Space Optics (2)
AstrOpt2016
Derive Etendu $A\Omega$.

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Transmittance, throughput, & vignetting

- How bright is my image?
- Can I record it?
- Parameters that describe the ability of the optical system to transmit power
- What is the diameter of my optical elements?
- Can they be fabricated or just designed!

Kirchoff's Laws

If a body of mass is at thermal equilibrium with its surrounding environment, conservation of energy requires that

$$\Phi_{incident} = \Phi_{absorbed} + \Phi_{reflected} + \Phi_{transmitted}$$

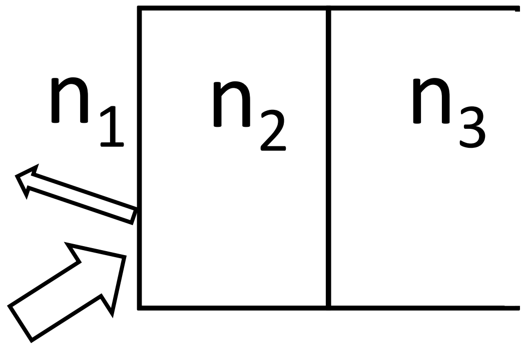
By dividing both sides by $\Phi_{incident}$, we write $\alpha + r + t = 1$, where α is absorbtance, r is reflectance, and t is transmittance. For an opaque body where there is no transmittance ($t = 0$), the radiation is either absorbed or reflected. Therefore,

$$\text{watts absorbed} = \alpha \cdot E \cdot \text{area} = \varepsilon \cdot M \cdot \text{area} = \text{watts radiated.}$$

Minimize reflectance loss to maximize transmittance

- Antireflection coat has limitations

$$R_i(\lambda) = \frac{I_{Reflected}(\lambda)}{I_{Incident}(\lambda)}$$



If $n_2(\lambda) = \sqrt{n_1(\lambda) n_3(\lambda)}$

Then reflectance is zero & transmittance is maximized

Often a physical material with just the right $n_2(\lambda)$ does not exist

Power at the focal plane

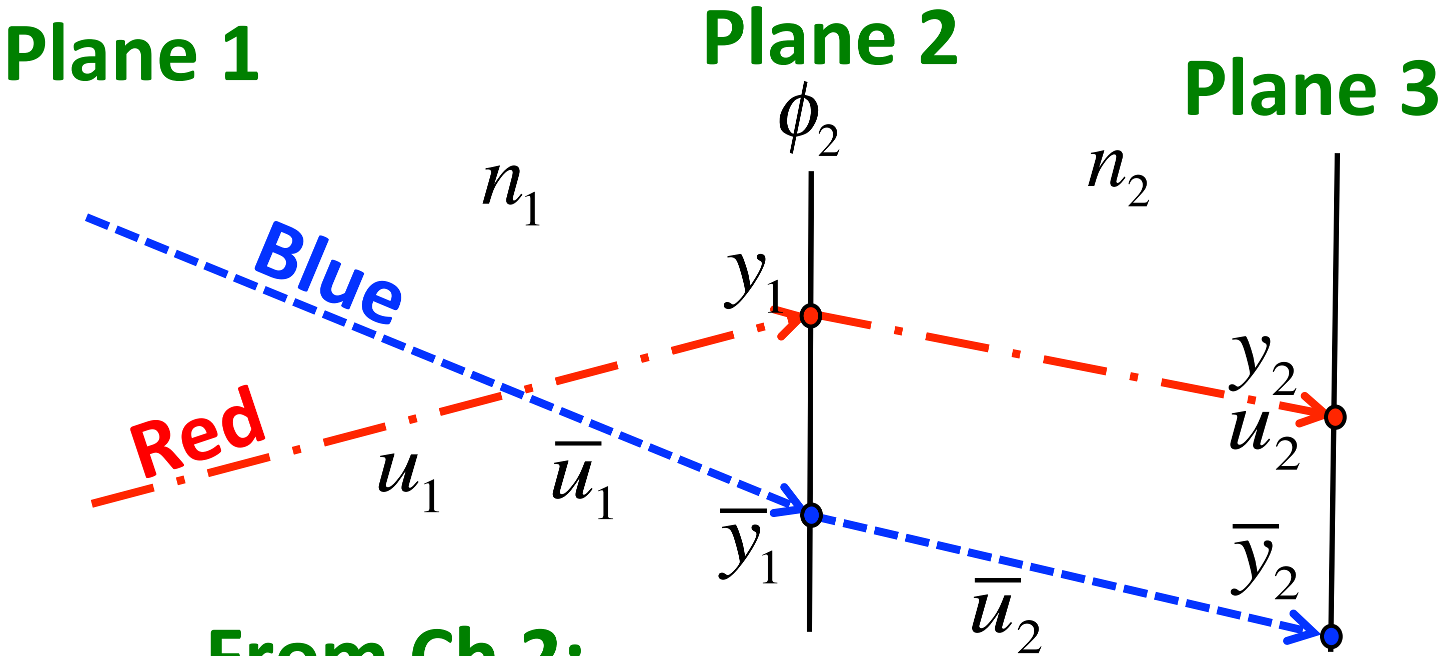
- Determined by
- Transmittance
- **Etendu or "through-put"**
- Polarization (discussed later)

Étendú or throughput [pp103]

- Calculated using tools of 1st order optics
- Expresses the geometric ability of an optical system to pass radiation from object space to image space
- Ray trace can indicate an excellent image but if no light gets through the system, there is no image – SNR=0.0!
- Consider 2 general rays, pass them through the optical system, then look at pupil and image planes

Étendú or throughput

Consider 2 general rays



From Ch 2:

$$n_2 u_2 = n_1 u_1 - y_1 \phi_2$$

$$n_2 \bar{u}_2 = n_1 \bar{u}_1 - \bar{y}_1 \phi_2$$

Étendú or throughput

Consider 2 general rays

$$n_2 u_2 = n_1 u_1 - y_1 \phi_2$$

$$n_2 \bar{u}_2 = n_1 \bar{u}_1 - \bar{y}_1 \phi_2$$

Recall that: $\phi_2 = (n_1 - n_2) C_2 = \frac{(n_1 - n_2)}{R_2}$

The optical power is the same for both rays

$$\frac{\left(n_2 u_2 - n_1 u_1\right)}{y_1} = \phi_2 = \frac{\left(n_2 \bar{u}_2 - n_1 \bar{u}_1\right)}{\bar{y}_1}$$

Re-group the terms

Then we discover that there is an invariant between any two planes in the optical system

Invariant on refraction

$$n_1 \bar{u}_1 y_1 - n_1 u_1 \bar{y}_1 = n_2 \bar{u}_2 y_1 - n_2 u_2 \bar{y}_1 = H$$

Étendú, Helmholtz, LaGrange Invariant

$$n_1\left(u_1\bar{y}_1 - \bar{u}_1 y_1\right) = n_2\left(\bar{u}_2 y_2 - u_2\bar{y}_2\right) = \mathrm{H}$$

Rewrite this equation with the object plane on the LHS

$$y_1 = 0$$

And and the pupil plane on the right hand side

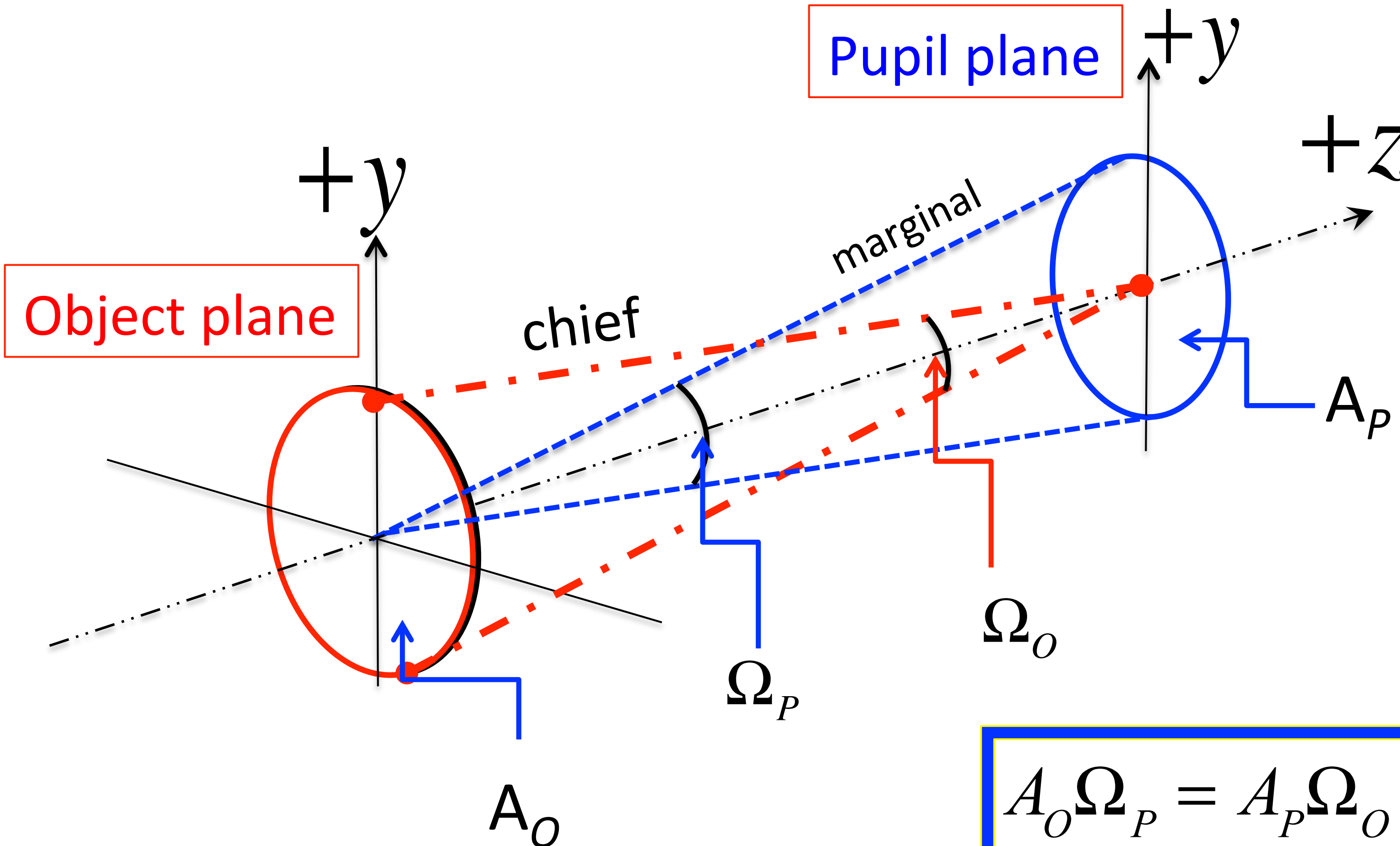
$$\bar{y}_2 = 0$$

Then

$$n u \bar{y}_1 = n \bar{u} y_2 = \mathrm{H}$$

H has units of angle × distance, e.g., radians × centimeters.

Area solid angle product



$$A_O \Omega_P = A_P \Omega_O$$

Confusion?

- Transmittance, transmission, transmissivity - **dimensionless**
- Throughput, etendu, Optical Invariant

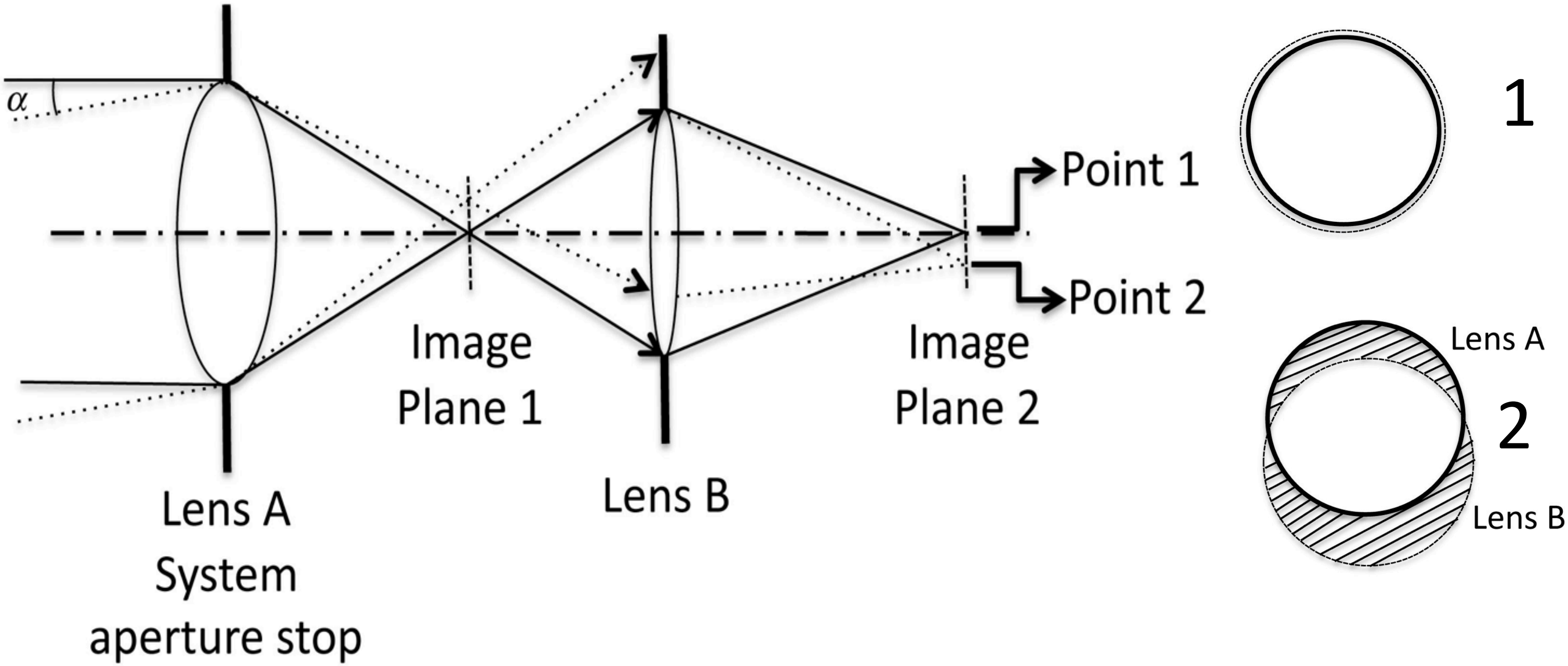
- Units of solid angle × Area, when calculating optical system capacity to transmit radiative power

- Units of radians × length for optical ray trace design

Useful relationships

$$\Omega = \frac{\pi}{4\left(f\#\right)^2}$$

Vignetting: Étendú is not conserved at field points



For no vignetting, the radius of the *k*th surface must be

$$R_k \geq |y_k| + |\bar{y}_k|.$$

Space Optics (2)
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Geometric Aberration Theory

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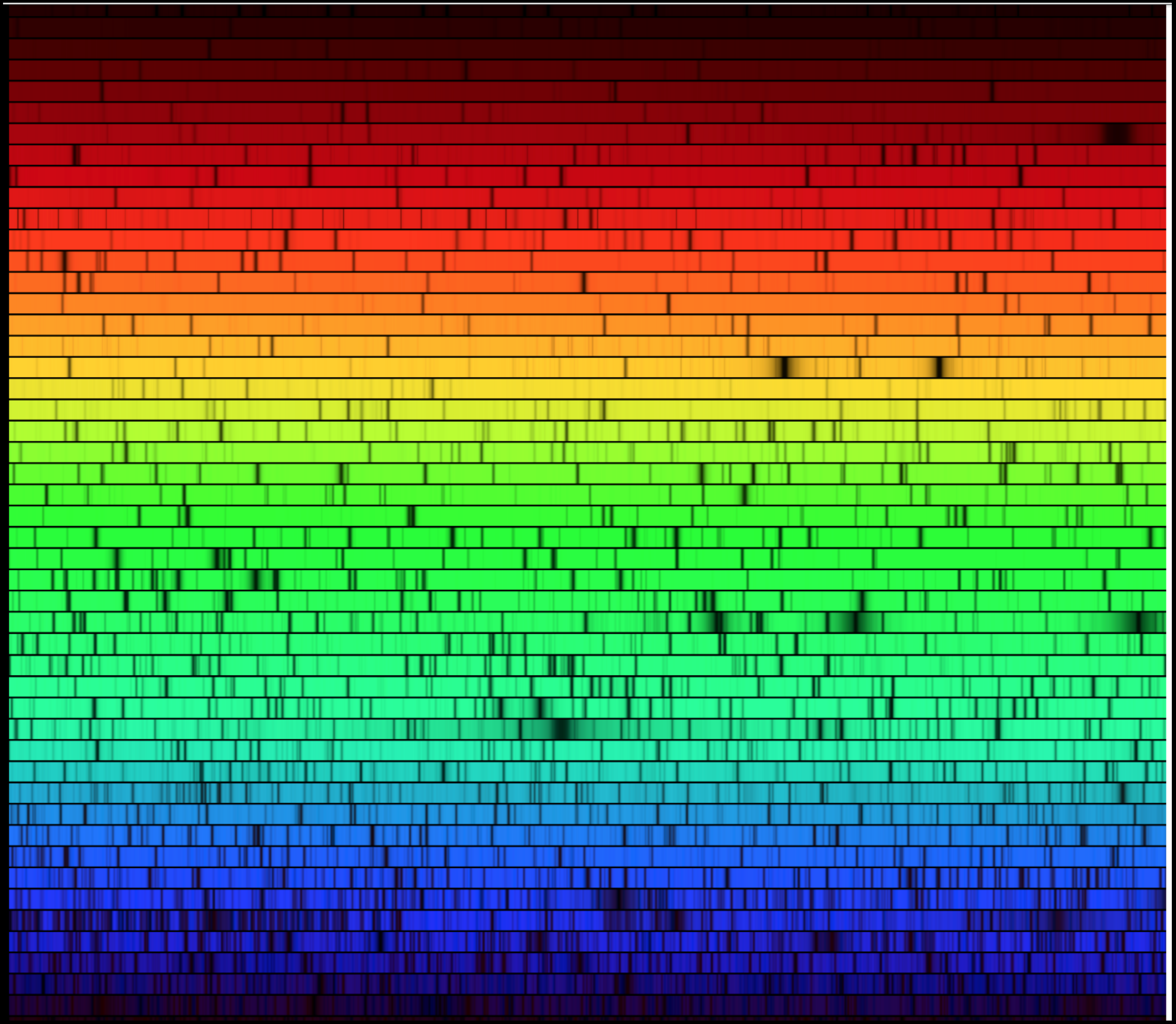
Class outline

- The challenges of space optics
- Derive Etendu, throughput, transmittance
  - Power to the focal plane
- **Geometric aberrations: thermal, structural, metrology, tolerancing & A/O**
- Scalar wave image formation
- Vector-wave image formation: polarization aberrations – partial coherence & correlated wave fields
- Hubble trouble



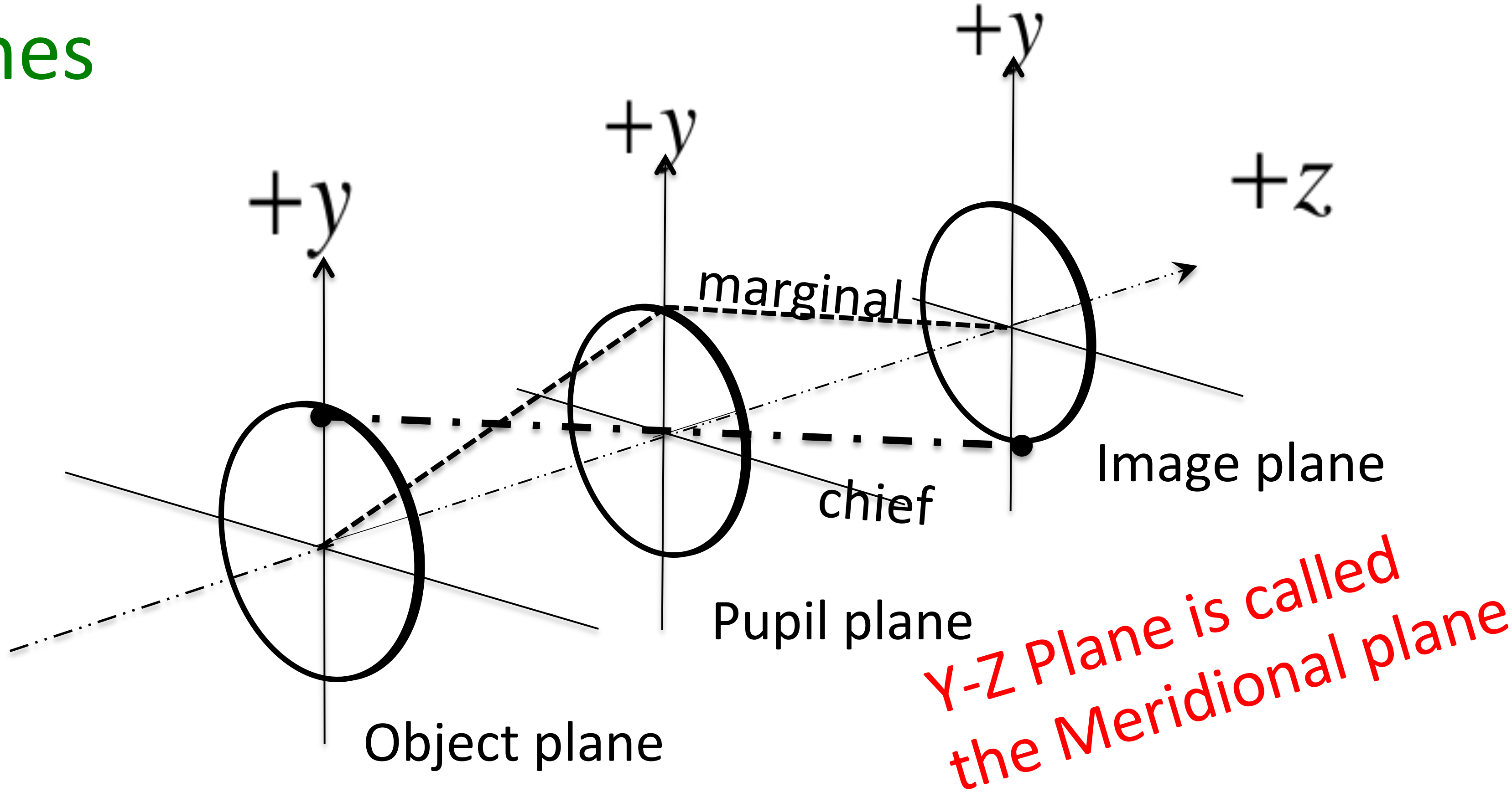
Space optics differ from ground optics

- No atmospheric turbulence
- High angular resolution astronomy
- Diffraction limited important
- Aberration really make a difference
  - Metering structure
    - Thermal (radiative, conduction, ~~convection~~),
    - Dynamics
  - Optical surfaces –
    - Static & dynamic wavefront control

1st order optics

- Two planes, normal to the axis are important in an optical system
  - **Pupil plane**
    - Entrance pupil
    - Exit pupil
  - **Image plane**
  - A complicated optical system can have several image planes, pupil planes, but ONLY one each:
    - Entrance pupil
    - Exit pupil

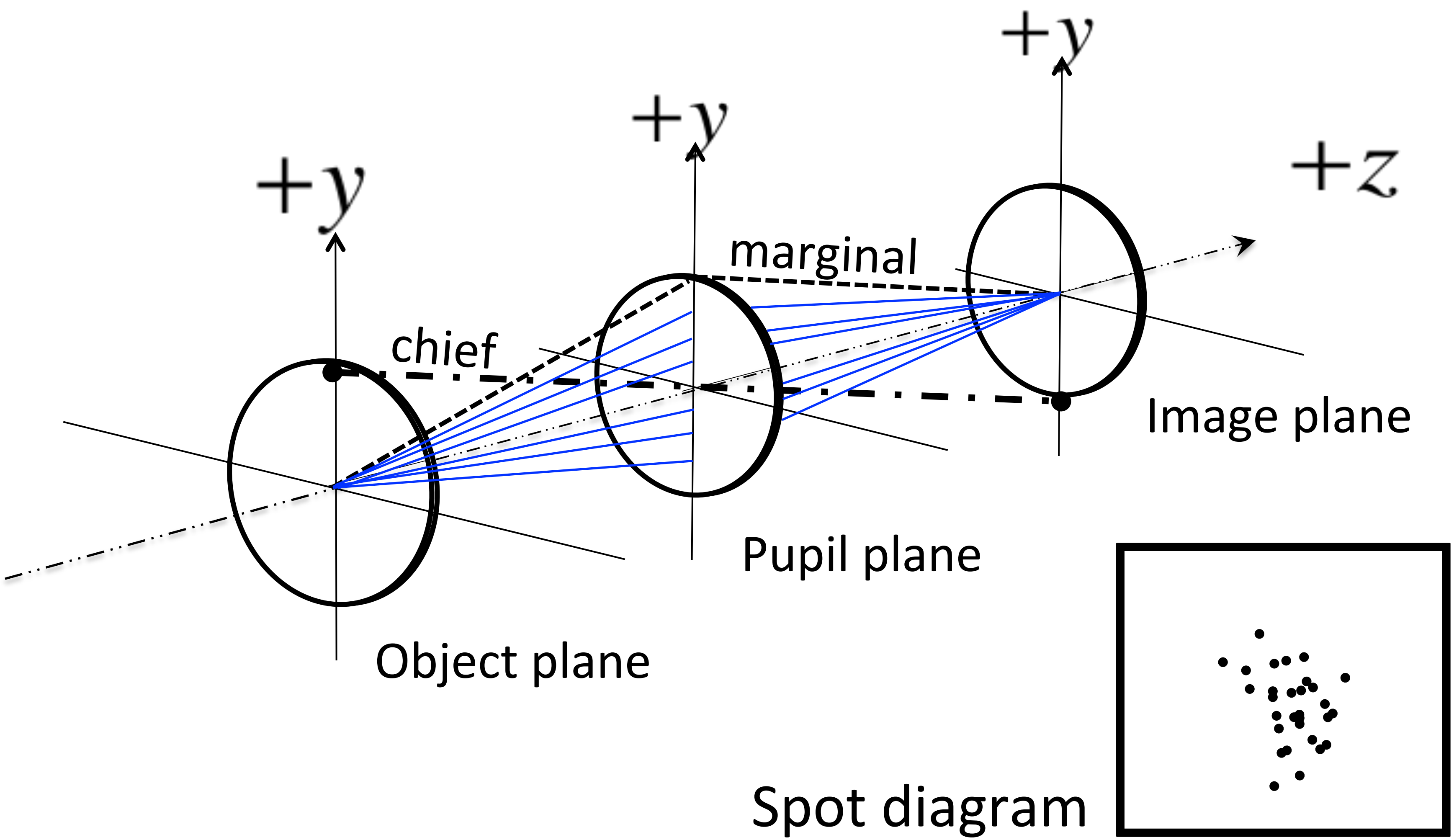
Two rays define location of image & pupil planes



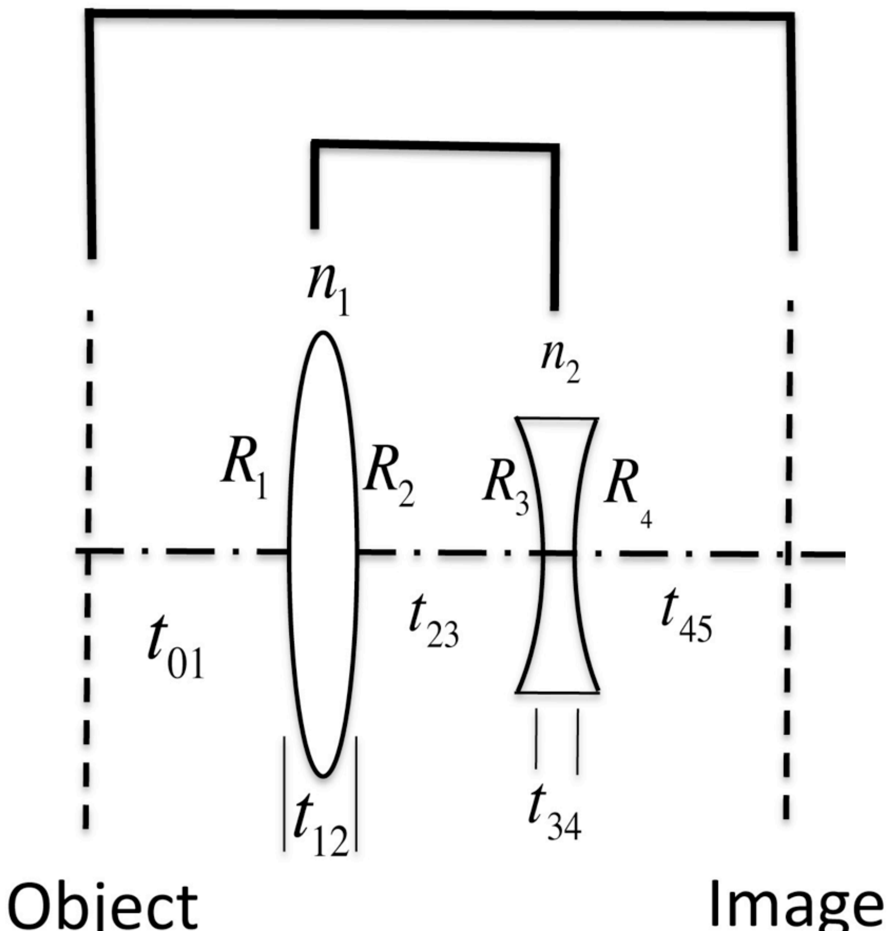
- The volume required for an optical system is defined by both the chief ray and the marginal ray

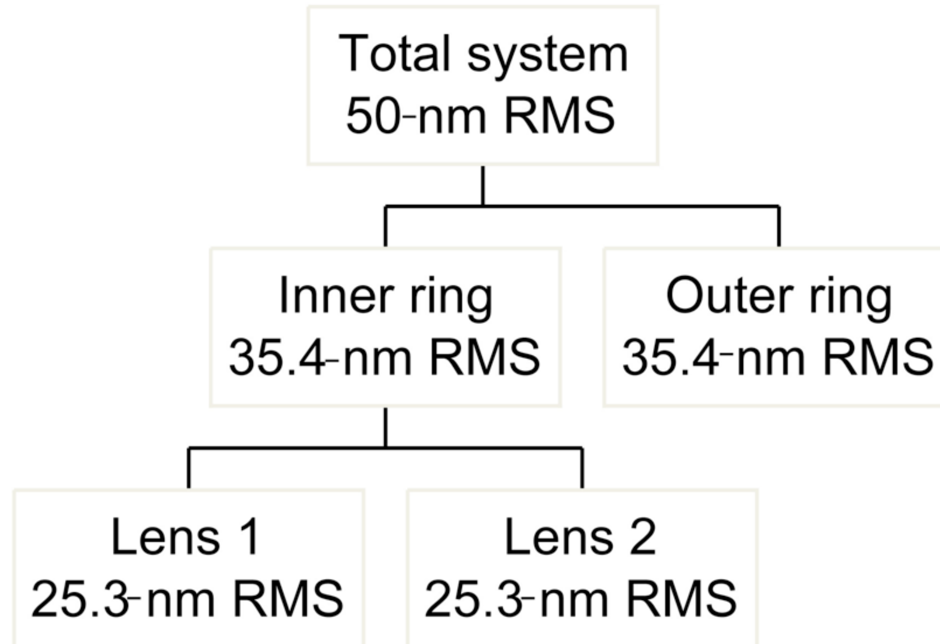
$$\text{radius of the clear aperture} = r(z) = |y| + |\bar{y}|$$

A fan of rays provides an estimate of image quality



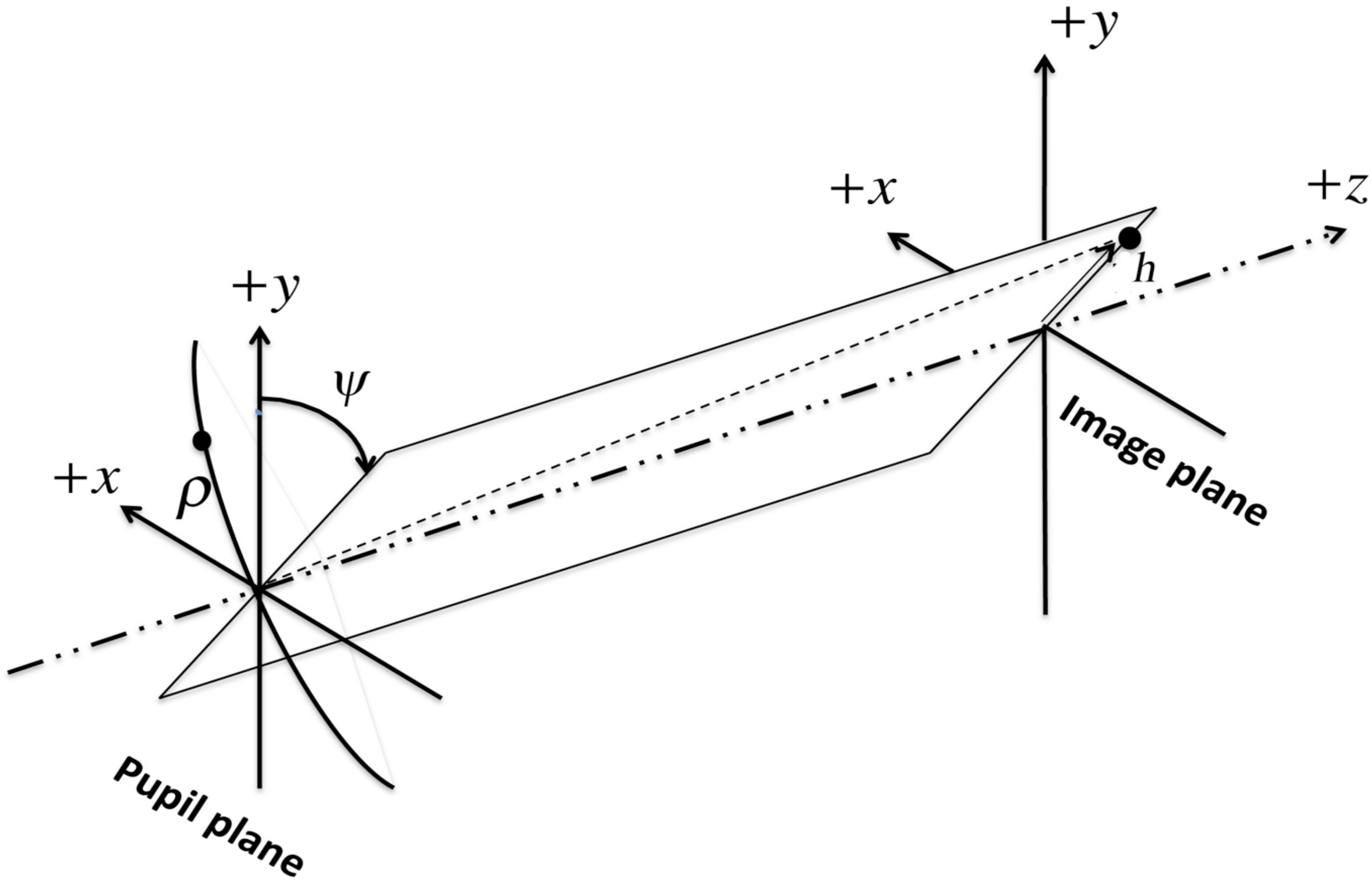
Tolerancing an optical system



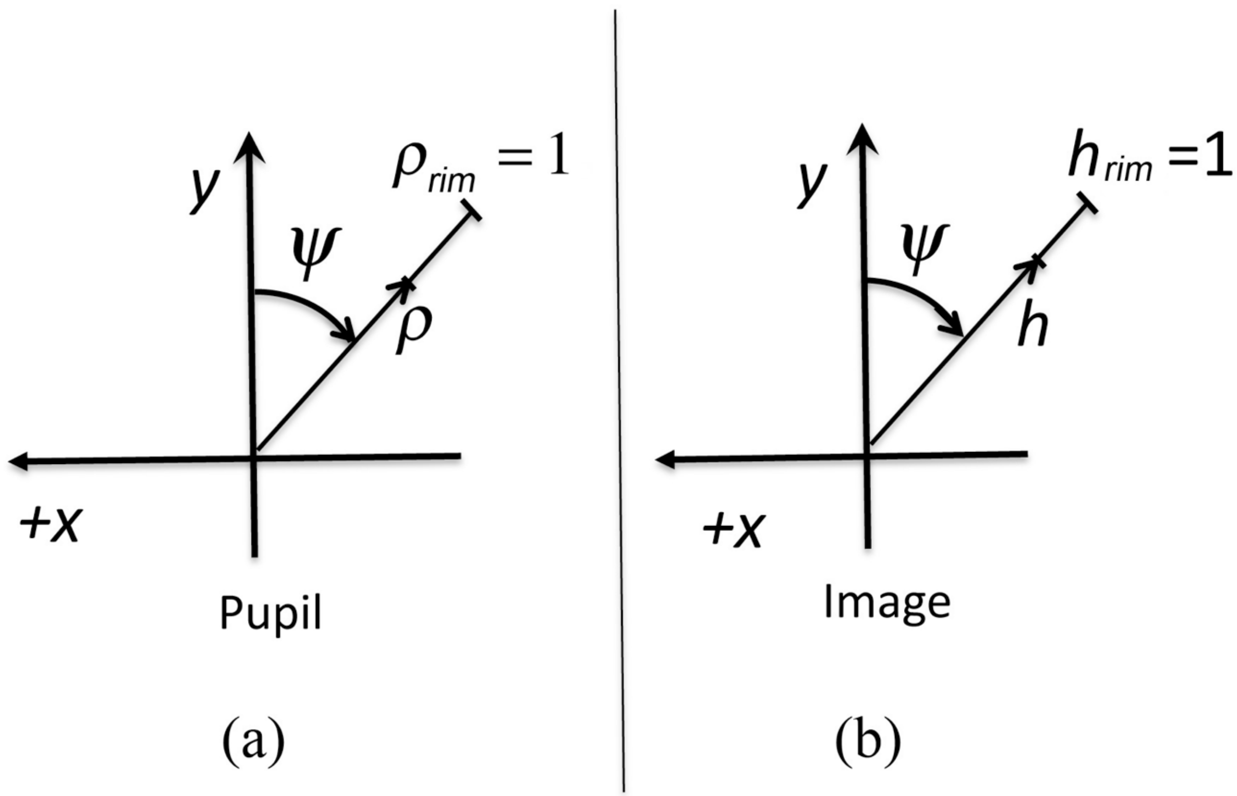


Sources of errors
Mechanical & structural

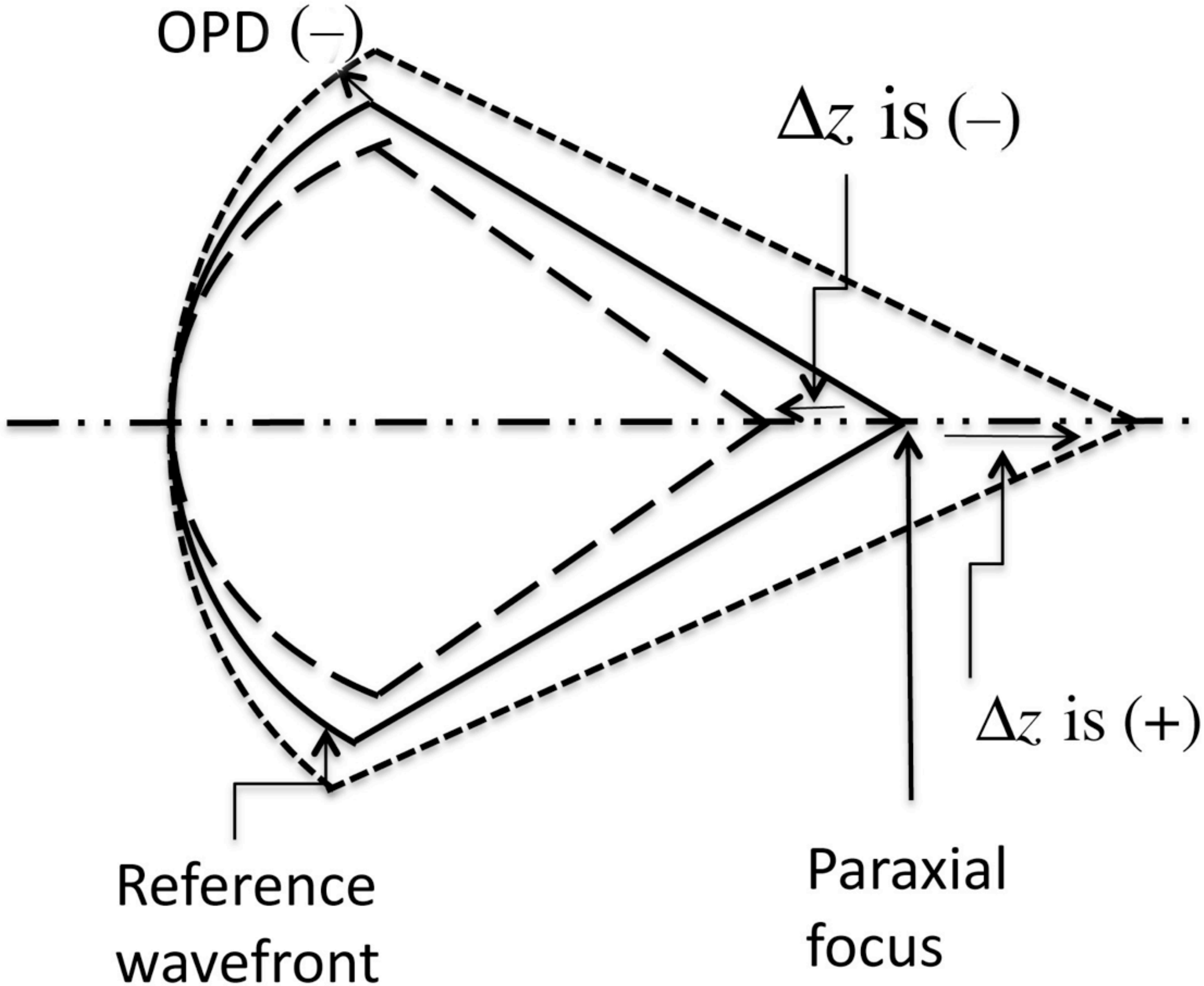
Coordinate system for geometric aberration analysis



Sign conventions

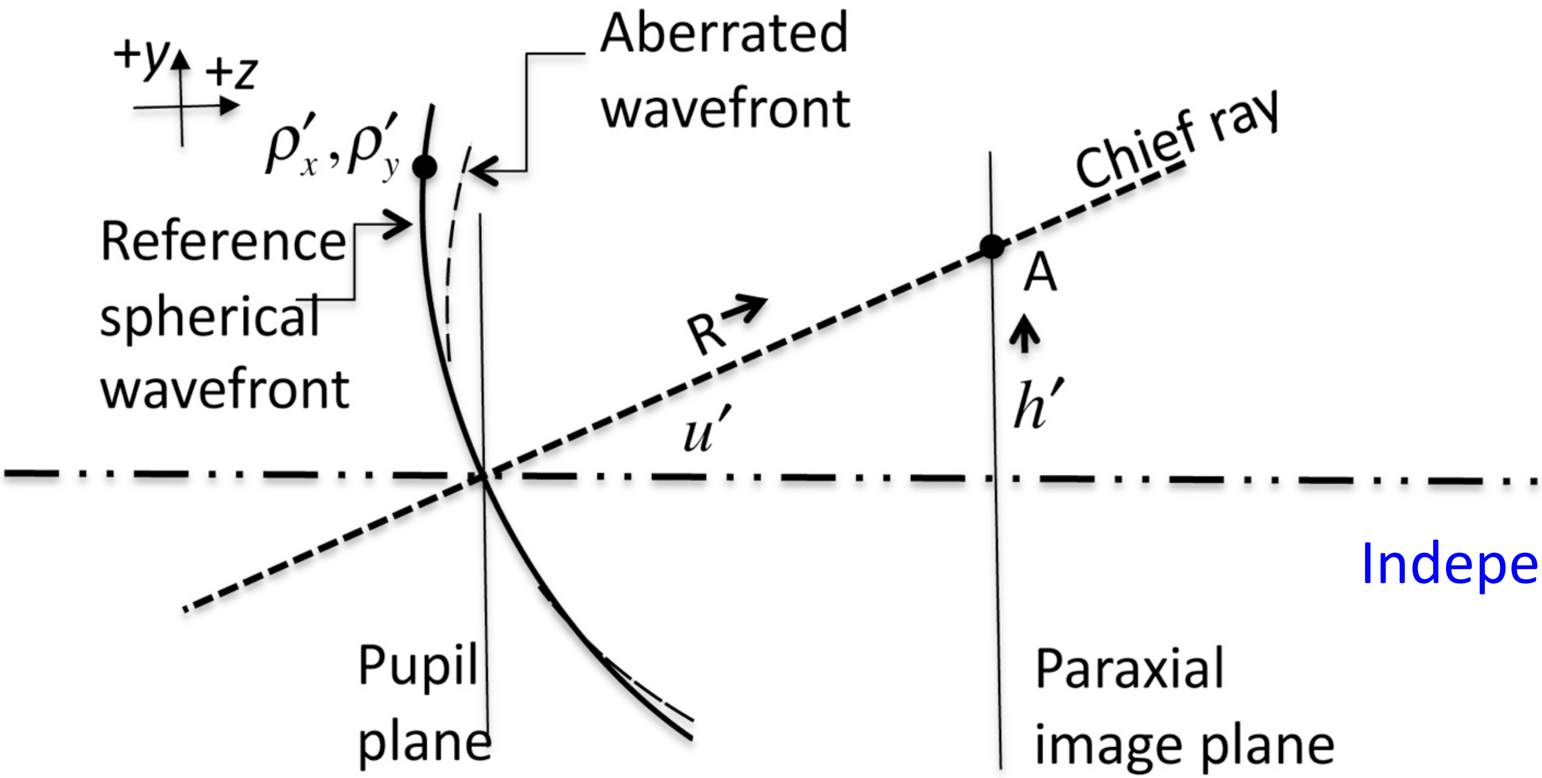


Sign convention



Rays and Geometric Waves

$$W\left(\rho'_x,\rho'_y\right)=\frac{\text{reference ray path}-\text{ray path}}{\lambda}=\frac{\text{OPD}\left(\rho'_x,\rho'_y\right)}{\lambda},$$



Independent of λ

Definition of Wavefront aberration W

The equation for a spherical wave converging to a point at a distance *f* from the pupil (lens) is

$$u'(x,y) = u(x,y) \cdot \exp\left[-j\frac{k}{2f}\left(x^2+y^2\right)\right], \qquad k = \frac{2\pi}{\lambda}$$

And *x* and *y* are Cartesian coordinates across the pupil plane.

An expression that includes the wave aberration term *W*.

$$E = -j\frac{k}{2f}\left(x^2+y^2\right)\left[1+W\left(x,y\right)\right],$$

Now what is $W(x,y)$?

Geometrical wavefront error

**The geometrical wavefront error is independent of the wavelength.
If one wanted to calculate how much material [as a function of position] is needed to be removed From a mirror surface, then one need to multiply by wavelength.**

Expand the wavefront error

- Zernike polynomials
  - Power series expansions on the unit circle
- Seidel aberrations
  - Defocus, tilt, spherical, coma, astigmatism, field curvature, Petzval

Zernike polynomials

Zernike polynomials are a set of orthogonal polynomials defined on a unit circle. They are expressed in either Cartesian (*x*, *y*) or polar (ρ, ψ) coordinates. There are both even and odd Zernike polynomials:

$$Z_n^m(\rho,\psi) = R_n^m(\rho)\cos(m\psi)$$

$$Z_n^{-m}(\rho,\psi) = R_n^m(\rho)\sin(m\psi)$$

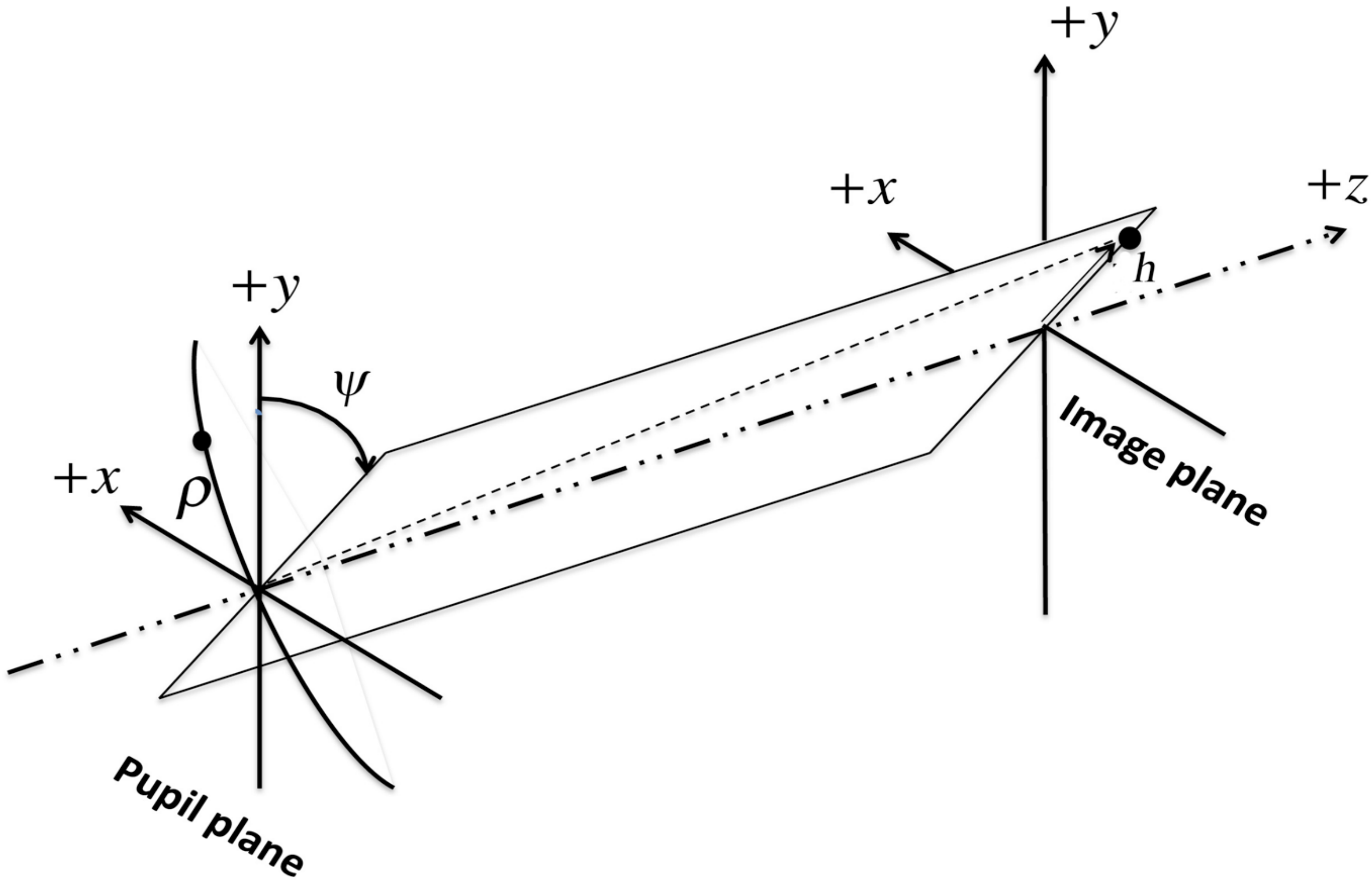
Zernike polynomials

where the radial polynomials are defined as

$$R_n^m(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k!\left[\frac{(n+m)}{2} - k\right]! \cdot \left[\frac{(n-m)}{2} - k\right]!} \rho^{n-2k}$$

We will not use the Zernike polynomials here, but rather will analyze using the Seidel aberrations, which lend themselves to an intuitive understanding of the physical origins of the aberrations.

Coordinate system for geometric aberration analysis



The wavefront series expansion for the aberrations yields the expression

$$W = \sum_{k,n,m} a_{knm} h^k \cdot \rho^n \left(\cos\psi\right)^m ,$$

By convention, first-order aberration terms are those for which $k + n - 1 = 1$,

Third-order aberration terms are those for which $k + n + 1 = 3$, and

Fifth-order aberration terms are those for which $k + n + 3 = 5$

The third-order aberration coefficients are:

a_{040}, a_{220}, a_{400}, a_{131}, a_{311}, and a_{222}.

The fifth-order aberration coefficients are:

a_{060}, a_{240}, a_{420}, a_{600}, a_{151}, a_{331}, a_{511}, a_{242}, and a_{422}.

Aberrations independent of the optics

Defocus a_{020}

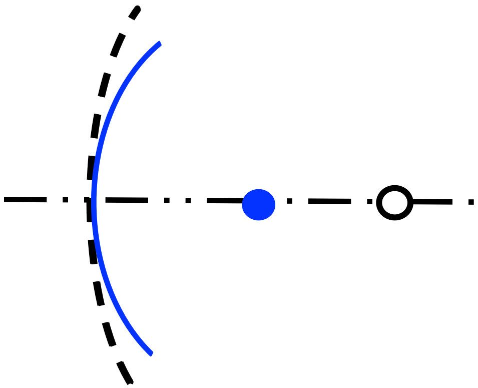
- The aberration a_{020} is introduced by an axial change in the focus and is called defocus. For defocus, the aberration is corrected by translating the image point along the axis until the sphere corresponding to the aberrated image point is superposed onto the reference sphere. This aberration, like a_{111}, is corrected by repositioning the focal plane; **it is not necessary to correct by refiguring any optical element.**

Aberrations independent of the optics

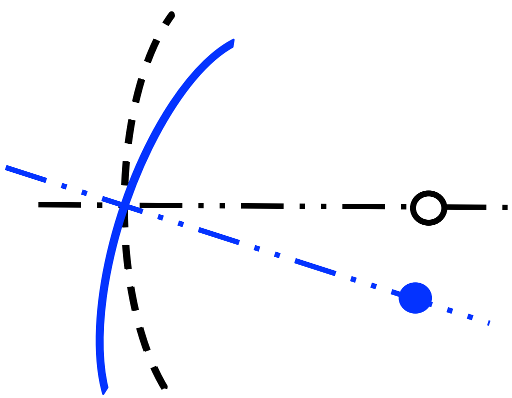
Tilt a_{111}

- The term a_{111} represents a lateral shift. The aberrated wave is tilted about the vertex of the reference wave. This aberration can be corrected by repositioning the focal plane. The aberration term a_{111} represents a tilt in the wavefront in the *y* direction. This aberration can be corrected by repositioning the focal plane; **it is not necessary to correct by refiguring any optical element.**

Defocus and Tilt



The actual surface is the dashed black sphere. Standing at the blue point looking left you have created the blue reference surface. To super-impose your reference surface to the actual surface you need to move to the right (+), or **refocus** the system to make: $a_{020} = 0$.



To super-impose your reference surface to the actual surface you need to tilt your reference surface up, or **tilt your focal plane** the system to make:

$$a_{111} = 0.$$

The 3rd order (Seidel) terms

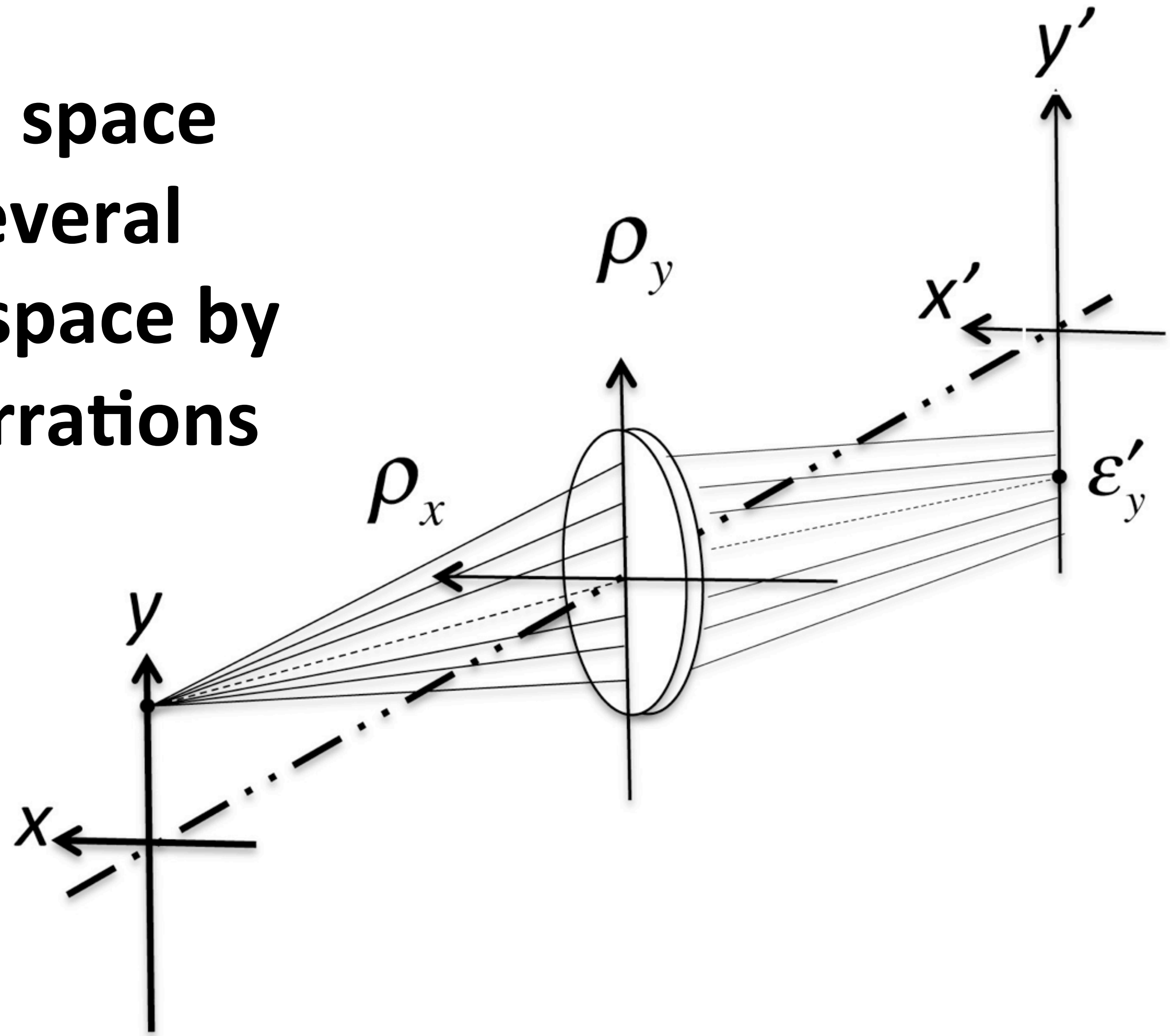
We will examine in detail the first and third order aberration

$$W = \sum_{k,n,m} a_{knm} h^k \cdot \rho^n \left(\cos\psi\right)^m$$

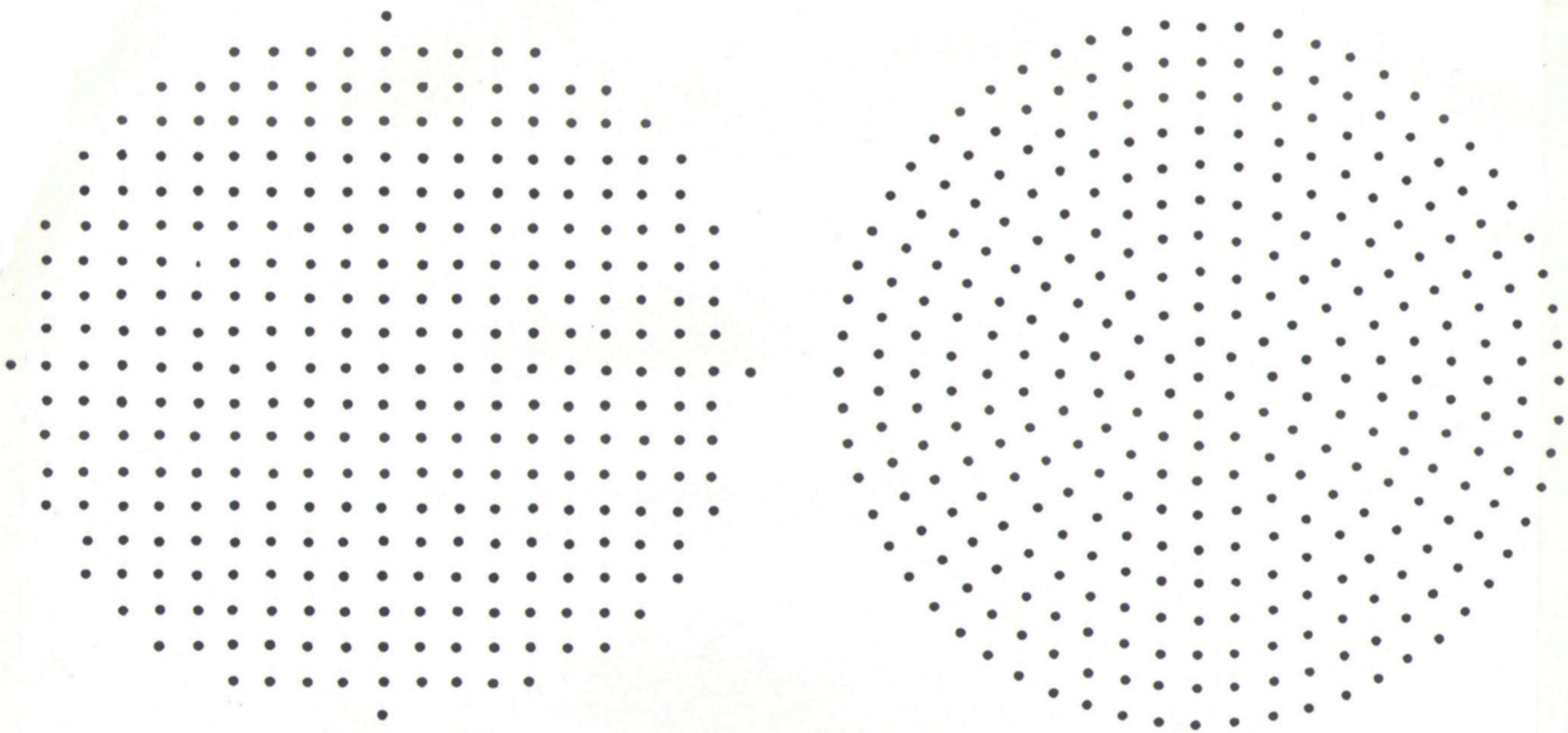
Coefficient [*k*, *l*, *m*]	Name
First order	
020	Axial (defocus)
111	Transverse (tilt)
Third order	
040	Spherical
131	Coma
222	Astigmatism
311	Distortion
400	Piston
220	Field curvature
Fifth order	
060	Fifth-order spherical
151	Fifth-order coma
242 and 240	Oblique spherical
333	Elliptical coma
422	Fifth-order astigmatism
511	Fifth-order distortion
600	Piston
420	Fifth-order field curvature

A point in object space is imaged …

A point in object space is imaged into several points in image space by a geometric aberrations



Tool for analysis: the spot diagram



Square pattern
(equally spaced rays)
(a)

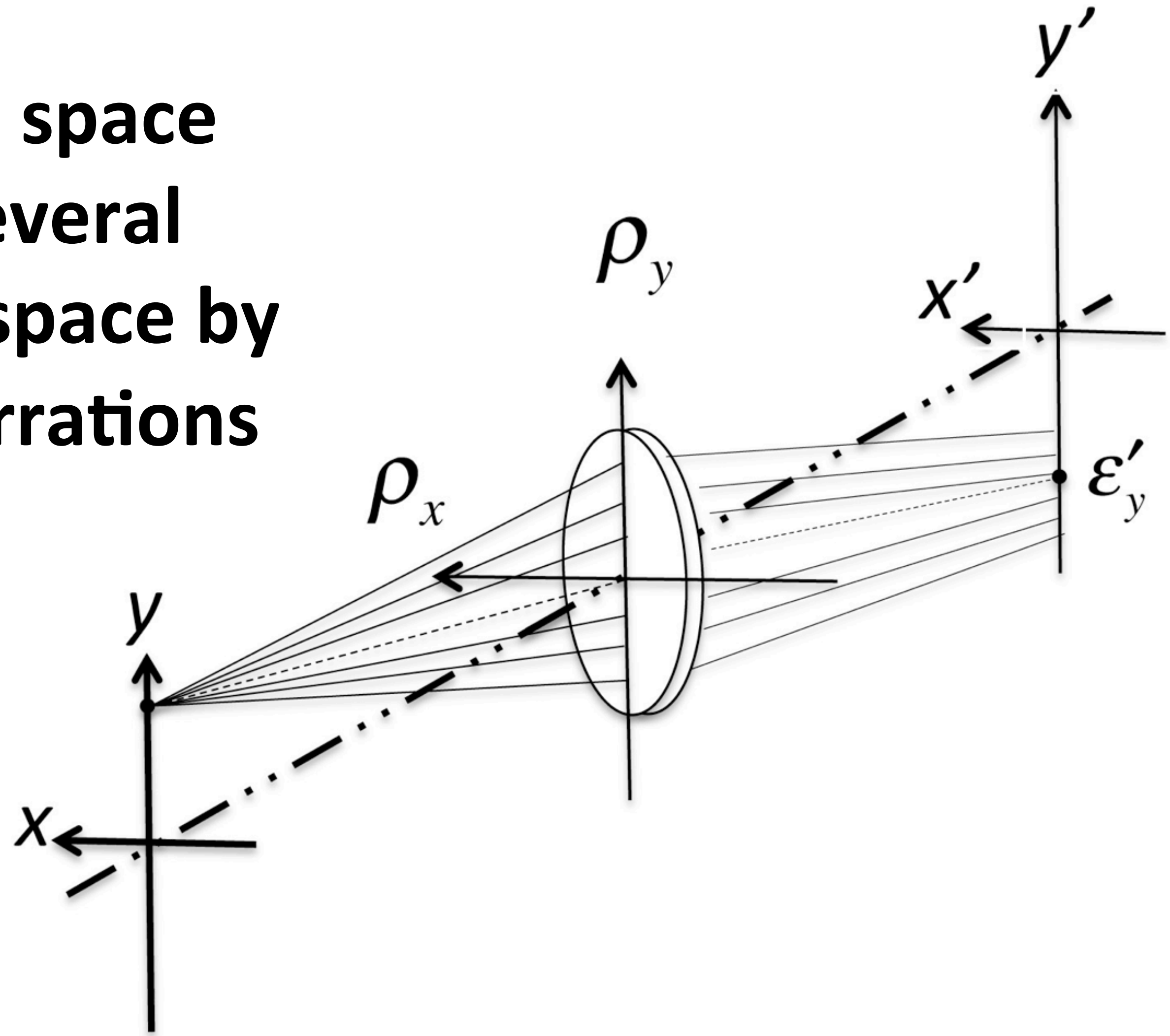
Hexapolar pattern
(b)

CAD ray-trace objectives

- **For each point [in object space]**
- Use trigonometry to trace rays from a point in object space
  - through a grid (hexapolar) on the physical surface of the lens,
  - refract and
  - then translate to the next surface until it
  - penetrates the focal plane at a point.

A point in object space is imaged …

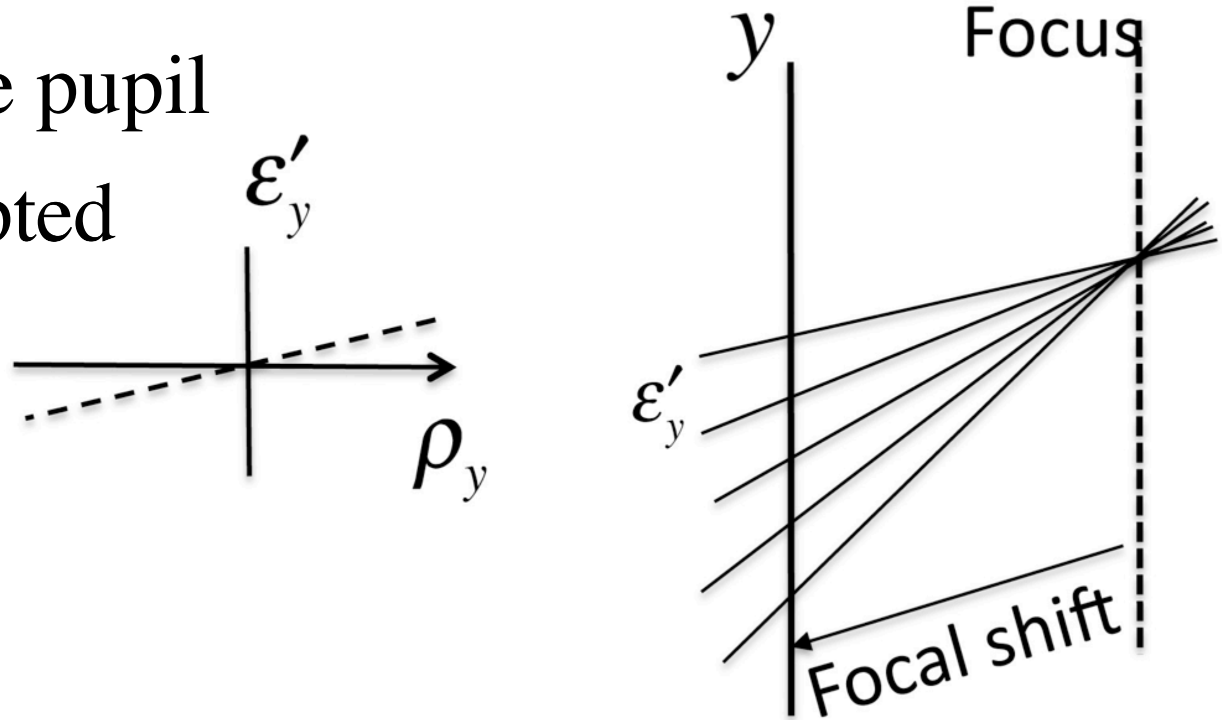
A point in object space is imaged into several points in image space by a geometric aberrations



The ray-fan plot

Plot of ε_y as a function of where on the pupil the ray intercepted it.

$$\varepsilon_y = f\left(\rho_y\right)$$



(a) (b)

CAD ray-trace objectives

- **For each point [in object space]**
- Use trigonometry to trace rays from a point in object space
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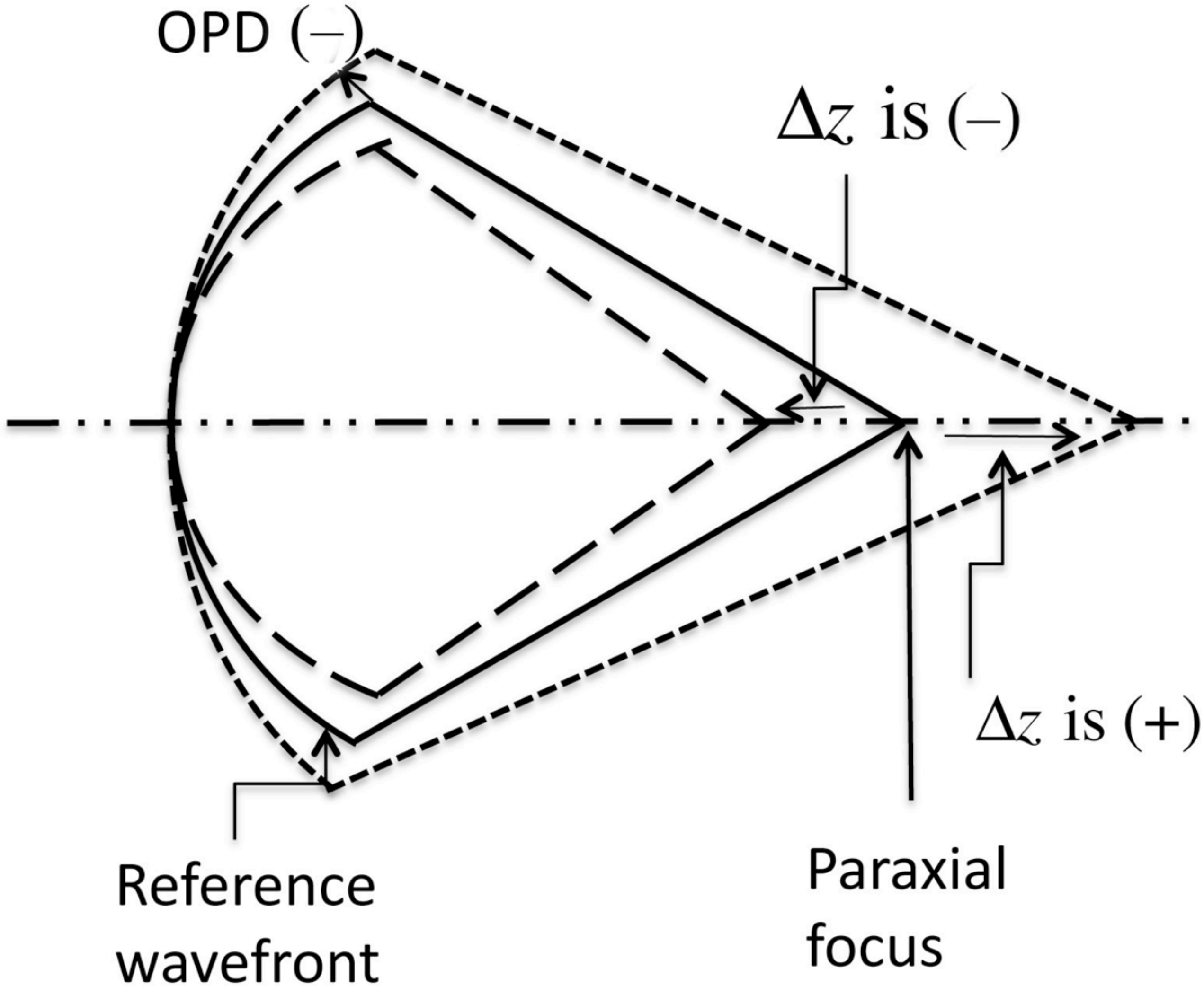
Sign convention



The third order monochromatic terms

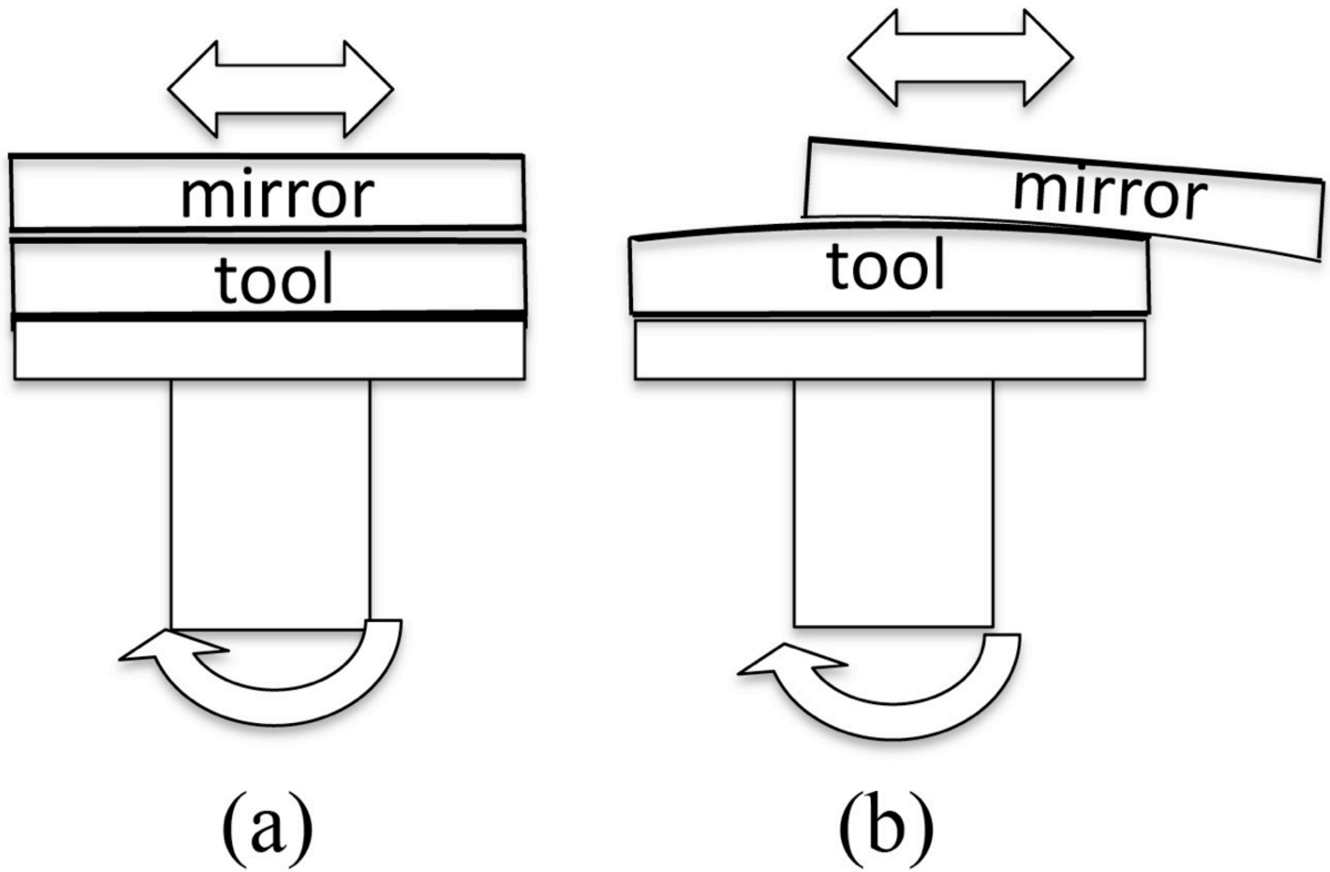
$$W =$$

$$a_{040}\rho^4 + a_{131}h\rho^3\cos\psi + a_{222}h^2\rho_2\cos 2\psi +$$

$$+a_{220}h^2\rho^2 + a_{331}h^3\rho\cos\psi$$

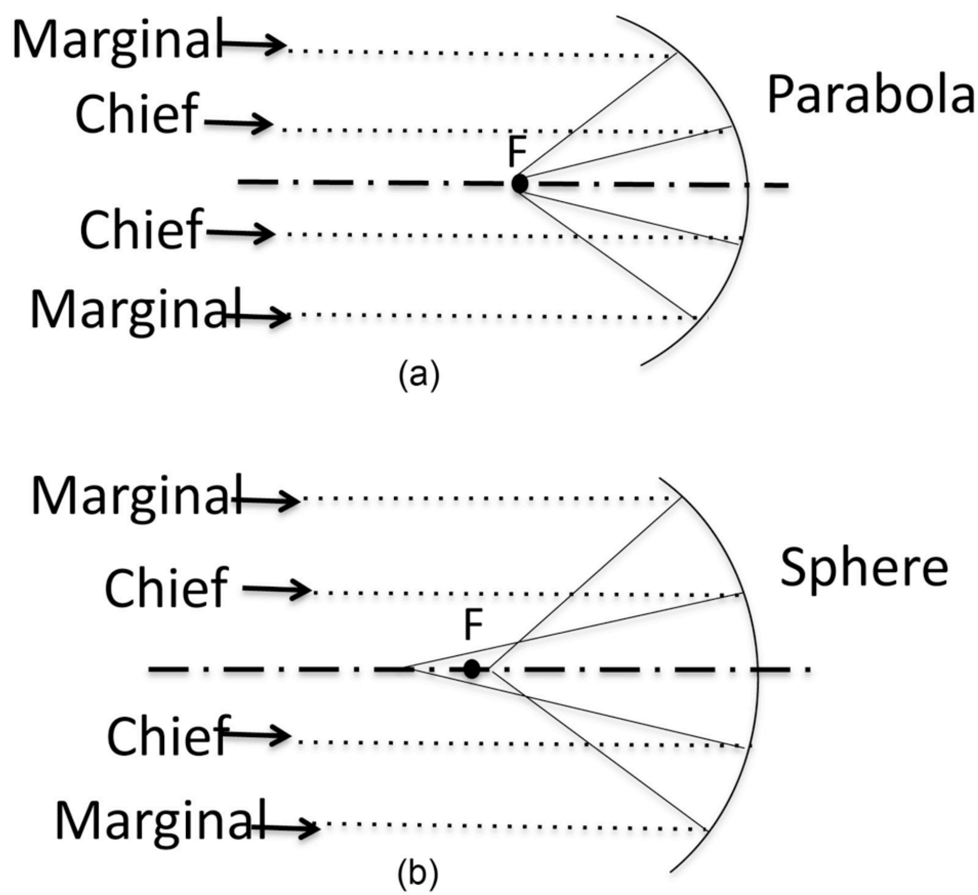
Spherical aberration

Spherical $a_{040}\rho^4$



Spherical in the presence of defocus

$$W = a_{040} \cdot \rho^4 + a_{020} \cdot \rho^2$$

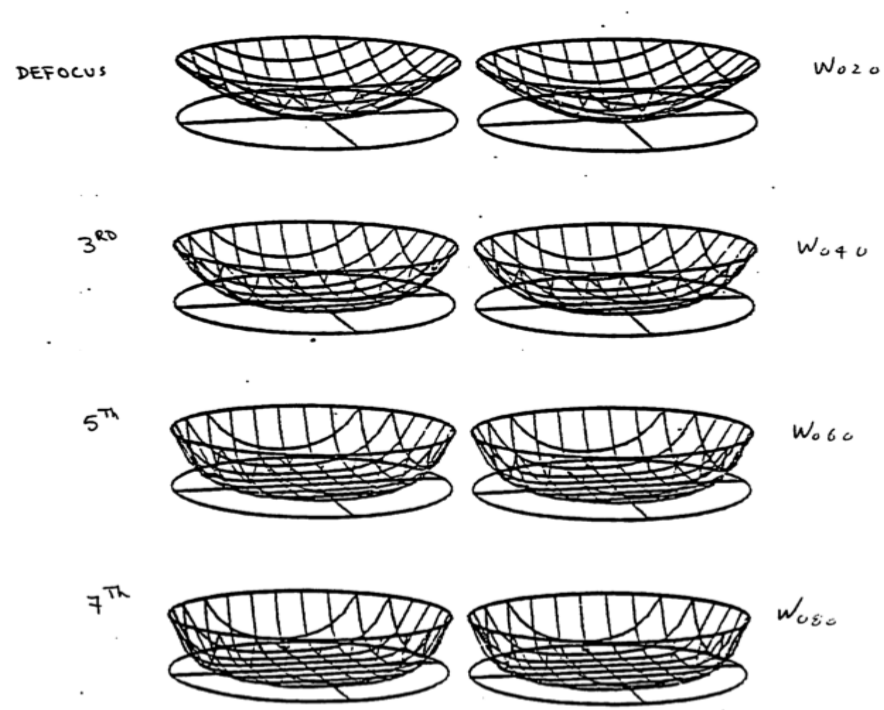


$$W = a_{020}\rho^2 + a_{040}\rho^4 + a_{060}\rho^6 + a_{080}\rho^8$$

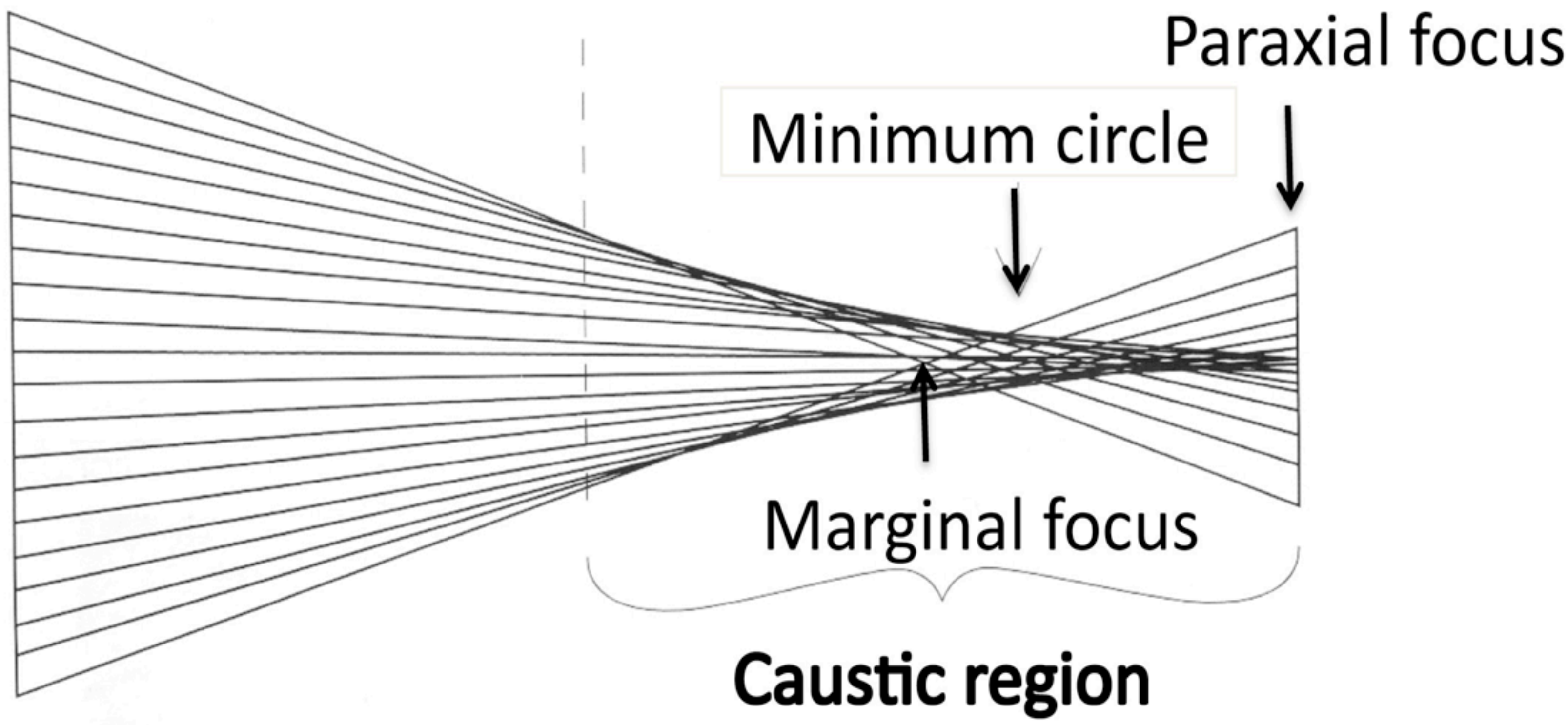
The only set of aberrations on axis for a properly aligned optical system

3 rd order spherical aberration

- spherical aberrations
  - defocus,
  - 3rd order,
  - 5th order
  - 7 th order



Spherical aberration region of the caustic



Change in the wavefront as a function of pupil position

$$\frac{\partial W}{\partial \rho_x} = \left(4a_{040}\rho_x^3 + 2a_{020}\rho_x\right)\sin\psi.$$

$$\frac{\partial W}{\partial \rho_y} = \left(4a_{040}\rho_y^3 + 2a_{020}\rho_y\right)\sin\psi$$

The radius of the circle at the focal plane is

$$\varepsilon_r = \sqrt{\varepsilon_x^2 + \varepsilon_y^2} = \frac{2R}{nr}\left(a_{040}\rho + 2a_{040}\rho^3\right).$$

$$\frac{R}{r} = \frac{2}{F^\#}$$

At the paraxial focus $a_{020}=0$

The diameter of the image is

$$d = \frac{8R}{nr} a_{040}$$

At the marginal focus $\rho = 1$ and $a_{020} = -2a_{040}$

$$\varepsilon_r = \frac{4R}{nr}\left(\rho - \rho^3\right) a_{040}. \qquad \frac{R}{r} = \frac{2}{F^{\#}}$$

At what value of ρ is ε_r a minimum

$$\frac{d\left(\varepsilon_r\right)}{d\partial} = \frac{4R}{nr}\left(1-3\rho^2\right)a_{040} = 0.$$

$$\rho^2 = \frac{1}{3}.$$

Spherical aberration spot size

Location along axis	Diameter	Distance from paraxial focus
Paraxial	$8\frac{R}{nr}a_{040}$	0.0
Minimum Circle	$2\frac{R}{nr}\cdot a_{040}$	$-3\frac{R^2}{nr^2}\cdot a_{040}$
Marginal Focus	$16\frac{\sqrt{3}}{9}\cdot\frac{R}{nr}a_{040}$	$-4\frac{R^2}{nr^2}\cdot a_{040}$

Depth of focus

The distance between the marginal and chief rays is

$$\Delta z = \frac{R^2}{nr^2}\left(4a_{040}r^2 + 2a_{020}\right)\lambda.$$

Radius of the marginal focus is

$$\varepsilon = \frac{8\sqrt{3}}{9} \cdot \frac{R}{nr} \cdot a_{040}$$

With no spherical aberration we find

$$\Delta z = \frac{R^2}{nr^2} 2a_{020}.$$

$$\Delta z = -8\left(f/\#\right)^2 a_{020}.$$

Depth of focus

$$\Delta z = -8\left(f/\#\right)^2\left(\frac{a_{020}}{\lambda}\right)\lambda,$$

The Rayleigh criterion is a quarter wave error

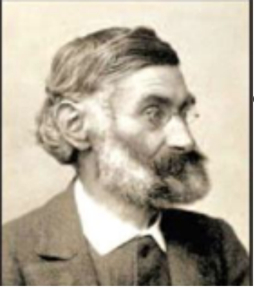
$$\frac{a_{020}}{\lambda} = \frac{1}{4},$$

The depth of focus for this criterion is therefore

$$\Delta z = \pm 2\left(f/\#\right)^2\lambda.$$

At 0.5 microns (mid visible)

$$\Delta z = \left(f/\#\right)^2 \mu\text{m}.$$



The third order monochromatic terms

$$W = a_{040}\rho^4 + a_{020}\rho^2$$

$$+a_{131}h\rho^3\cos\psi$$

$$a_{222}h^2\rho^2\cos 2\psi +$$

$$a_{220}h^2\rho^2$$

$$+a_{311}h^3\rho\cos\psi$$

We just finished looking at Spherical aberration as a function of defocus

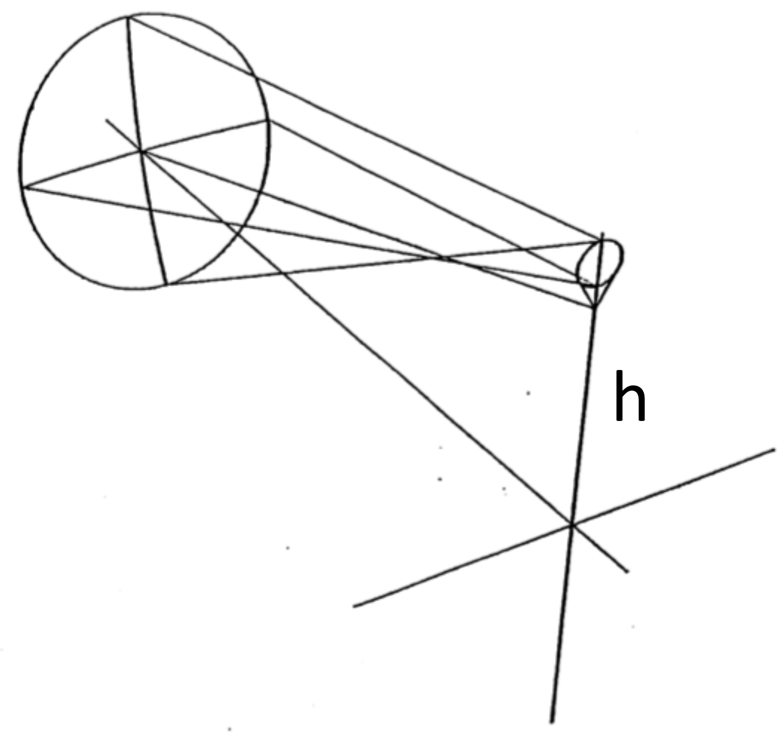
Coma

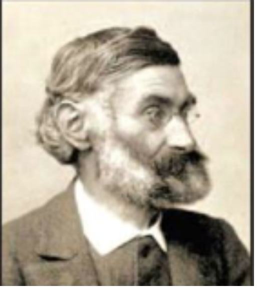
Examine the behavior of the term

$$W = a_{131}h\rho^3\cos\psi.$$

Coma is a field aberration

- As you move off axis the exit pupil appears shorter in the y direction than it does in the x direction





Abbe

Coma

$$W = a_{131} h \rho^3 \cos\psi .$$

To see the way in which this maps to the image plane, we calculate the slopes of the wavefronts-

Remember the focal plane maps slopes (angles)

$$\frac{\partial W}{\partial \rho_x} = a_{131} h \left(2 \rho_x \rho_y\right) = a_{131} h \rho^2 \left(2 \cos\psi \sin\psi\right).$$

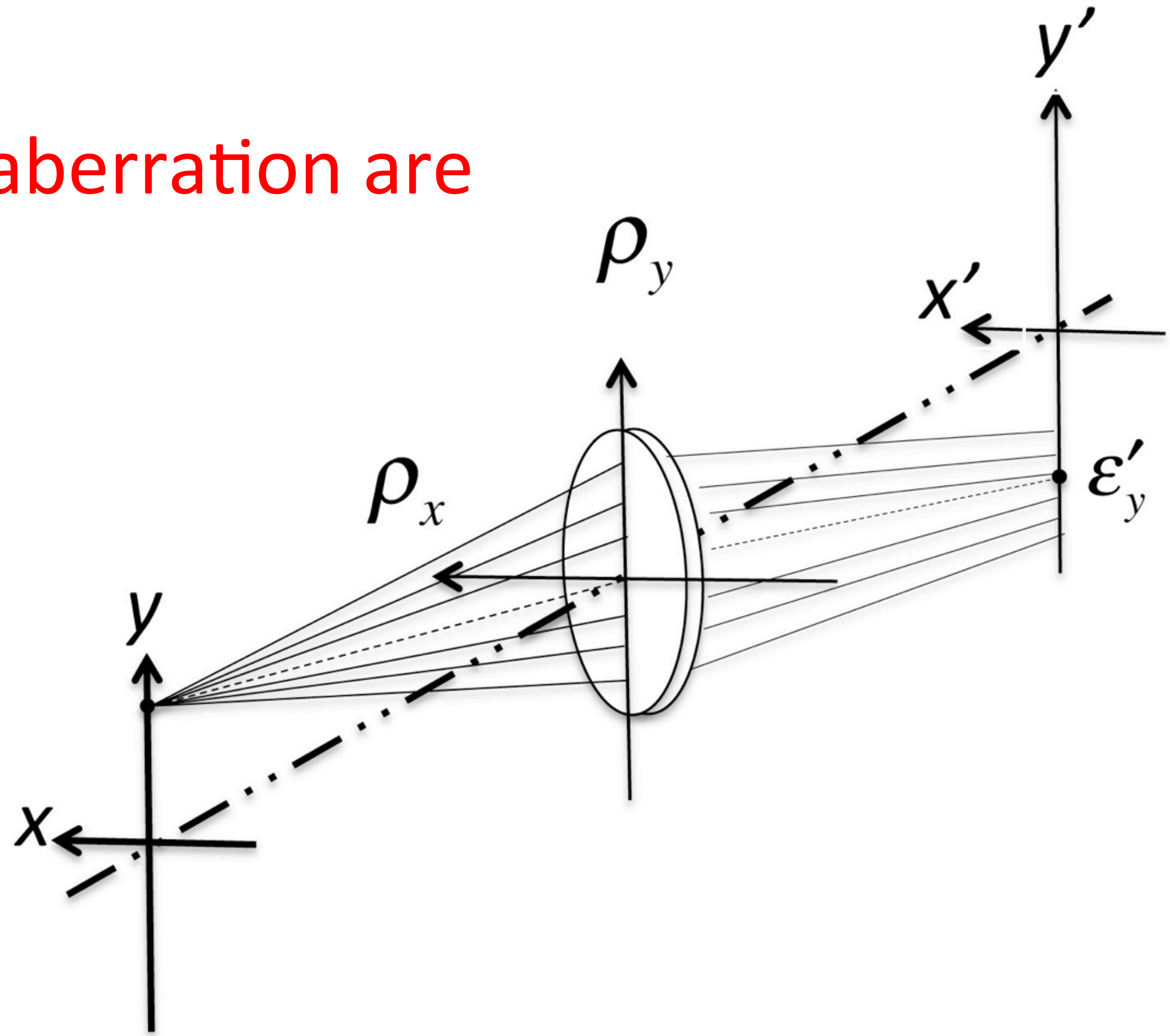
$$\frac{\partial W}{\partial \rho_x} = a_{131} h \rho^2 \sin\left(2\psi\right).$$

$$\frac{\partial W}{\partial \rho_y} = a_{131} h \left(\rho_x^2 + 3\rho_y^2\right) = a_{131} h \rho^2 \left(2 + \cos 2\psi\right).$$

A point in object space is imaged into many points in image space

Transverse ray aberration are

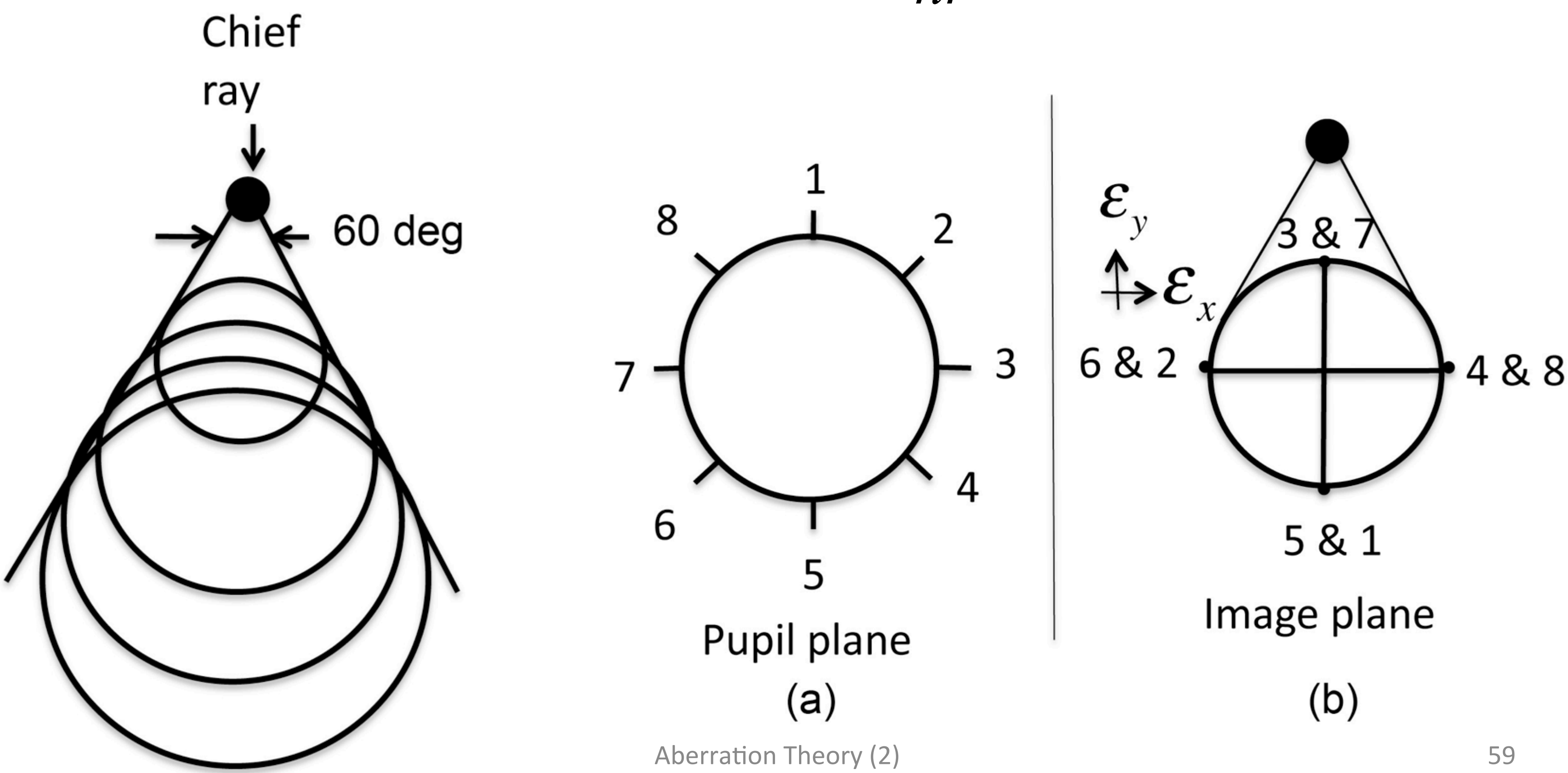
$$\varepsilon_x', \varepsilon_y'$$



Coma $W = a_{131} h \rho^3 \cos\psi.$

The transverse ray aberrations are then

$$\varepsilon_x = -\frac{R}{nr}\left(a_{131} h \rho^2 \sin 2\psi\right), \quad \varepsilon_y = -\frac{R}{nr}\left[a_{131} h \rho^2 \left(2 + \cos 2\psi\right)\right].$$



Coma

Now lets examine coma in the presence of defocus

$$W = a_{020}\rho^2 + a_{131}h\rho^3\cos\psi$$

Then we find the ray intercept at the focal plane is

$$\varepsilon_x = -\frac{R}{nr}\left[2a_{020}\rho\cdot\sin\psi + a_{131}h\rho^2\sin(2\psi)\right]$$

Coma

$$\varepsilon_x = -\frac{R}{nr}\left[2a_{020}\,\rho\sin\psi + a_{131}\,h\rho^2\sin\left(2\psi\right)\right]$$

$$\varepsilon_y = -\frac{R}{nr}\left[a_{131}\,h\rho^2 + \left(2a_{020}\rho + 2a_{131}h\rho^2\cos\psi\right)\cos\psi\right].$$

$$a_{131}\,h\rho^3\cos\psi = a_{131}h\left(\rho_x^2\rho_y + \rho_y^3\cos\psi\right)$$

This aberration is clearly not rotationally symmetric

Coma

The sign on the coma term affects its appearance

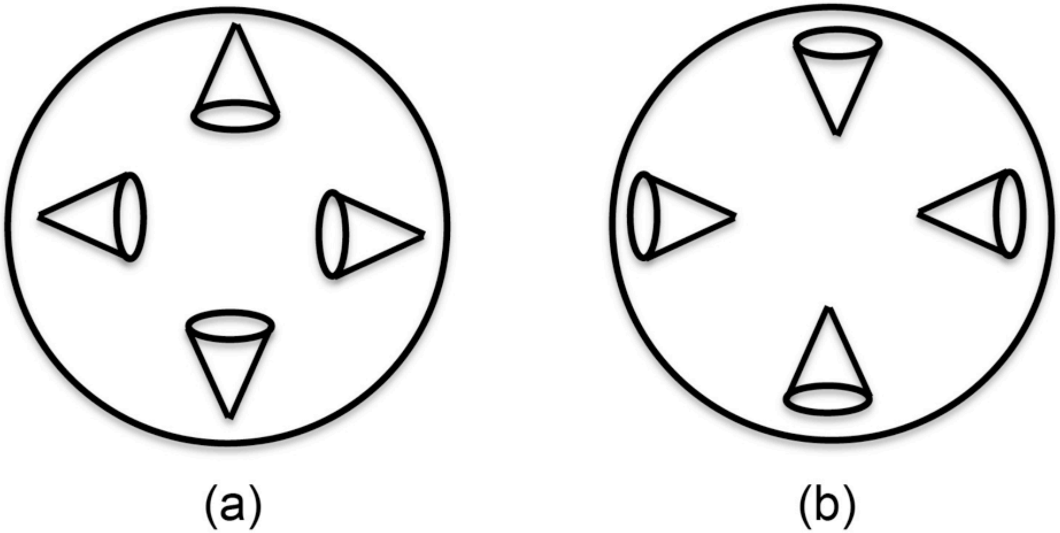
After calculating the the ray intercept,
the displacements in the image plane are given by

$$\varepsilon_x = \frac{R}{nr}\left(-a_{131}\, h\rho^2 \rho_y\right) \qquad \varepsilon_y = \frac{R}{nr}\left(-a_{131}\, h\rho_x^2 + 3\rho_y^3\right).$$

If we let $C = \frac{R}{nr} a_{131}\, h,$ then

$$\varepsilon_x = C\rho^2 \sin 2\psi \qquad \varepsilon_y = C\rho^2\left(2 + \cos\psi\right).$$

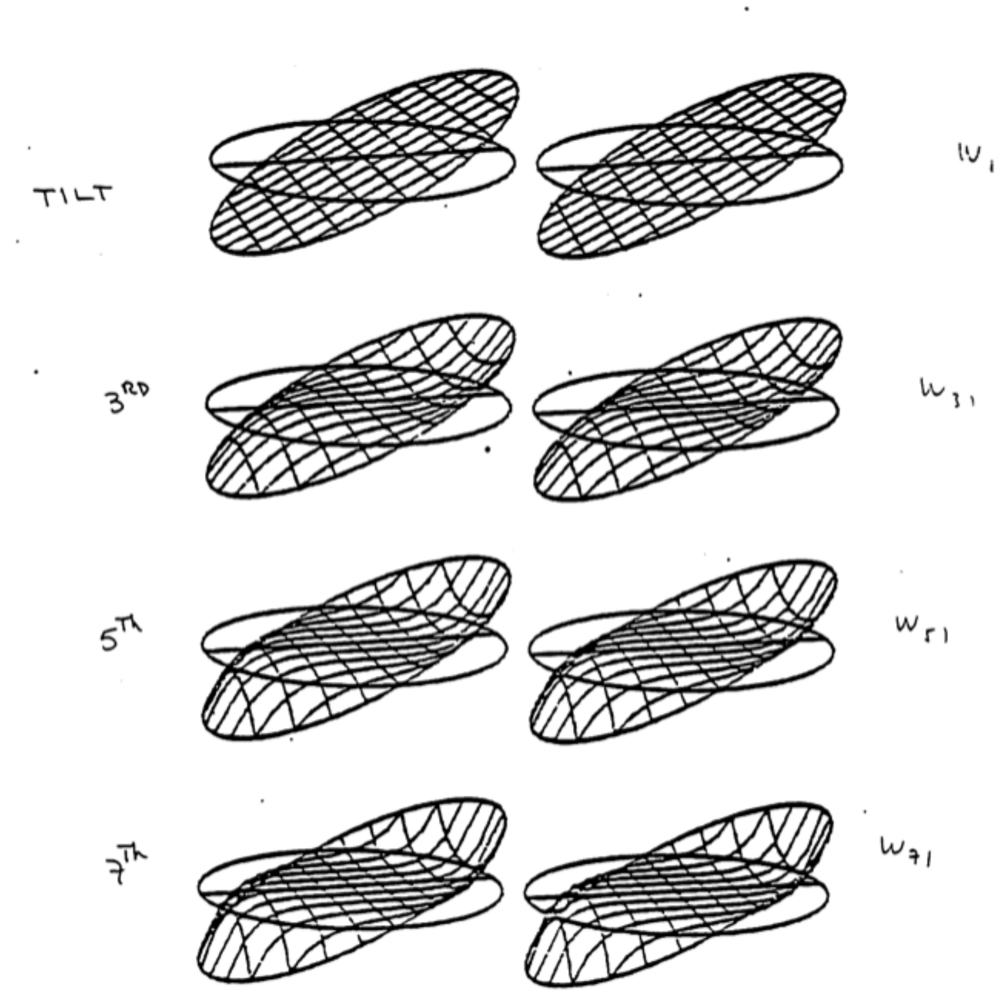
Coma



The image plane appearance for (a) positive coma in the image plane on the left and (b) negative coma in the image plane on the right.

Coma related aberrations

- Coma
  - –Tilt a_{11}
  - –3rd order a_{31}
  - –5th order a_{51}
  - –7th order a_{71}



The third order monochromatic terms

$$W = a_{040}\rho^4$$

$$a_{020}\rho^2 + a_{131}h\rho^3\cos\psi +$$

$$a_{222}h^2\rho^2\cos^2\psi +$$ Field curvature

$$a_{220}h^2\rho^2$$ Astigmatism

$$+a_{311}h^3\rho\cos\psi$$

Layout the next three VG

Investigate field curvature and astigmatism in the presence of defocus

$$W = a_{020}\rho^2 + a_{222}h^2\rho^2\cos^2\psi + a_{220}h^2\rho^2.$$

In the meridional plane $\psi = 0 \Rightarrow \cos\psi = 1$

$$W = a_{222}h^2y^2 + a_{220}h^2y^2 + a_{020}y^2.$$

From the first two terms in this equation, we see that if we add a little field curvature, that is, $a_{222} \neq 0$ such that $a_{220} = -a_{222}$, and with $a_{020} = 0$, we have a cylinder that is curved in y direction and flat in the x direction:

$$W = -a_{020}h^2\rho_x^2.$$

Source of Astigmatism

If one sits at the focal plane, at a point off axis, looking back into the exit pupil, one sees that the optical power (lens curvature) for this tilted lens (off axis) in the meridional plane becomes less than that in the $\psi = 90$ deg or the x-z plane. It is this difference in optical power that gives rise to astigmatism.

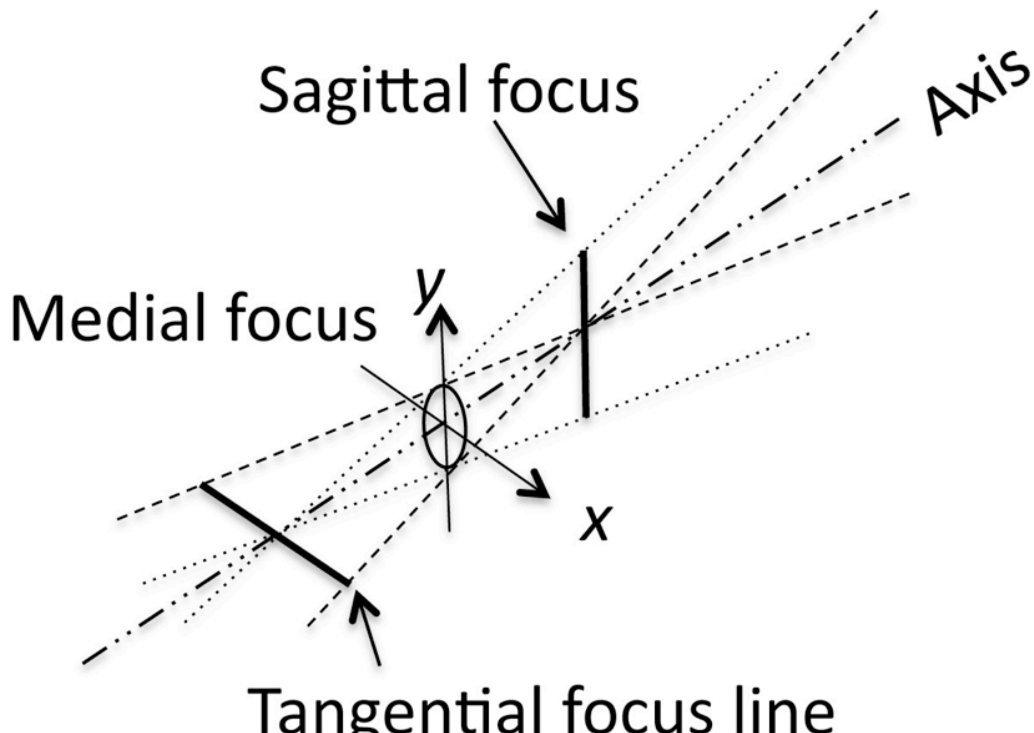
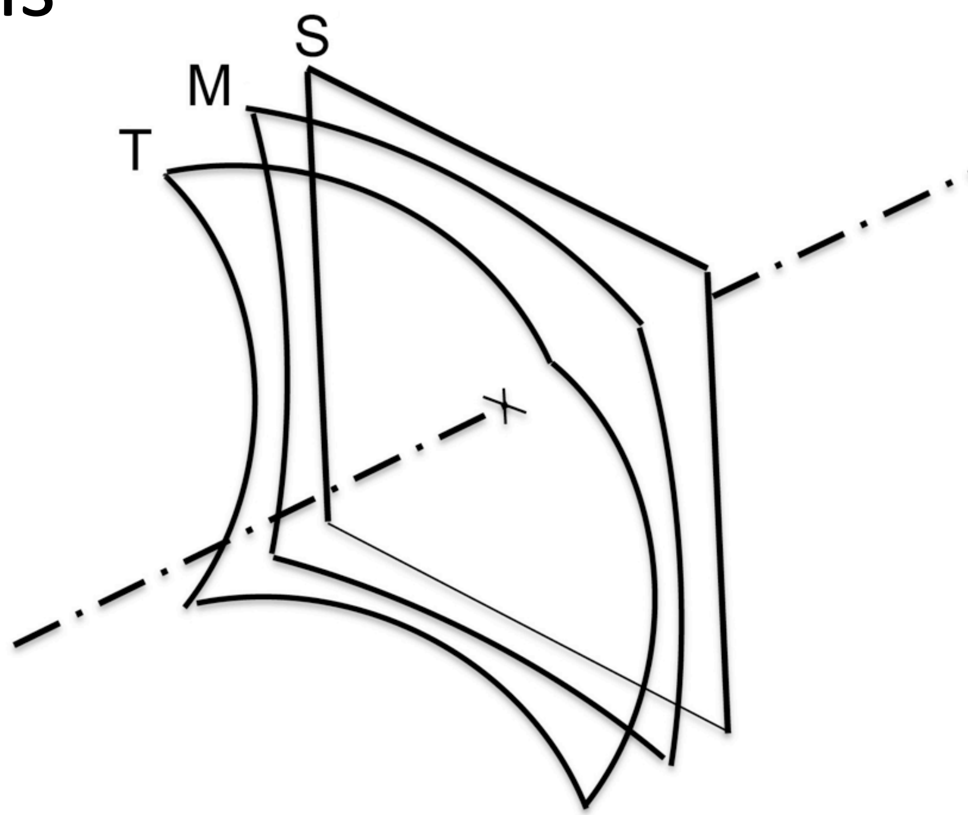
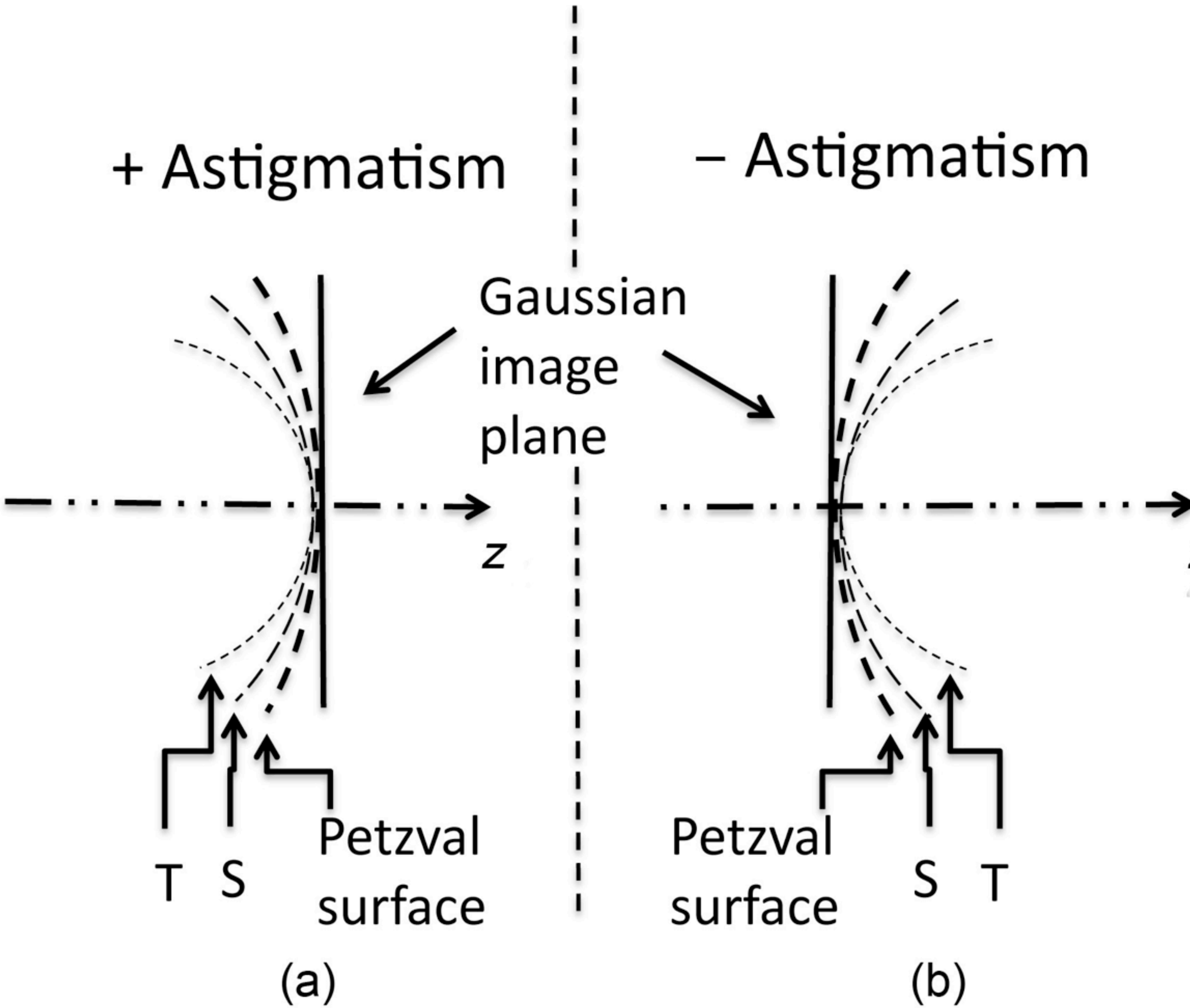


Image plane in the presence of astigmatism

- The image plane falls on three surfaces:
- Tangential
- Medial
- Sagittal



Petzval (field) Curvature



Petzval (field) Curvature

Field Curvature

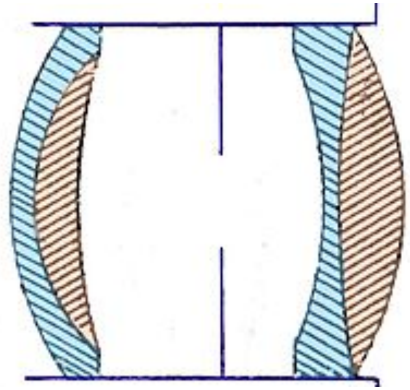
$$W = a_{020}\rho^2 + a_{222}h^2\rho^2\cos^2\psi + a_{220}h^2\rho^2.$$

The best focus as a function of field point will, in general, lie on a paraboloidal surface

If the astigmatism is zero, the image, in general, falls on a curved surface

- Petzval developed the design methods to eliminate field curvature; thus
- **Photography with flat emulsion-coated glass plates was made practical**

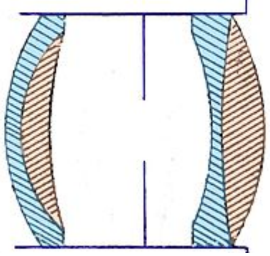
Petzval (field) Curvature



It is possible to eliminate field curvature by using two separated elements, positioning the stop properly, and satisfying the Petzval condition:

$$n_1 f_1 + n_2 f_2 = 0,$$

$$\frac{\phi_1}{n} + \frac{\phi_2}{n} + \frac{\phi_3}{n} + \frac{\phi_4}{n} + \frac{\phi_5}{n} + ... = 0$$



Petzval (field) Curvature

To obtain an estimate for the geometric spot size, we now calculate the ray intercept plot for defocus, astigmatism, and field curvature.

$$W = a_{020}\rho^2 + a_{222}h^2\rho^2\cos^2\psi + a_{220}h^2\rho^2.$$

$$\varepsilon_x = -\frac{\partial W}{\partial x} = -\frac{2R}{nr}\left(a_{020}\rho + a_{220}h^2\right)\rho\sin\psi.$$

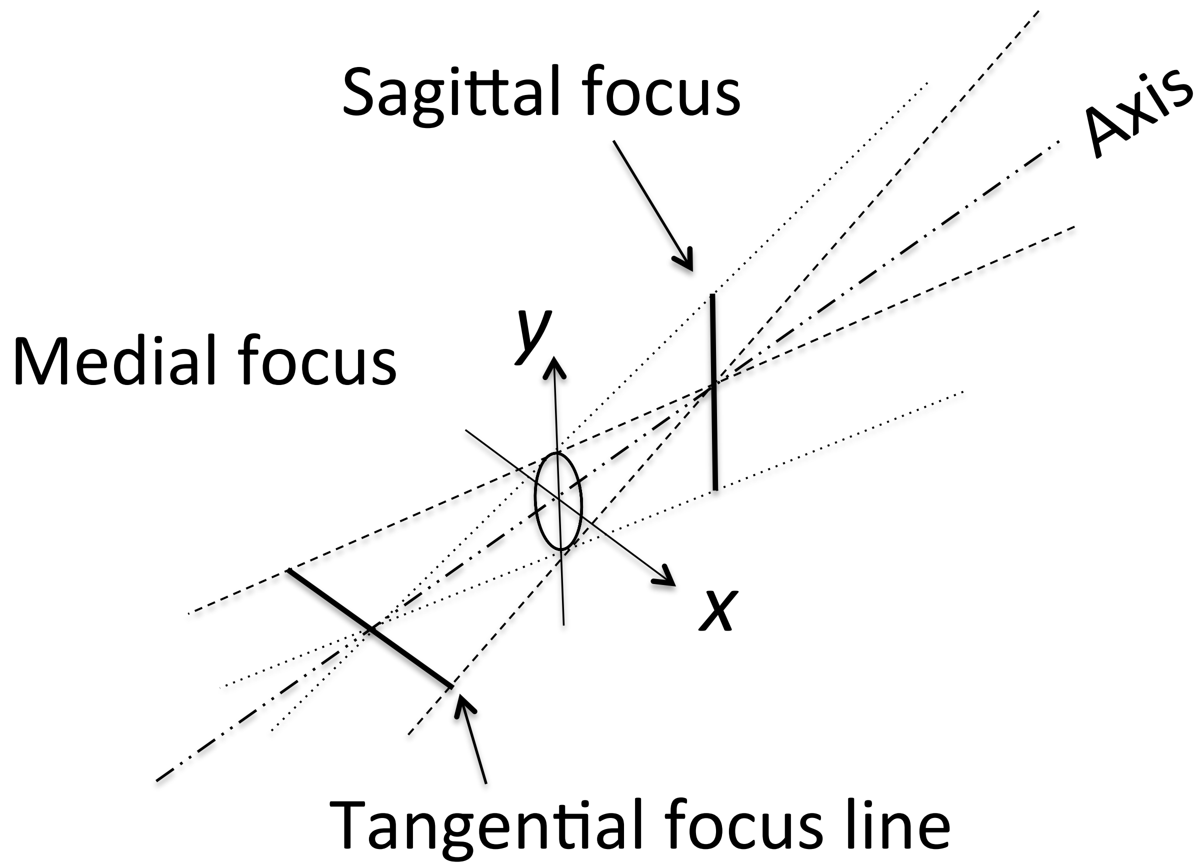
$$\varepsilon_y = -\frac{\partial W}{\partial y} = -\frac{2R}{nr}\left[a_{020} + \left(a_{220} + a_{222}\right)h^2\right]\rho\cos\psi$$

Field Curvature

Recognizing that $y = \cos\psi$ and using $\sin^2\psi + \cos^2\psi = 1$, we find that the zonal image is an ellipse centered on the chief ray, given by

$$\frac{\varepsilon_x^2}{\left[\left(2R/nr_c\right)\left(a_{020}+a_{220}h^2\right)r\right]^2}+\frac{\varepsilon_y^2}{\left\{\left(2R/nr_c\right)\left[a_{020}+\left(a_{220}+a_{222}\right)h^2\right]r\right\}^2}=1.$$

Astigmatism



Field Curvature and Astigmatism

Tangential Focus

$$W = a_{020}\rho^2 + a_{222}h^2\rho^2\cos^2\psi + a_{220}h^2\rho^2.$$

By definition, the Tangential focus condition is when

$$a_{020} = -\left(a_{220} + a_{222}\right)h^2.$$

In this case

$$\varepsilon_x = -\frac{2R}{nr}a_{222}h^2\rho\sin\psi, \;\; \varepsilon_y = 0$$

The full length, *d* of the tangential image is

$$d = 2\varepsilon_y = \frac{4R}{nr}a_{222}h^2.$$

$$\frac{R}{r} = \frac{2}{F^{\#}}$$

Field Curvature and Astigmatism

Medial Focus

$$W = a_{020}\rho^2 + a_{222}h^2\rho^2\cos^2\psi + a_{220}h^2\rho^2.$$

By definition, the medial focus condition is when

$$a_{020} = -\left(a_{220} + \frac{1}{2}a_{222}\right)h^2$$

$$\varepsilon_x = +\frac{2R}{nr}a_{222}h^2\rho\sin\psi.$$

$$\varepsilon_y = +\frac{2R}{nr}a_{222}h^2\rho\cos\psi$$

Field Curvature and Astigmatism

Sagittal Focus

$$W = a_{020}\rho^2 + a_{222}h^2\rho^2\cos^2\psi + a_{220}h^2\rho^2.$$

By definition, the Sagittal focus condition is when

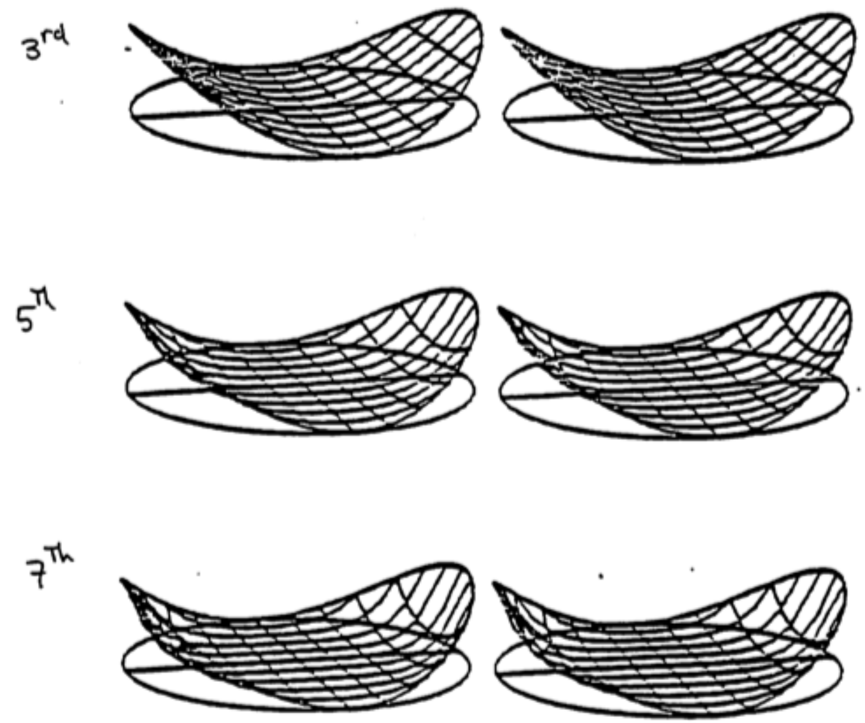
$$a_{020} = -a_{220}h^2.$$

In this case

$$\varepsilon_y = -\frac{2R}{nr}a_{222}h^2\rho\cos\psi, \text{and } \varepsilon_x = 0$$

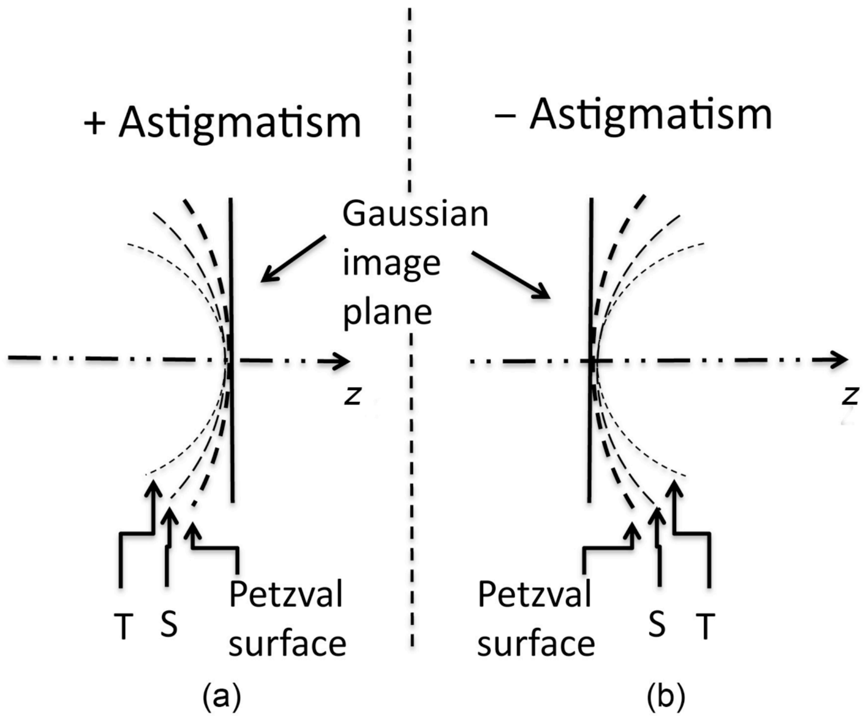
Astigmatism aberrations

- Astigmatism
  - 3rd order a_{22}
  - 5th order a_{42}
  - 7th order a_{62}

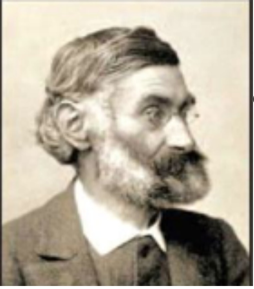


Field Curvature (review)

An in-focus "image" lies on a curved surface



The designer, by changing the power on optical surfaces can control astigmatism, but not completely depending on what the other requirements are.



The third order monochromatic terms

$$W = a_{020}\rho^4$$

$$a_{020}\rho^2 + a_{131}h\rho^3\cos\psi$$

$$a_{222}h^2\rho^2\cos 2\psi +$$

$$a_{220}h^2\rho^2$$

$$+a_{311}h^3\rho\cos\psi$$

Distortion

Examine the behavior of the term

$$W = a_{311}h^3\rho\cos\psi.$$

Distortion $W = a_{311} h^3 \rho \cos\psi.$

Distortion in the presence of tilt is

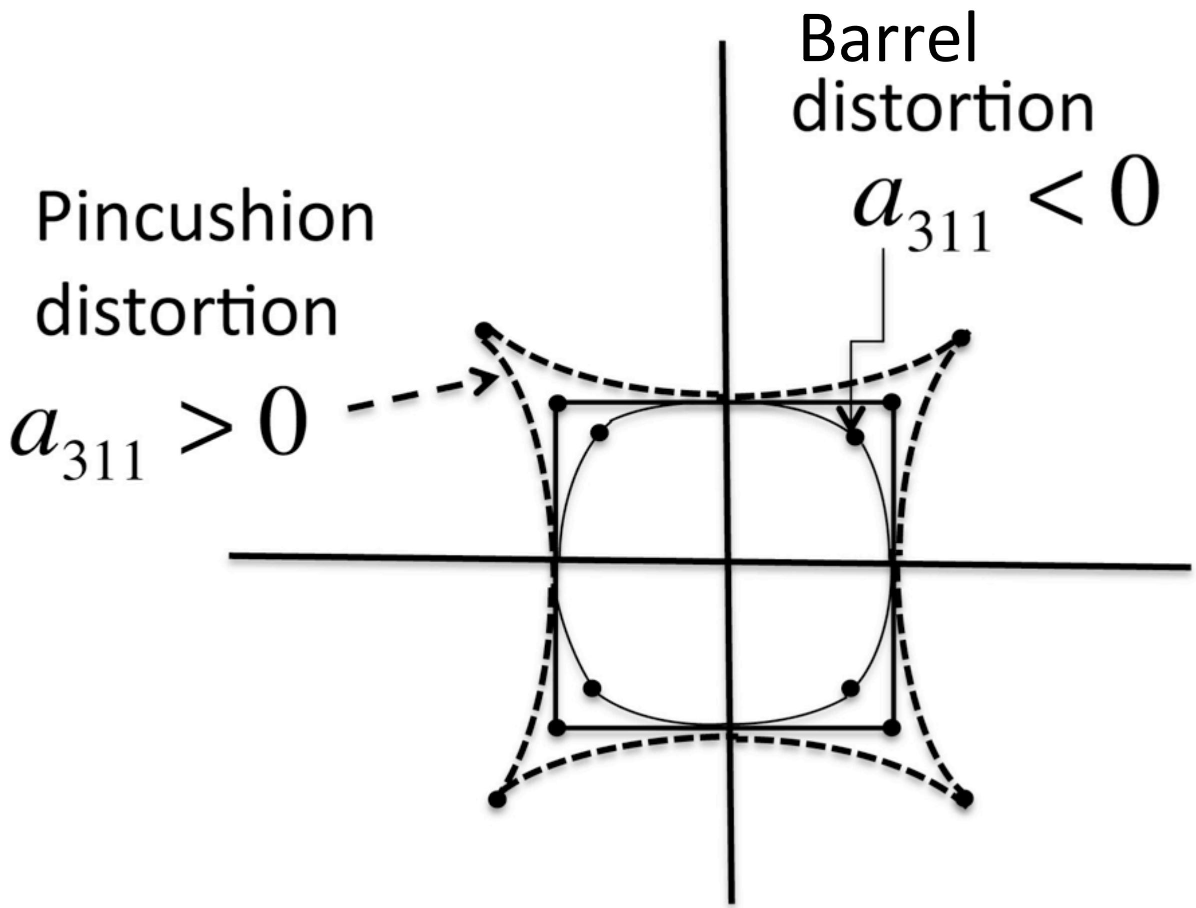
$$W = \left(a_{111} + a_{311} \cdot h^3 \right) \rho \cdot \cos\psi$$

The wavefront error *W* equals zero provided that

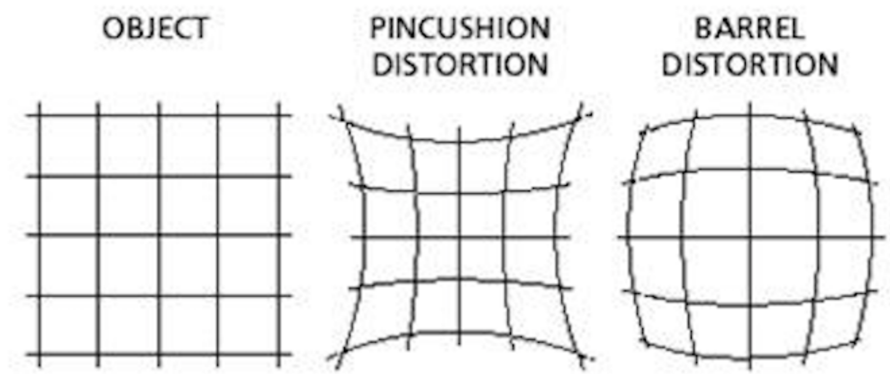
$$a_{111} = -a_{311} \cdot h^3$$

$$\varepsilon_y = -\frac{R}{nr} a_{311} \cdot h^3, \varepsilon_x = 0$$

Distortion rescales object space as a function of field point



Distortion rescales object space as a function of field point



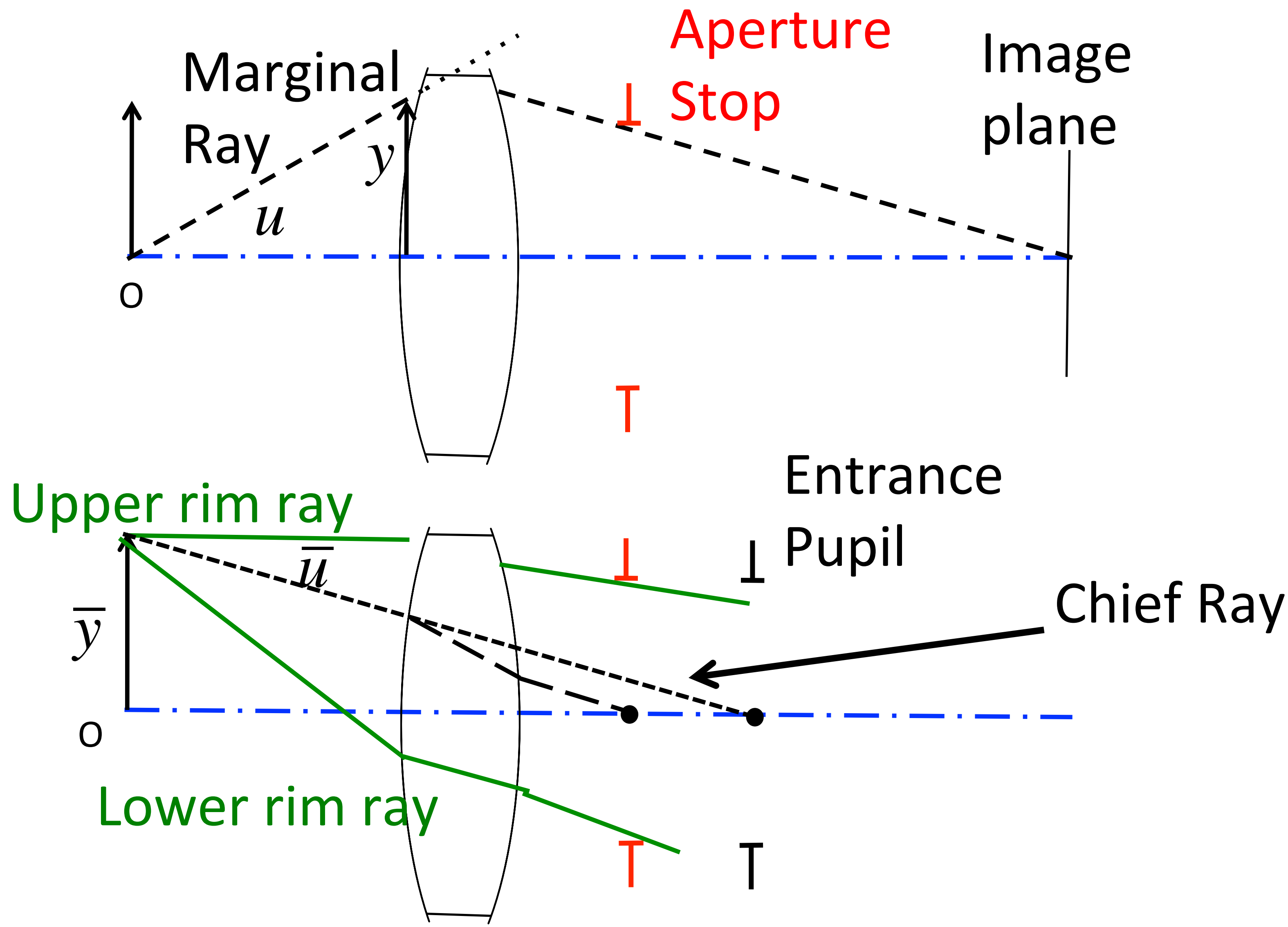
Pincushion shifts the image position a distance

$$\varepsilon_y = -\frac{R}{h} a_{311} y^3$$

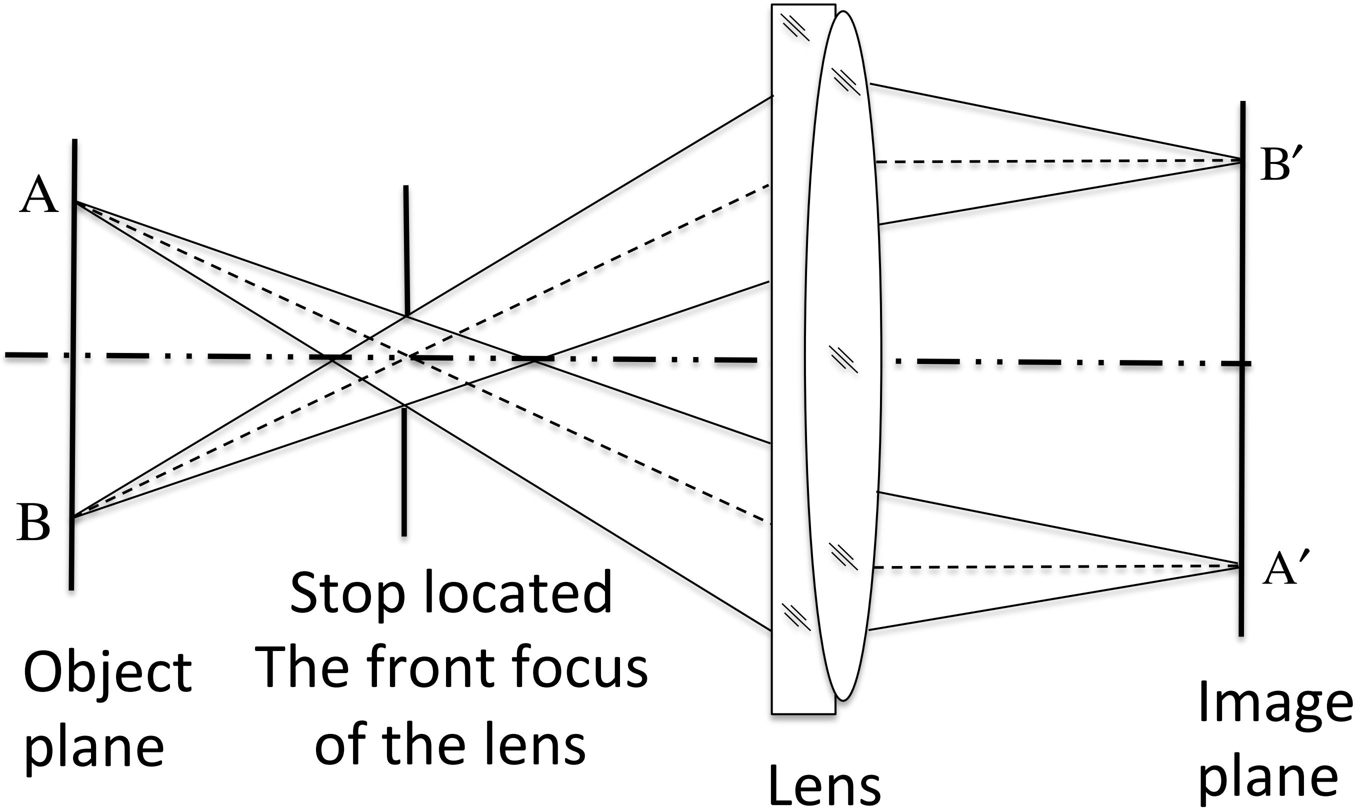
This is not a change in magnification because the shift is dependent on the cube of the field position

Example of wide angle lens with barrel distortion

Stop shift – controls aberrations



Telecentric optical system

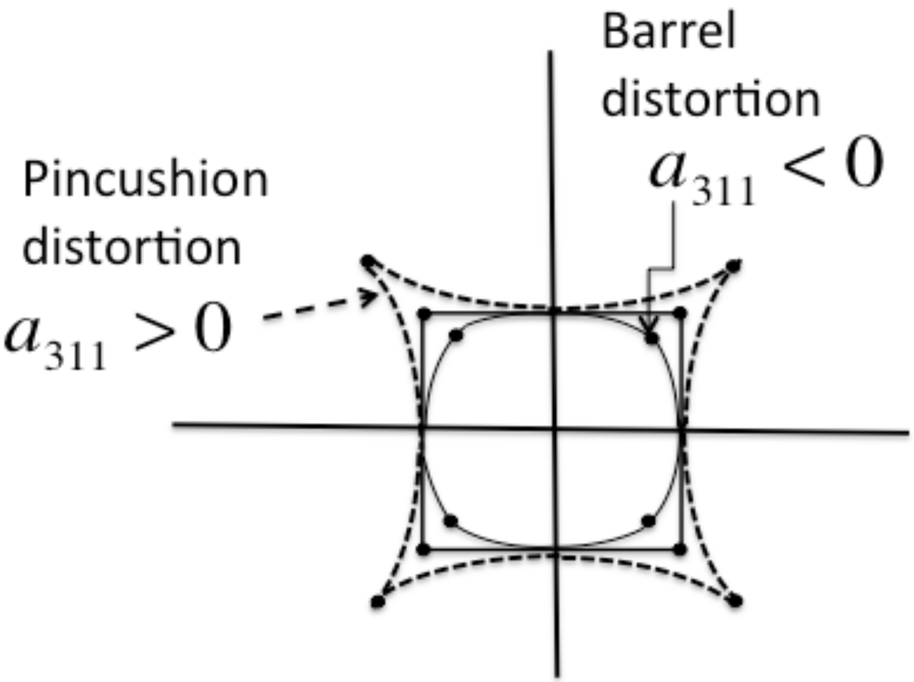


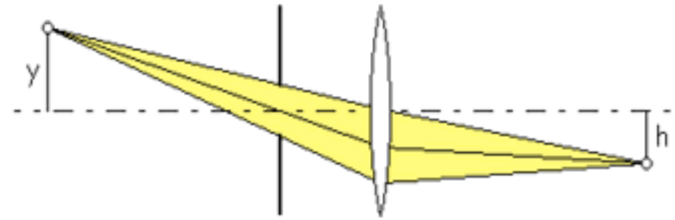
Desensitize the image quality to shifts in the BFD (thermal, flexure, creep, etc.)

Changing focus does not not change image plane scale or the image centroid

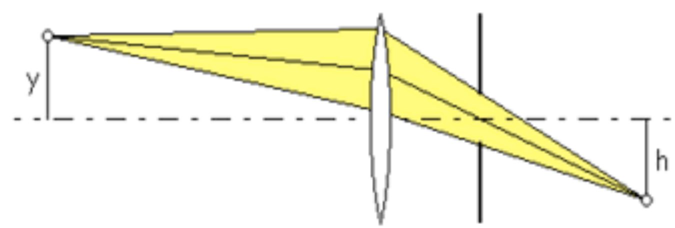
The role of stop shift in distortion

$$\varepsilon_y = -\frac{R}{h} a_{311} y^3$$





Barrel

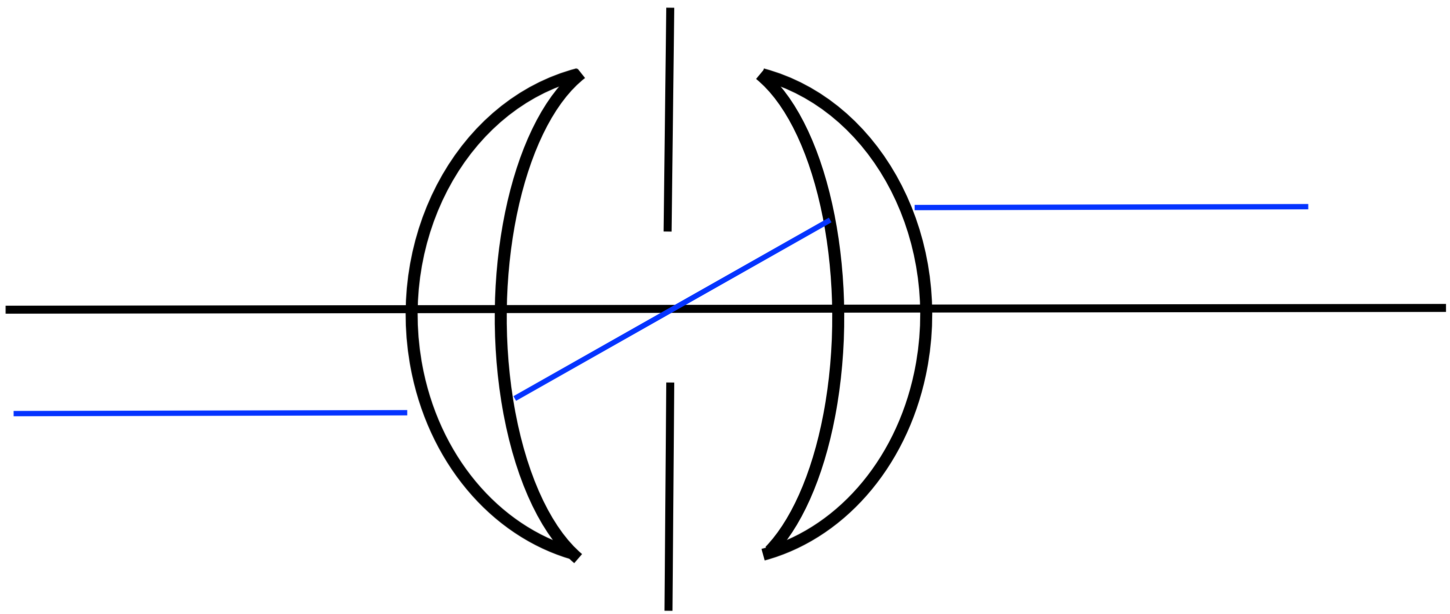


Pincushion

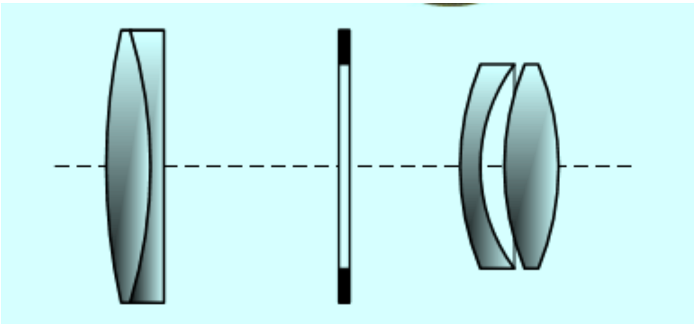
Distortion is controlled by a stop shift

Simple zero distortion lens

1840

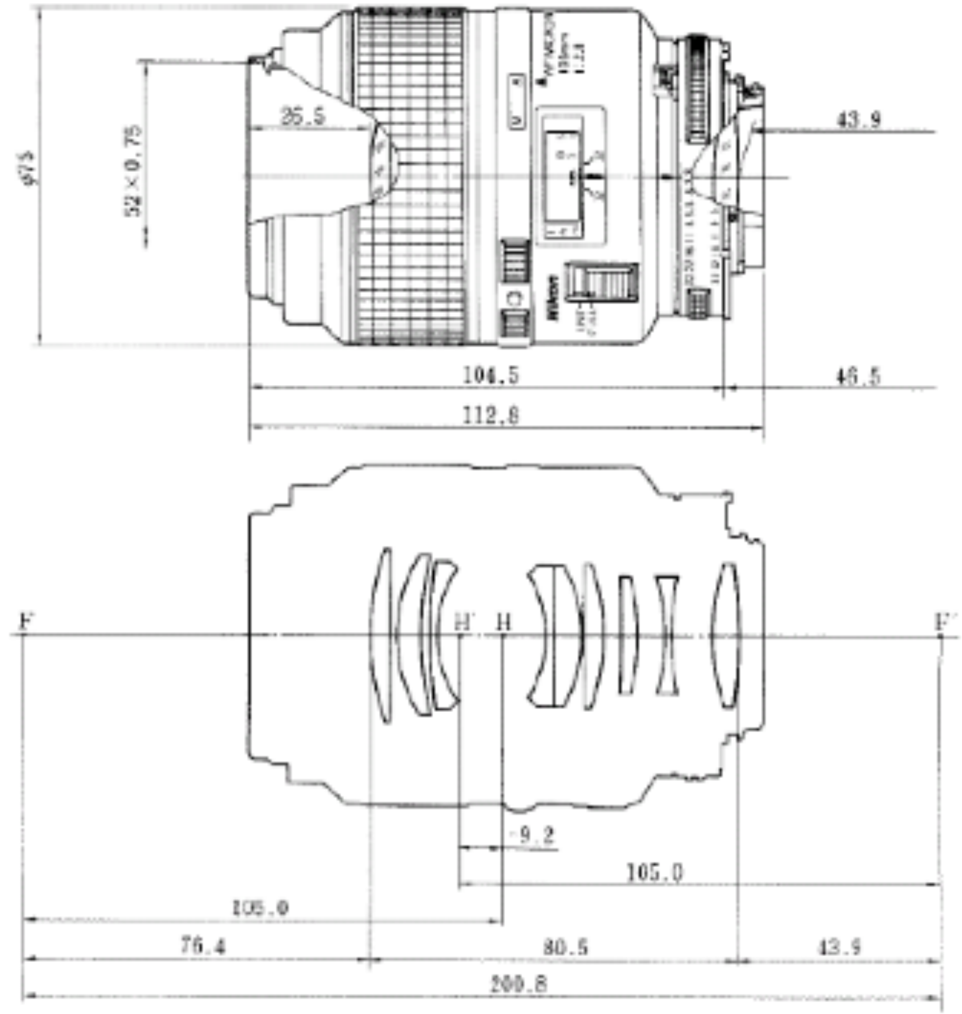


Petzval lens



- Front doublet is well corrected for spherical but introduces coma.
- The second doublet corrects coma.
- The position of the stop corrects for astigmatism, but introduces additional field curvature and vignetting. Total FOV is restricted to 30 degrees.
- F# as low a 3.7 were achieved.
- Ideal portrait lens for the new invention of photography.

2014 State of the art Commercial Canon lens layout is derived from the Petzval lens of 1840



Lens design resources:
Arthur Cox book &
Patent literature

3rd order aberration summary

- Spherical aberration depends on 4th power of the aperture height

$$\rho^4$$

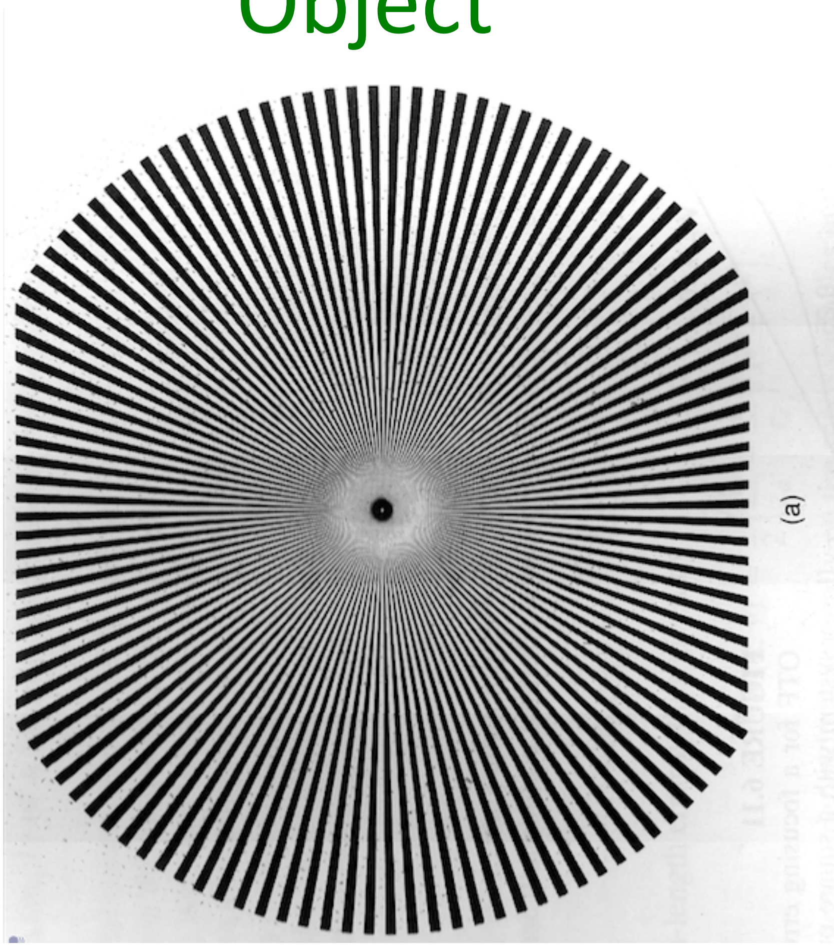
- Coma depends on the square of the aperture height and is linear with field. $\rho^2 h$
- Astigmatism is caused by the difference in curvature between sagittal and tangential ray fans. It varies linear with aperture and the square of the field. ρh^2
- Petzval curvature

$$\propto \sum_{j=1}^{N} \left(\frac{\phi_j}{n_j} \right)$$

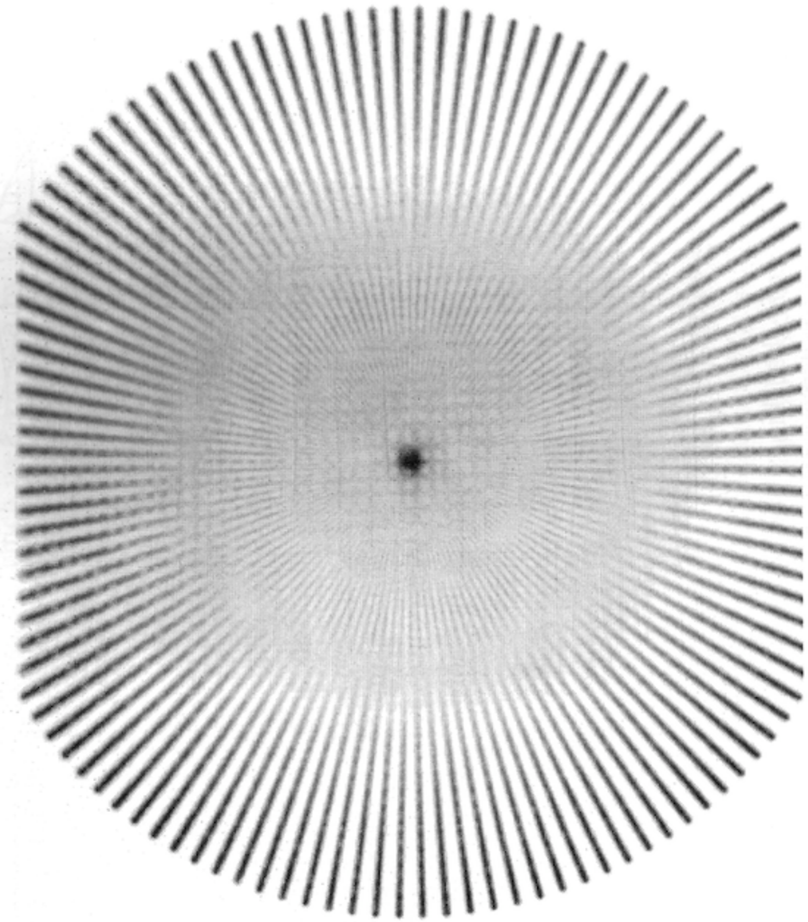
- Distortion shifts image location and is dependent on the cube of the field. h^3

Spherical aberration shown on Spoke Target

Object



Image



Coma through focus

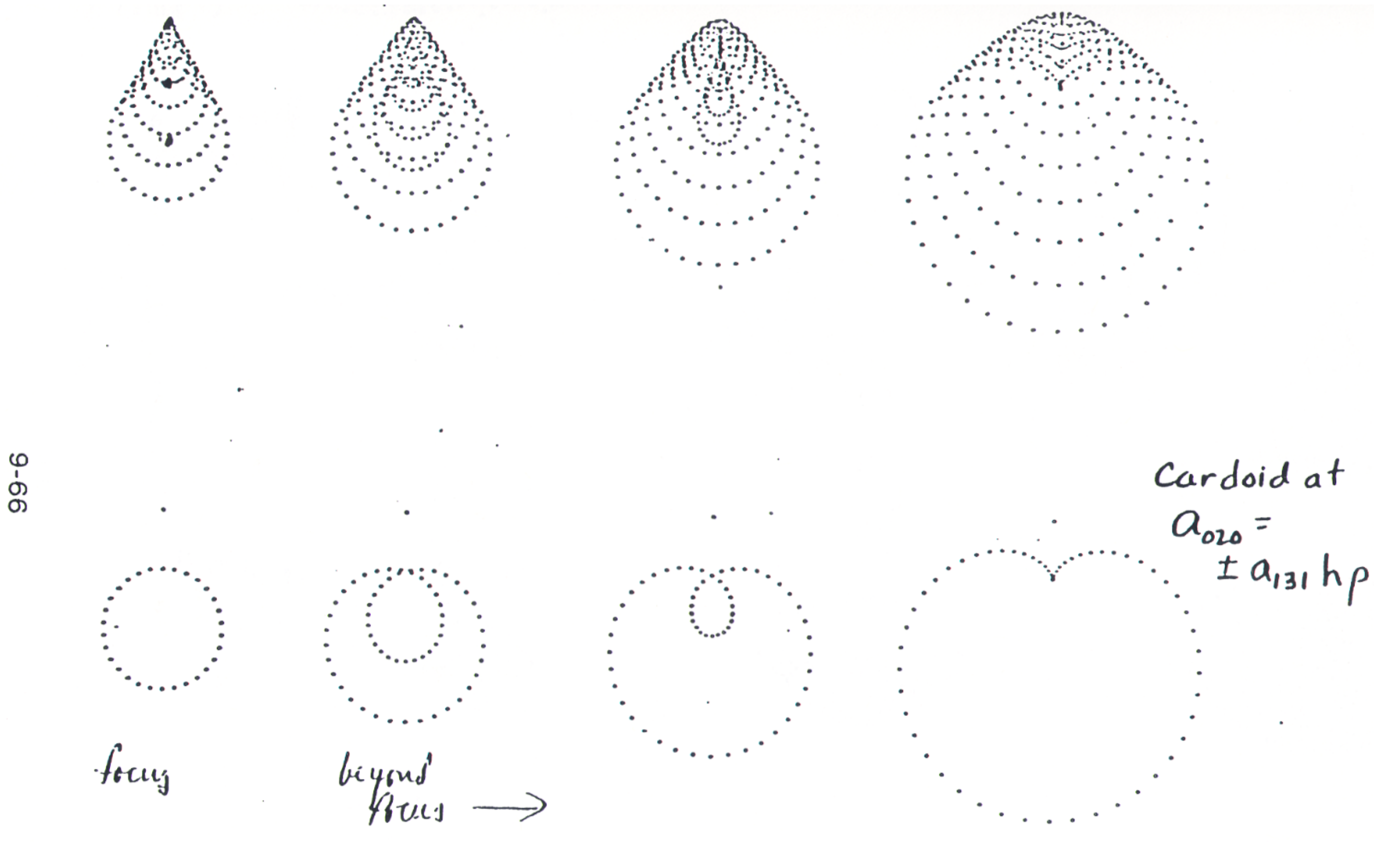


Figure 9-25. Spot diagrams for coma (hexapolar grid).

Applications of aberration theory

- In this section we describe the way in which the theory of aberrations is applied to a few optical systems to show the reader the utility of this analysis approach.

- Optical ray-trace design computer programs require input of the first-order properties on an optical system.

- The closer these first-order properties are to an optimized design, the faster and more accurately the CAD program will converge to an optimized design

Applications of aberration theory

- The structural aberration coefficients were developed to simplify the design process.
- We will show how the structural coefficients are used to determine system aberrations.
- Here we discuss aberrations introduced by a plane-parallel plate, derive the aberration coefficients a_{040}, a_{131}, a_{222}, a_{220}, and a_{311}, analyze the effects of lens bending, and describe the Schmidt camera design.