by pixel)

Image name (used through this document)
Operation performed on images or data
Performance as function of telescope size and FOV (baseline system)

1 % area coverage on PM

$m_v = 3.7$ target

Galactic pole observation

2 day per observation

Larger telescope diameter :
- more light in spikes ($D^2$), finer spikes ($1/D$) → spike calibration accuracy goes as $D^{-2}$
- more light in background stars ($D^2$), and smaller PSF ($1/D$) → position measurement goes as $D^{-2}$

**Astrometric accuracy goes as $D^{-2} \ FOV^{-0.5}$**

Number of pixels goes as $D^{-2} \ FOV$

**At fixed number of pixels, larger D is better (FOV can be reduced as $D^{-4}$)**

<table>
<thead>
<tr>
<th></th>
<th>FOV = 0.03 sq deg</th>
<th>FOV = 0.1 sq deg</th>
<th>FOV = 0.25 sq deg</th>
<th>FOV = 0.5 sq deg</th>
<th>FOV = 1.0 sq deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>D = 1.4 m</td>
<td>0.62 μas</td>
<td>0.34 μas</td>
<td>0.22 μas</td>
<td>0.15 μas</td>
<td>0.11 μas</td>
</tr>
<tr>
<td>D = 2.0 m</td>
<td>0.30 μas</td>
<td>0.17 μas</td>
<td>0.11 μas</td>
<td>0.07 μas</td>
<td>0.053 μas</td>
</tr>
<tr>
<td>D = 3.0 m</td>
<td>0.14 μas</td>
<td>0.074 μas</td>
<td>0.047 μas</td>
<td>0.033 μas</td>
<td>0.023 μas</td>
</tr>
<tr>
<td>D = 4.0 m</td>
<td>0.076 μas</td>
<td>0.042 μas</td>
<td>0.026 μas</td>
<td>0.019 μas</td>
<td>0.013 μas</td>
</tr>
</tbody>
</table>
# Error budget (baseline)

<table>
<thead>
<tr>
<th>Error on background stars position (photon noise, zodi, sampling)</th>
<th>Assumption(s)</th>
<th>Mitigation(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.128 μas</td>
<td>Galactic pole pointing</td>
<td>Reduce other terms → brighter stars can be used with smaller photon noise</td>
</tr>
<tr>
<td>Detector flat field error (static)</td>
<td>0.033 μas</td>
<td>1% RMS, 6% Peak</td>
</tr>
<tr>
<td>Optical distortions (static)</td>
<td>0.083 μas</td>
<td>1.5 nm optics for M2 &amp; M3</td>
</tr>
<tr>
<td>Detector distortions (static)</td>
<td>0.0153 μas</td>
<td>0.2% of pixel size</td>
</tr>
<tr>
<td>Variation in detector flat field (dynamic)</td>
<td>0.0289 μas</td>
<td>0.1 % RMS</td>
</tr>
<tr>
<td>Variation of optical distortions (dynamic)</td>
<td>0.0629 μas</td>
<td>40 pm on surfaces</td>
</tr>
<tr>
<td>Variation of detector geometry (dynamic)</td>
<td>0.0755 μas</td>
<td>~20 mK uncalibrated variations across FPA</td>
</tr>
<tr>
<td>Photon noise on spikes</td>
<td>0.0478 μas</td>
<td>Includes zodi photon noise mV = 3.7 star</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>0.20 μas</strong></td>
<td><strong>Obtained with 0.29 sq deg FOV</strong></td>
</tr>
</tbody>
</table>
Error budget: overview
(baseline system)

- Photon noise on background stars: 44%
- Detector flat field (static): 15%
- Optical distortions (static): 11%
- Detector distortions (static): 6%
- Variation in detector flat field (dynamic): 3%
- Variation of optical distortions (dynamic): 18%
- Variation of detector geometry (dynamic): 2%
- Photon noise measurement on spikes: 1%
Detector static errors are expected to be very small in the roll-averaged angular coordinate.

The combined measurement error is \( \sim 0 \) in angular coordinate.

Radial coordinate (not measured)

Angular coordinate
finite detector sampling, polychromatic PSF

Detector saturation

poor detector calibration, unstable optical system and focal plane array

stable optical system and focal plane array

good detector calibration

accuracy floor due to distortions & detector limits

finite detector sampling, polychromatic PSF

shorter observation time

longer observation time

1.4m telescope

0.1 deg field radius

(0.03 sq deg)
galactic pole

2 day observation

single star photon noise limited error

photon noise limit, monochromatic PSF, infinite sampling, no background

actual measurement error

measurement error (1-sigma)
Enhancements

Main sources of error:
1. **optical distortions** (static and dynamic)
   Why? → because of high spatial frequencies in distortions which are not sampled by the spikes (they fall between the spikes)
2. **variations in detector geometry**

**Apodizing the edges of M1** (mitigates issue #1)
→ beam walk effect on M2 and M3 becomes unable to produce distortions that change rapidly with sky position
→ astrometric calibration by spikes becomes much better

Issues:
- Small loss in throughput (few % ?)
- Manufacturing of apodized edge on M1
- Many issues to check (chromaticity, compatibility with coatings, etc.)

**Projecting interference fringes on the detector** (mitigates issue #2)
→ allows measurement of detector geometry any time

Issues:
- Takes time away from observation
- Complexity (add laser, fibers)

Warning: as instrumental errors become smaller, risk and impact of possible companions on field stars increase
Static distortion map due to M2 & M3 optical surfaces

Distortion maps shown below is for 0.46x0.46 deg field. Unit is arcsec; left map is x, right map is y. Distortion map is computed at 220000 positions on the sky with raytracing software written in C (cross-checked with code-V), then interpolation is used to compute the full map. Total number of rays used = 7e11 (122 day CPU of execution time on 2 GHz CPUs)

Distortion amplitude is ~1 mas, dominated by low order modes. The differential distortion over ~1” is much smaller.
Exoplanet science with coronagraphy + astrometry + wide field imaging

Provides a complete picture of a planetary system:

CORONAGRAPHER:
• Planets orbits
• Planet atmospheres (spectra, polarization from coronagraph)
• Rotation periods (time photometry from coronagraph)
• Zodiacal cloud: morphology, spectra, polarization (coronagraph)

ASTROMETRY:
• Planet masses

CORONAGRAPHY + CORONAGRAPHY:
• Good sensus of planets in a system (astrometry + coronagraphy)

WIDE FIELD IMAGING:
• Very distant planets, possibly ejected
• Debris disks at large separation
• Occultations of field stars by Kuiper belt objects

Also: high precision photometry of field stars
• Microlensing program possible (with pointing to galactic bulge)  
  stable sharp PSF with good astrometry valuable
• Transit observations
Deep wide field imaging science

Wide field + stable diffraction limited PSF is scientifically valuable for many scientific programs, and will be unique in visible:
• Cosmology: weak lensing, type Ia supernovae
• Galactic astronomy
• Planetary astronomy: search for small & distant objects (asteroids, comets, KBOs)

The dots on PM do not significantly impact sensitivity
Loss in sensitivity is due to 3 effects:
• Light absorbed by the dots → 1% loss in throughput
• Light diffracted out of the PSF core by the dots → 1% loss in flux
• Additional background due to diffraction spikes of central star
  - spikes occupy a tiny fraction of the FOV, and are sufficiently stable to be efficiently removed from images by postprocessing
  - for a mV=3.7 central source, over 95% of the field, additional diffracted light is less than 1% of zodi background
  - mean value for additional diffracted background over the field = 6 ph/pix/day (unfiltered), vs 20000 ph/pix/day for zodi
Conclusion

Coronagraphy, astrometry and wide field imaging can be combined for simultaneous observation without compromising performance.
- rich science for both exoplanet science and general astrophysics
- could be key to gain support and funding for large (>1m) space mission for spectroscopy of habitable exoplanets

Future work …

- Lab testbed at UofA: demonstrate performance and algorithms, validate error budget
- Test with coronagraph at NASA Ames and NASA JPL
- We are investigating ground-based system and doing science with it (Funding from University of Arizona and NASA)

More info …

- See Eduardo Bendek's poster today (lab testbed at UofA)
- J. Catanzarite's talk today “The synergy of Direct Imaging and Astrometry for Detection of Exo-Earths”
- Website (includes detailed error budget, algorithms, C source code): http://www.naoj.org/staff/guyon/
  → research → coronagraphic astrometry