Modern Astronomical Optics

1. Fundamental of Astronomical Imaging Systems

OUTLINE:

A few key fundamental concepts used in this course:
- Light detection: Photon noise
- Diffraction: Diffraction by an aperture, diffraction limit
- Spatial sampling

Earth's atmosphere: every ground-based telescope's first optical element
- Effects for imaging (transmission, emission, distortion and scattering) and quick overview of impact on optical design of telescopes and instruments

Geometrical optics: Pupil and focal plane, Lagrange invariant

Astronomical measurements & important characteristics of astronomical imaging systems:
- Collecting area and throughput (sensitivity)
  - flux units in astronomy
- Angular resolution
- Field of View (FOV)
- Time domain astronomy
- Spectral resolution
- Polarimetric measurement
- Astrometry

Light detection: Photon noise

Poisson noise
Photon detection of a source of constant flux F. Mean # of photon in a unit dt = F dt.
Probability to detect a photon in a unit of time is independent of when last photon was detected → photon arrival times follows Poisson distribution
Probability of detecting n photon given expected number of detection x (= F dt):
$f(n,x) = x^n e^{-x}/(n!)$
x = mean value of f = variance of f

Signal to noise ration (SNR) and measurement uncertainties
SNR is a measure of how good a detection is, and can be converted into probability of detection, degree of confidence

Signal = # of photon detected

Noise (std deviation) = Poisson noise + additional instrumental noises (+ noise(s) due to unknown nature of object observed)

Simplest case (often valid in astronomy): Noise = Poisson noise = $sqrt(N_{ph})$
Most of the time, we assume normal distribution (good approximation of Poisson distribution at high flux)

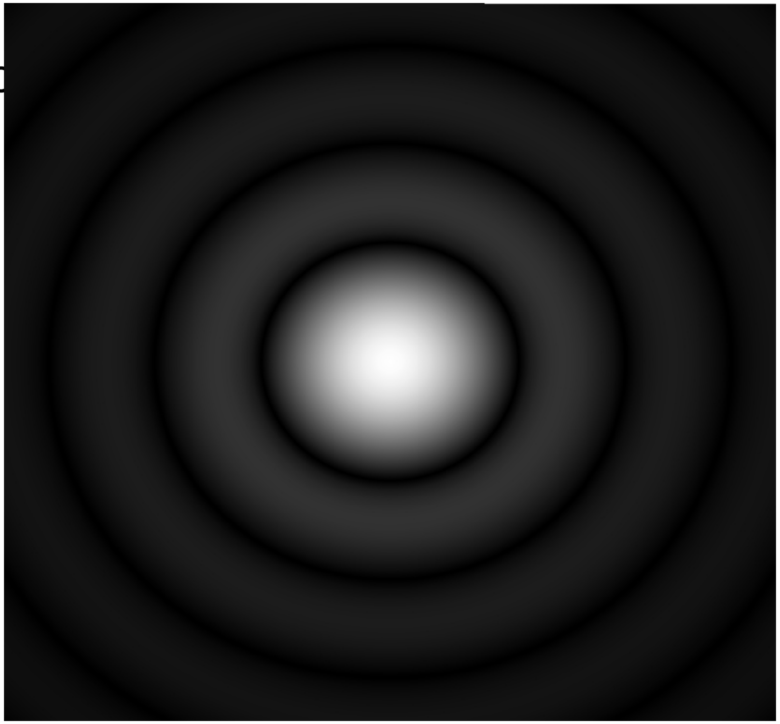
For example:

Telescope observes source for 5s, and detects 200 photon → measured source flux is 40 ph/s with a 3-σ measurement error of 3xsqrt(200)/5 = 8.5ph/s → 99.7% probability that actual flux is between 31.5 ph/s and 48.5 ph/s

Diffraction by an aperture – telescope diffraction limit

Fresnel diffraction integral: $E(x,y,z) = \frac{e^{ikz}}{i\lambda z}\iint E(x',y',0)e^{\frac{ik}{2z}[(x-x')^2+(y-y')^2]}dx'dy'$

In imaging telescope, focal plane is conjugated to infinity
Fraunhofer is far field approximation of the Fresnel diffrac
computed as a Fourier transform.



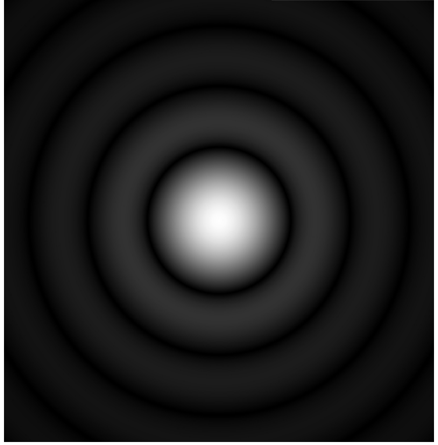
For circular aperture without obstruction : Airy pattern
First dark ring is at ~1.22 λ/D
Full width at half maximum ~ 1 λ/D
The “Diffraction limit” term = 1 λ/D

D=10m, λ=0.55 μm → λ/D = 0.011 arcsec

On large telescopes, image angular resolution is limited by atmospheric turbulence on the ground, at about 1 arcsecond
→ Adaptive optics required for < arcsecond imaging

Note: In astronomy, we use arcsecond (1/3600 deg) as unit for small angles

Spatial sampling of images



Astronomical imaging systems use arrays of pixels.
How many pixels across image to capture signal ?

Nyquist-Shannon sampling theorem:
If a function contains no spatial frequency of period smaller than P, then it is fully specified by its values at interval P/2

The Optical Transfer Function of a telescope goes to zero at λ/D: an noiseless image is band limited (telescope acts as a low pass filter in spatial frequencies)
→ Nyquist limit:
2 pixels per resolution element (= λ/D if diffraction limited)

Sampling and physical size of pixels defines F/ratio of optical beam onto the detector

Example:
Diffraction-limited telescope with Adaptive Optics
D=5m, λ=1.0 μm → λ/D = 0.04 arcsec
Nyquist limit : 20 mas (0.02 arcsec) per pixel

With 20 μm pixels, 1 mas / μm on the detector: 1 mas x f = 1 μm
f = 206m → f/D = 40

Increasing sampling beyond Nyquist limit doesn't bring new information.

Flux units in optical astronomy

At optical wavelengths, the most common unit is the astronomical magnitude scale. Historically, from 0 (brightest stars in sky) to 6 (faintest stars visible to the eye in night sky).
Large number = faint source !!!

Magnitude scale has since been defined for different colors, and extends beyond visible light to both IR/near-IR and near-UV.

Magnitude scale is logarithmic:
5 magnitudes = 100x flux (1 magn = $100^{1/5}$ ratio = 2.512 ratio in flux)

$m = -2.5 \log_{10}(F/F_0)$
$F = F_0 \, 2.512^{-m}$
With F_0 given in table below

Conversion between Jy and $ph.s^{-1}.m^{-2}.\mu m^{-1}$:
1 Jy = 1e-26 $W.m^{-2}.Hz^{-1}$
(Johnson-Cousins-Glass)

Band	B	V	R	I	J	H	K
effective wavelength (μm)	0.436	0.545	0.638	0.797	1.22	1.63	2.19
zero mag flux (Jy)	4000	3600	3060	2420	1570	1020	636
zero mag flux ($ph.s^{-1}.m^{-2}.\mu m^{-1}$)	1.38E11	9.97E10	7.24E10	4.58E10	1.94E10	9.44E9	4.38E9

Flux units in optical astronomy

V magnitudes:

Sun : -26
Full moon : -13
Brightest star (Sirius) : -1.4
Faintest naked eye stars: 7
Faintest stars imaged by Hubble Space Telescope: 30

Magnitude scale also used for surface brightness: magn.arcsec^{-2}

Absolute Magnitude

Astronomical unit (AU) = Sun-Earth distance = 1.496e7 m
parallax = amplitude of apparent motion of a source on background sky due to Earth's orbit
parsec (pc) = parallax of one arcsecond = 3.0857e16 m = 3.26156 light year (ly)
Absolute magnitude (M): apparent magnitude an object would have if located 10 pc from Earth

If object is at 10pc, M=m
If object is at D_L pc, apparent flux = $(D_L/10)^{-2}$
$m = M + 5 (\log_{10}(D_L) -1)$
$M = m – 5 (\log_{10}(D_L) -1)$

Problem #1:
How big a telescope does it take to image an Earth-like planet at 10pc (32.6 lyr) in 1hr ?

Assume:
detection SNR = 5
0.1 μm bandpass filter at 0.55 μm (V band)
50% efficiency
no background
Star light has been completely removed by coronagraph
Sun V band absolute magnitude = 4.83
Earth is 1e10 fainter than Sun

Solution to problem

How many photons needed ?

SNR = 5 is reached with 25 photons, for which signal (S) = 25 and noise (N) = sqrt(25) = 5

Zero point of the system as a function of collecting area

According to the table of magnitude zero points, in one hour, a 0.1 μm wide filter around V band gives for a magnitude zero source :

N_0 = 0.1 μm x 3600s x 9.97E10 $ph.s^{-1}.m^{-2}.\mu m^{-1}$ = 3.59E13 $ph.m^{-2}$

With the 50% efficiency, the number gets reduced to zp = 1.79E13 $ph.m^{-2}$

Apparent magnitude of the planet

The apparent magnitude of the star is:

$m = M + 5 \times (\log_{10}(D_L)-1) = M = 4.83$

The planet is 25 mag fainter (= 1e-10 flux ratio)

→ m = 29.83

Number of photon collected per hour from the planet

$N = zp \times 2.512^{-m}$ = 21.0 $ph.m^{-2}$

Telescope diameter required

Collecting area required = 25/45.9 = 1.19 m^2

→ telescope diameter required = 1.2 m

(Note: other effects ignored here are the star halo, background, detector noise etc...)

The first optical element in every ground-based telescope: Earth's atmosphere

Transmission

Atmosphere is fairly transparent in optical when not cloudy

nearIR: windows of transparency exist, main absorber is water vapor

→ choose right wavelenght bands for observations

Emission: the sky is not fully dark

In visible light: airglow (~100km altitude)

→ optical filtering and/or calibration

In IR: blackbody emission from water vapor

→ high altitude, dry and cold sites better

Wavefront distortions

fluctuations in refractive index (temperature, humidity, pressure, water content) introduce wavefront errors

Atmospheric turbulence

typical angular distortion = 1” = diffraction limit of 10cm telescope in visible

→ ***Adaptive optics can mitigate this issue***

Atmospheric refraction

refraction is chromatic: stars turn into spectra at low elevation

→ ***Can be compensated by atmospheric dispersion compensator***

Rayleigh Scattering

Daytime sky too bright for observations

Moonlight increases sky brightness in visible light (but near-IR is OK)

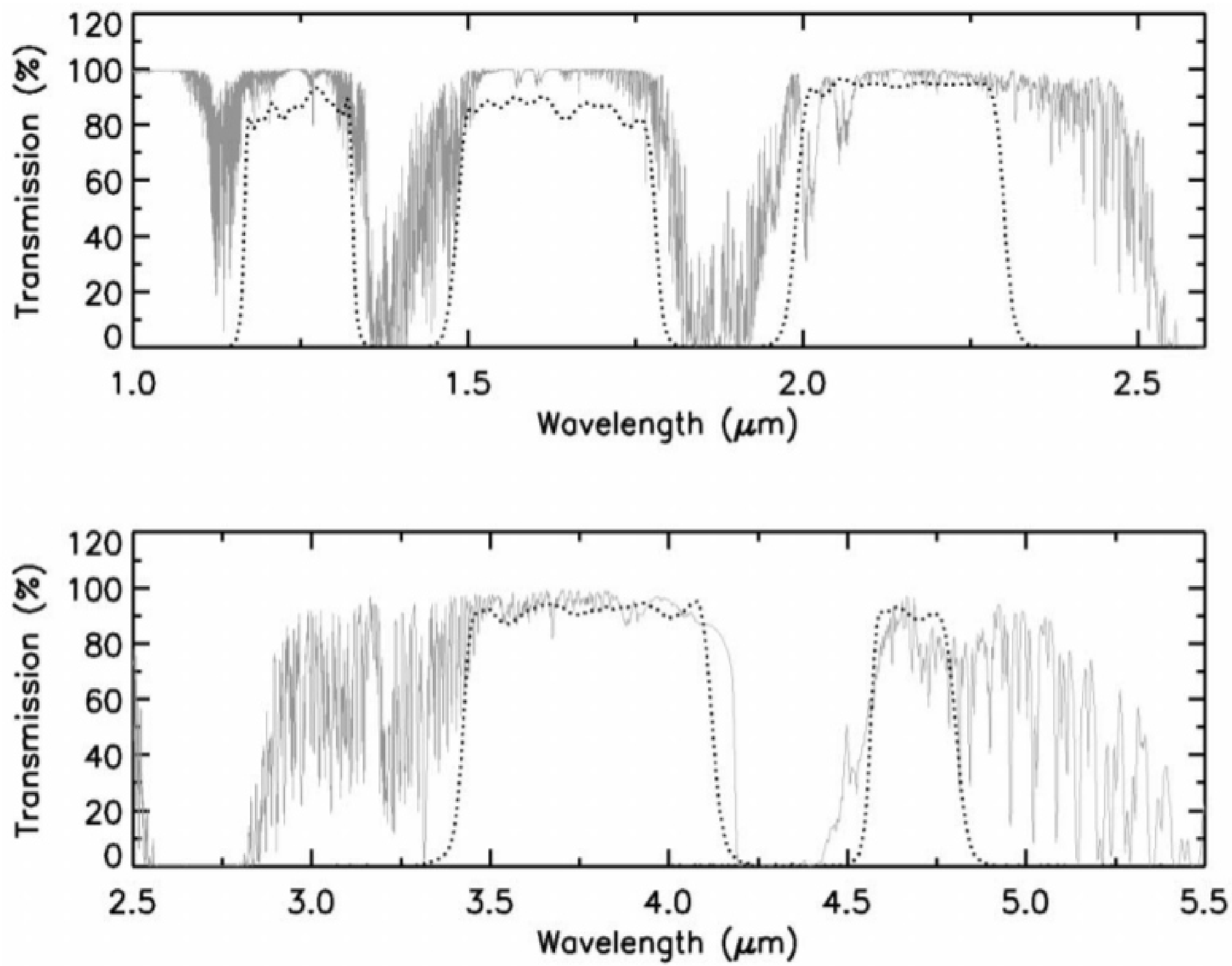
→ observe in the near-IR / IR during bright time, visible during dark time

The first optical element in every ground-based telescope: Earth's atmosphere

Transmission & Emission in near IR

In IR: poor transmission = high thermal emission (sky is glowing)

→ IR filters for ground-based observations chosen to match high transmission windows



J, H, Ks, L', and M' filter profiles superposed on the atmospheric transmission at Mauna Kea kindly provided by G. Milone for 1 mm precipitable water vapor and an air mass of 1.0

Tokunaga, Simons & Vacca, 2002

The first optical element in every ground-based telescope: Earth's atmosphere

Transmission

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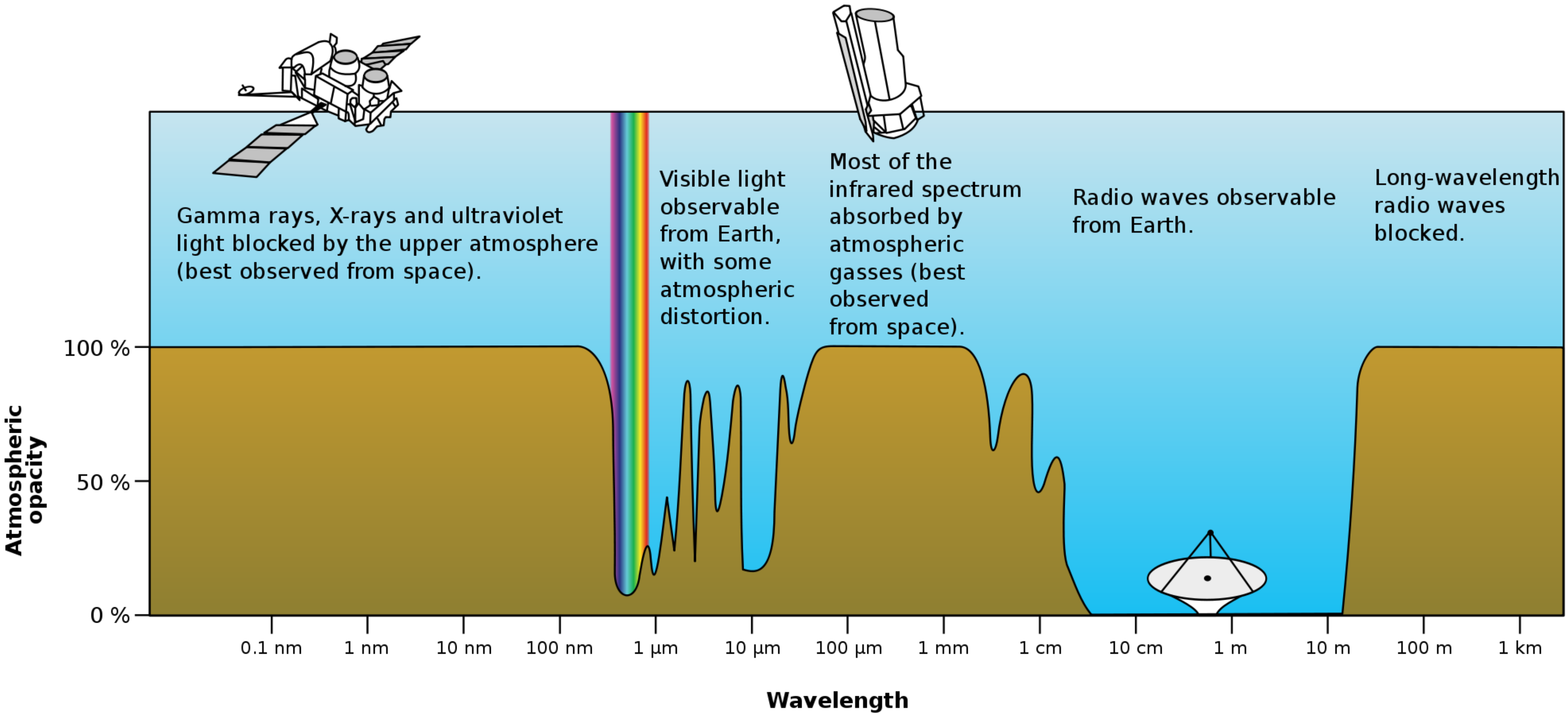
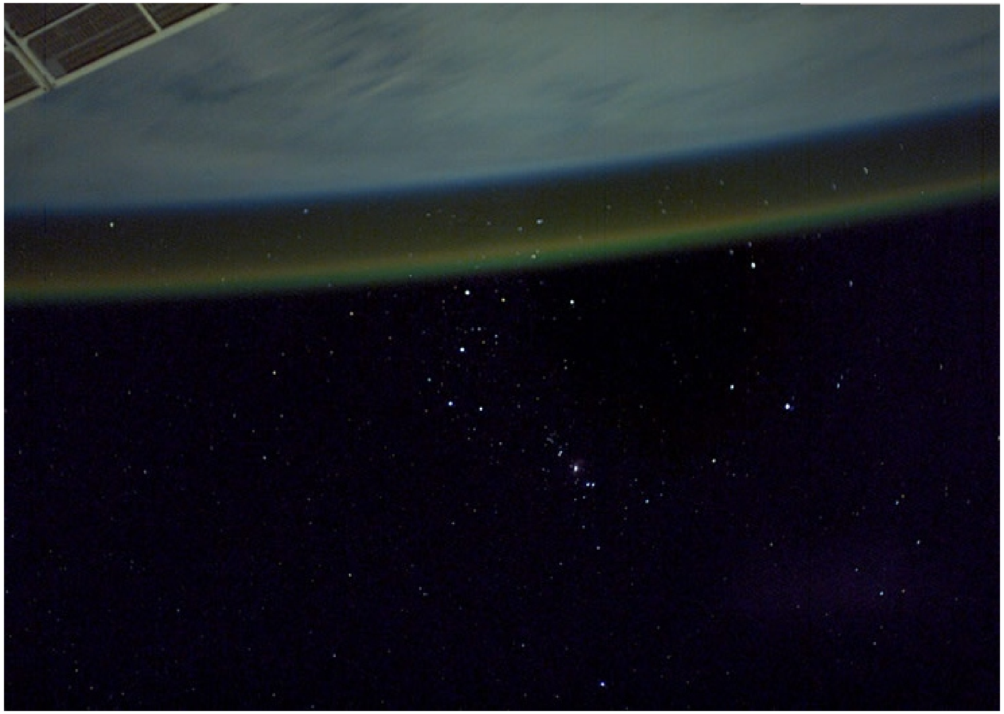


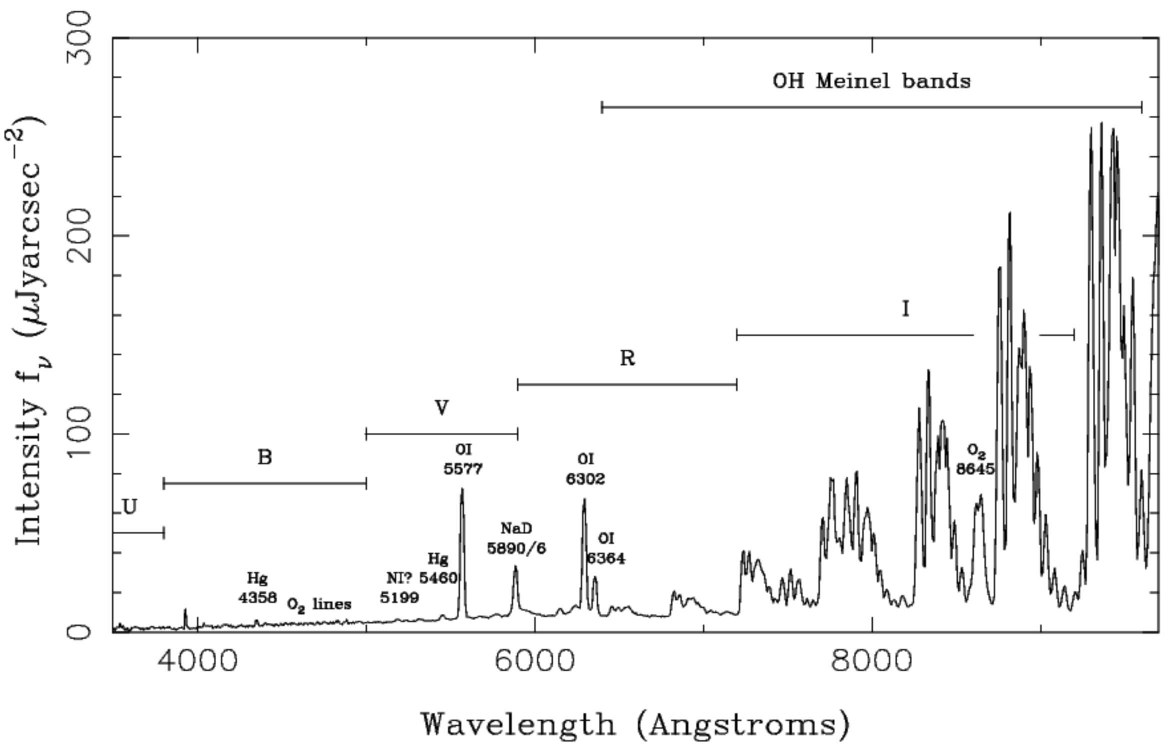
The first optical element in every ground-based telescope: Earth's atmosphere



Optical emission : airglow

Emission from OH (red & nearIR), O (visible green line) and O_2 (weak blue light) at ~90km

Airglow is time-variable, has structure over wide angles: it is very important for spectroscopy to either optically filter it out or have a good scheme to calibrate it and subtract it from the spectra



The first optical element in every ground-based telescope: Earth's atmosphere

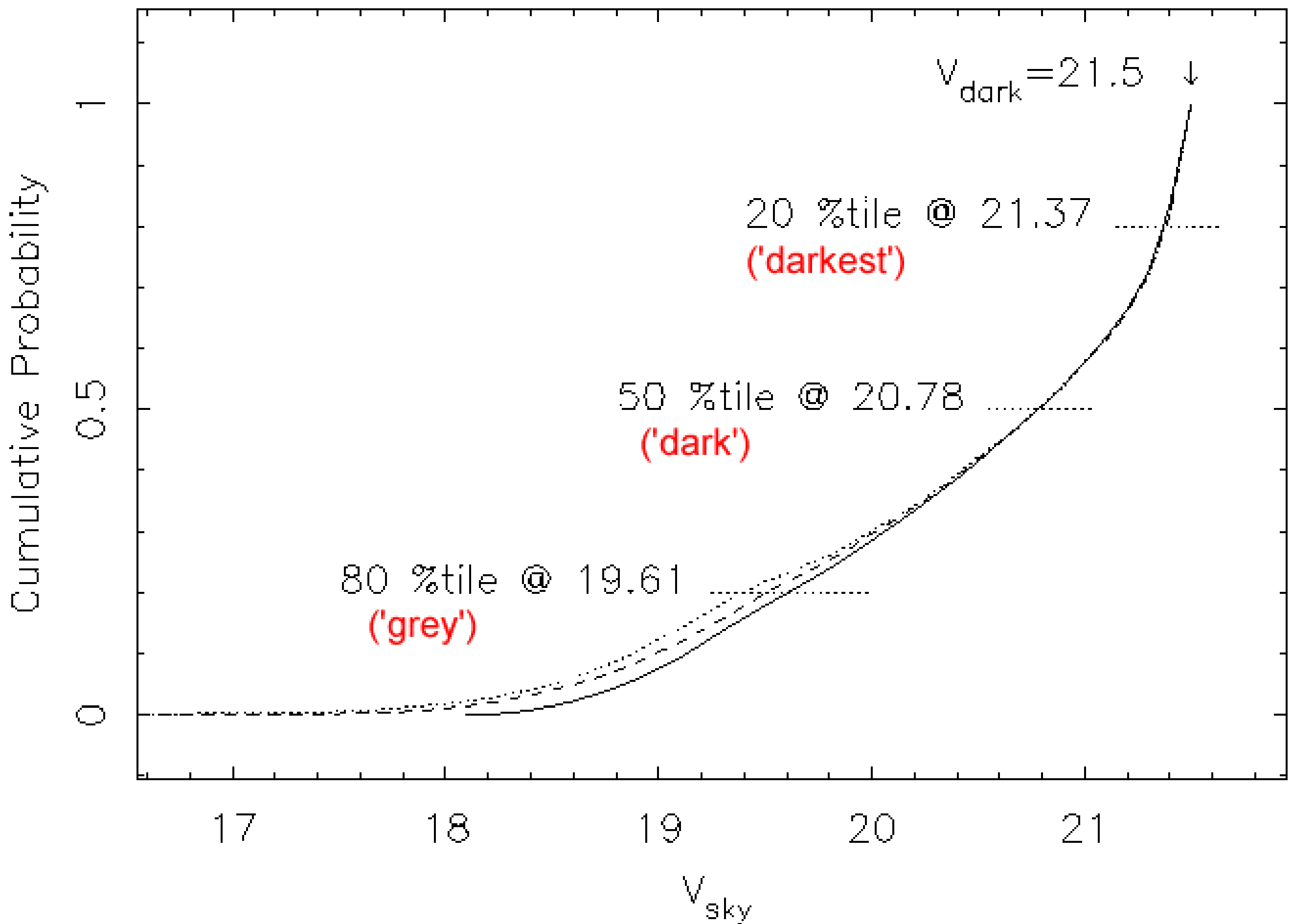
Moonless sky background in the optical (V band):

Airglow : $m_V = 22.4$ arcsec^{-2}

Zodiacal light : $m_V = 23.3$ arcsec^{-2} (brighter closer to ecliptic)

+ scattered starlight (much smaller)

Total darkest sky background ~ $m_V = 21.9$ arcsec^{-2} (rarely achieved from ground)



Cumulative probability distributions of V-band sky brightness at an arbitrary phase in the solar cycle for three model observation scenarios
Gemini North Telescope

The first optical element in every ground-based telescope: Earth's atmosphere

Credit: D. Duriscoe, C. Duriscoe, R. Pilewski, & L. Pilewski, U.S. NPS Night Sky Program
Full resolution image on Astronomy Picture of the Day (APOD), 2009 Aug 27

Geometrical Optics: Lagrange Invariant

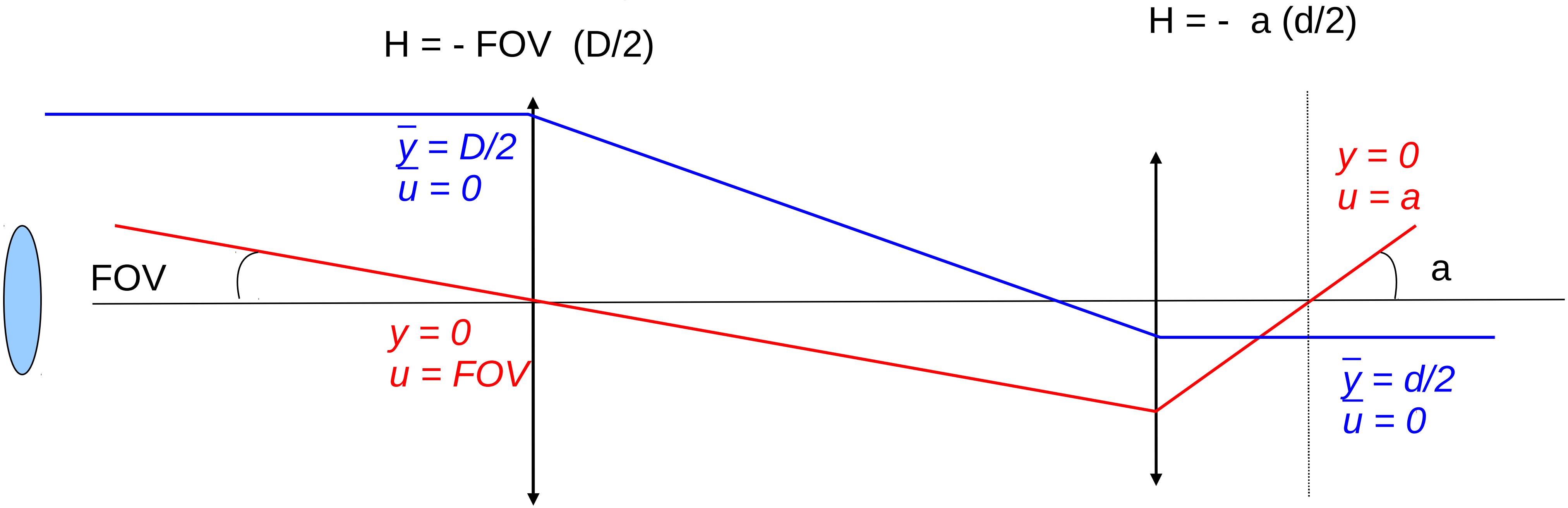
$H = n\,\bar{u}\,y - n\,u\,\bar{y}$

n = ambient refractive index (= 1 in most cases, unless H is computed inside a lens)
y = chief ray height
u = chief ray angle
$\bar{y}$ = marginal ray height
$\bar{u}$ = marginal ray angle
(see next slide for visual representation of these terms)

→ **large field of view and large collecting area requires large optics**
example: 10-m telescope, 1 deg field of view
if beam in compressed to 10cm (100x compression), angle = 100 deg → very difficult to design re-imaging optics of sufficiently high quality

→ **small beam = compressed propagation distances, lots of beam walk and diffraction effects at fixed physical distance from pupil**
Example: 10-m diameter beam compressed to 10mm (1000x lateral compression)
In this beam, lateral compression = 1e6: 10mm along the small beam is 10 km along the 10-m diameter beam

Example: afocal telescope (= beam reducer), input diameter D → output diameter d
In Astronomy, object to be imaged is at infinity



Chief ray (starts at edge of object, crosses center of aperture)

PUPIL= where chief ray intersects optical axis = conjugated to aperture stop

Marginal ray (starts at center of object, crosses aperture at its edge)

FOCAL PLANE = where marginal ray intersects optical axis = conjugated to infinity

a = FOV (D/d)

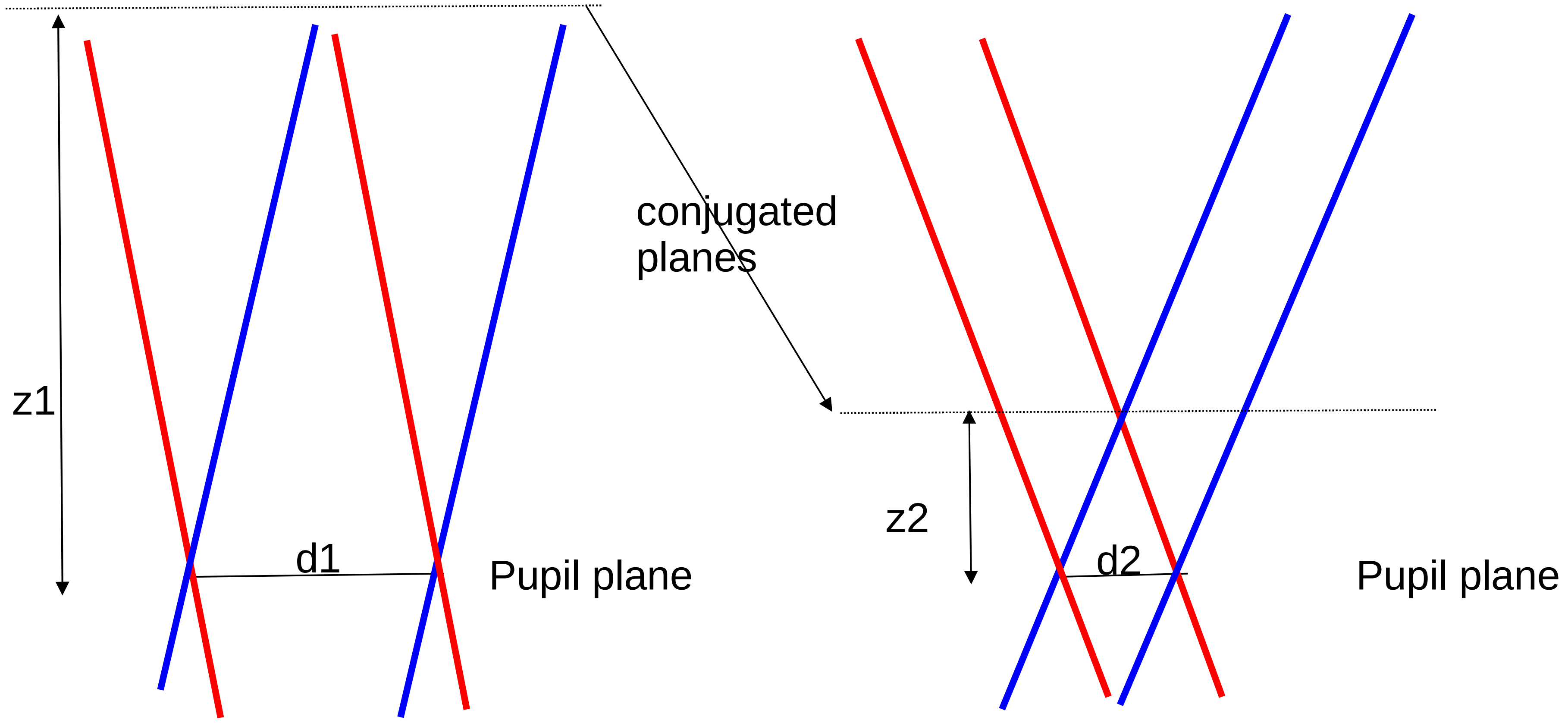
Compressing the beam by factor x = multiplying angles by factor x

Note: The Lagrange invariant can also be seen as conservation of retardation (unit: waves, or m) between one size of the beam and the other: reducing the beam size conserves this retardation, and therefore amplifies angles.

→ impossible to build a wide field of view large telescope using small relay optics !!

→ large FOV & large diameter telescopes are challenging to build and have very large optics

Smaller beam : angles get larger



Lagrange invariant → $d1^2 / z1 = d2^2 / z2$

Reducing beam size by x compresses propagation distances by x^2

Drawing above provides physical illustration by looking at overlap between beams

Note:

Diffractive propagation equations (Talbot distance) show same beam volume compression effect: Talbot distance goes as f^{-2}, where f is the spatial frequency. If the beam is compressed by x, spatial frequencies are also multiplied by x, and the Talbot distance is divided by x^2

Telescope sensitivity to faint sources: Importance of collecting area

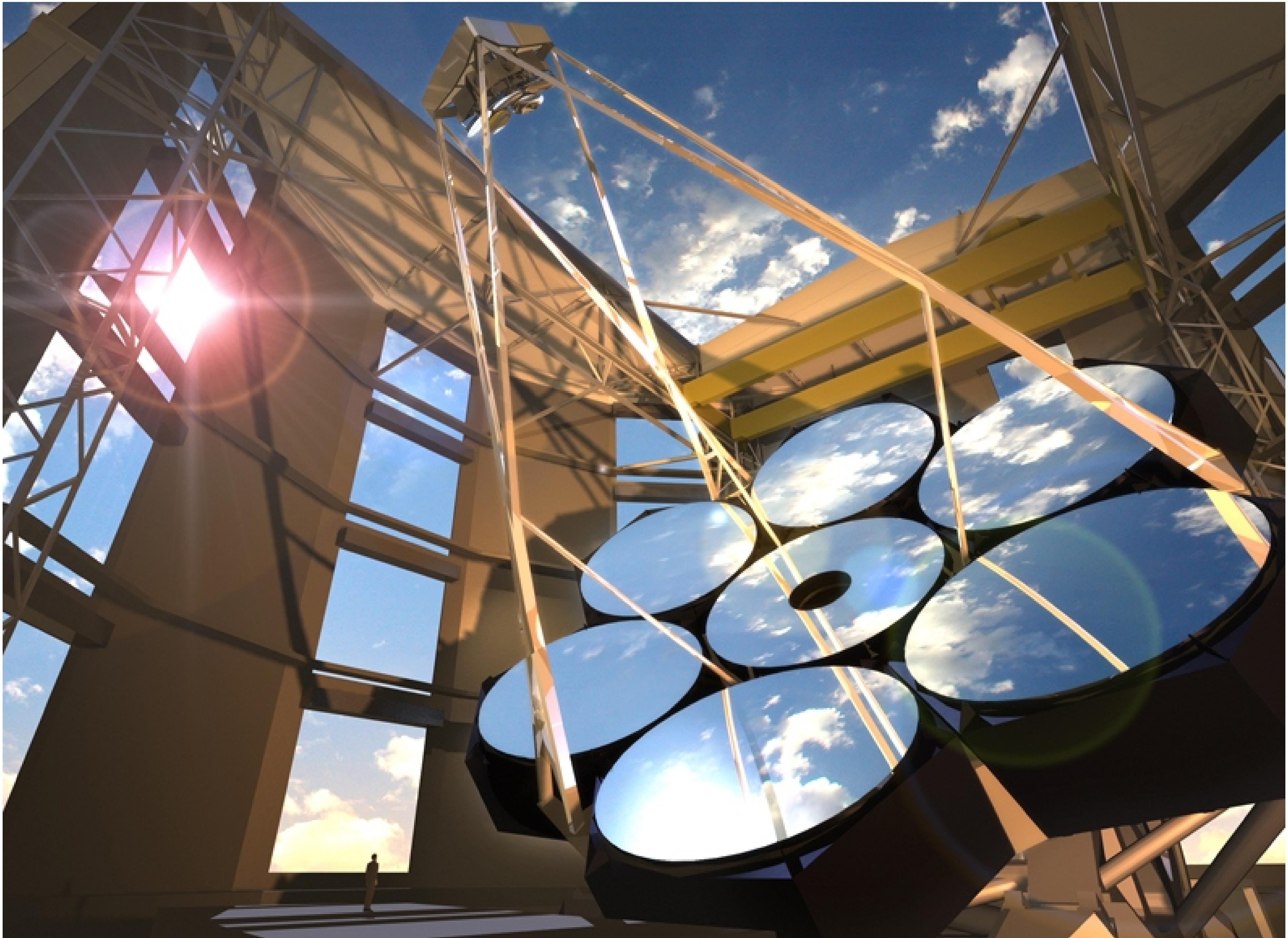
Astronomical measurements are very often flux-limited: limited number of photon available from the source.
Example:
Typical nearby (by cosmological standards) galaxy : $m_V \sim 15$
In V band, with 0.1 μm bandpass, a 1 m^2 telescope with 50% efficiency: 5000 $ph.s^{-1}$
Imaging structure in the galaxy (details in spiral arms, bright clusters), will require >> 5000 ph

Detailed study of astrophysical objects requires spectroscopy and/or angular resolution → total number of photon required grows larger to take advantage of spectral and angular resolution

Study of faint objects is essential for astronomy, pushing for larger telescopes.
With larger telescopes:
- a given type of object can be imaged further away
    For example, imaging stars in nearby galaxy for stellar population studies allows better understanding of the galaxy architecture and history
- statistical studies require large samples. Sample size for a given type of object increases as sensitivity improves
- Cosmology requires the study of the distant (=young) universe. Finite speed of light used to look back in time

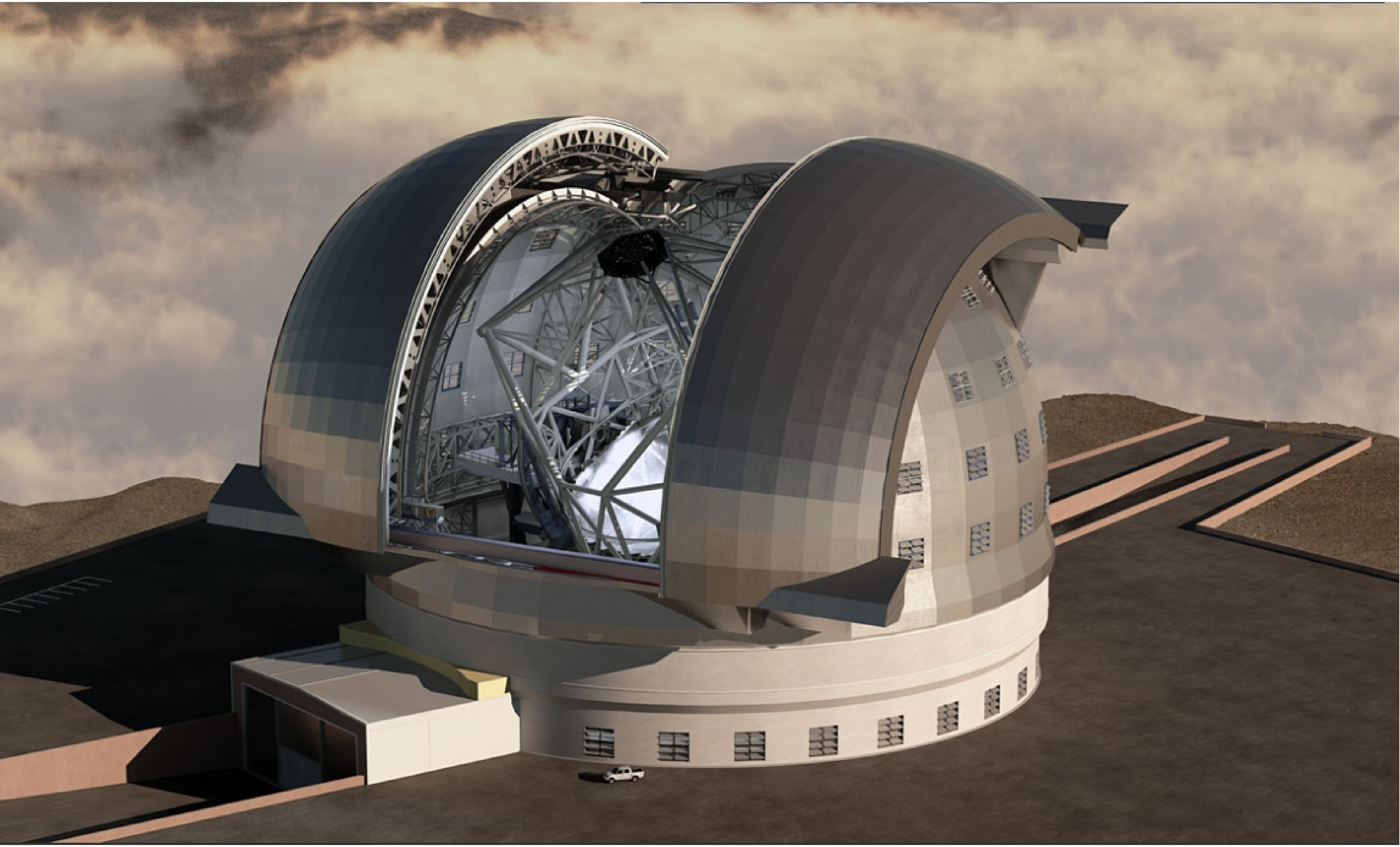
Current large telescope at 8m to 10m diameter, next generation will be 25m to 42m diameter

Giant Magellan Telescope (GMT) project: 24.5 m



Thirty Meter Telescope (TMT) project

European Extremely Large Telescope (EELT) project



Angular resolution

Without adaptive optics, angular resolution is limited to ~0.5” in good sites in optical.
Hubble space telescope diffraction limit = 0.05” (10x better)

With Adaptive Optics (AO), diffraction limit is reached in near-IR (> 1 um) → 0.05” on the current generation of large telescopes (up to 10m diameter).
Next generation of large telescopes (GMT, TMT, EELT) all include adaptive optics.

The two drivers for high angular resolution are:

(1) Ability to resolve small structures:
- Mitigate confusion limit problem (too many sources too close together)
- Astrometry (measure position of stars accurately)
- Image and study complex structures (map star forming clouds in galaxy)

(2) Sensitivity
At the faint end of the detection limit, the detection is a background limited problem: more light comes from the background (airglow, zodiacal light, thermal emission of telescope and sky) than from the source itself. Background is constant spatially, so it can be removed, but it photon noise (Poisson noise) is left.
This is especially important in the IR (higher background)
No background: sensitivity goes as D^2
Background-limited: sensitivity goes as D^4 if diffraction limited

Problem #2:

Repeat problem #1 with zodi + airglow background, assuming no AO (1” image) and AO (diffraction limited image), assuming a dark sky at m_V = 21 $mag.arcsec^{-2}$

Solution to problem #2

We have already computed in problem #1:
Zero point: zp = 1.79E13 ph./hr/m^2
Apparent magnitude of a Sun-like star in Andromeda: m_V = 28.98
Number of photon collected per hour from the star: N = zp x $2.512^{-28.98}$ = 45.9 $ph.m^{-2}$
With D the telescope diameter in [m], N = 45.9 ($\pi D^2/4$) [ph] = 36 D^2 [ph]

Number of photon collected per hour from the background
Without AO, the image is 1” wide (we assume that the photons are collected over a 1 $arcsec^2$ area, corresponding to a 1” by 1” square)
N_b = zp x 2.512^{-21} = 7.12E5 [$ph.m^{-2}$] = 7.12E4 ($\pi D^2/4$) [ph] = 5.59E4 D^2 [ph]
With AO, the image is λ/D wide (we assume that the photons are collected over a λ/D x λ/D square region). In $arcsec^2$, the area is:
$A_{AO} = ((\lambda/D)/\pi*180*3600)^2$ [$arcsec^2$] = 0.0129 D^{-2} [$arcsec^2$]
and the total number of background photons collected in the area in 1h is:
N_{bAO} = 0.0129 zp ($\pi/4$) 2.512^{-21} = 700 [ph]   (note: this is independent of telescope diameter)

Telescope diameter required for SNR = 5
Signal = photons collected from the star = 36 D^2 [ph]
Noise = square root of number of photons
= sqrt(36 D^2 + 5.59E4 D^2)   without AO
= sqrt(36 D^2 + 700)   with AO
Solving for SNR = 5 gives:
D = 33 m (without AO)
D = 2.0 m (with AO) → Huge gain in sensitivity offered by improved resolution !!!

Imaging Field of View (FOV)

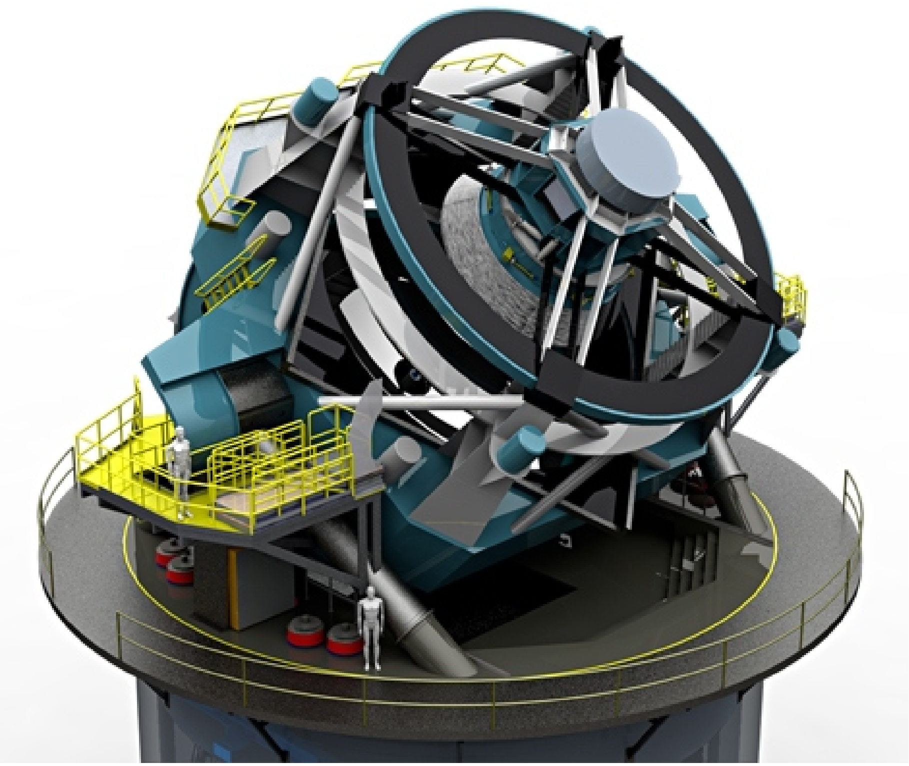
Field of view is important for survey astronomy.

Large sample of object needed.
Discovering new objects, compiling large catalogs of objects for statistical studies.
Examples:
- map 3-D large scale structure of the universe
- identify potentially dangerous asteroids
- galactic archeology through statistical study of our galaxy's stars

Most important is the survey efficiency, also called étendue, or AΩ product:

étendue = collecting area [m^2] x field of view [deg^2]

Time required to carry out a given survey = 1/étendue

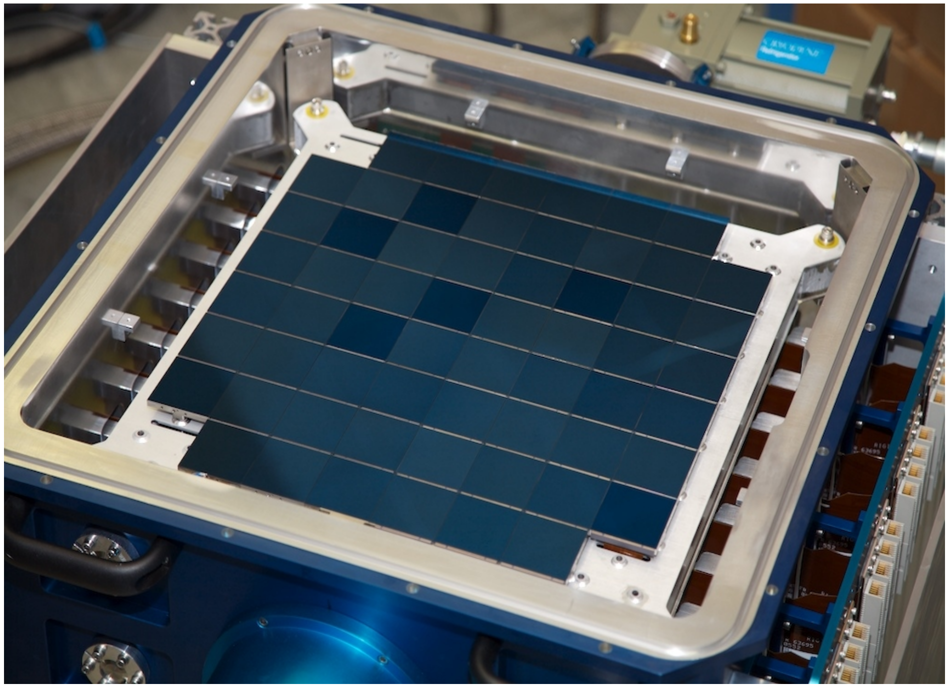


LSST telescope design (8.4m wide field telescope $9.6\ deg^2$ FOV)

Large étendue requires special telescope designs, with large corrector optics (see Lagrange invariant), and large focal plane arrays. Modern large étendue systems are becoming optically very different from conventional telescopes.

Imaging Field of View (FOV)

Large FOV + high angular resolution
= lots of pixels !



Pan-STARRS 1 focal plane array:
1.4 billion pixel

Time domain astronomy – measuring flux as a function of time

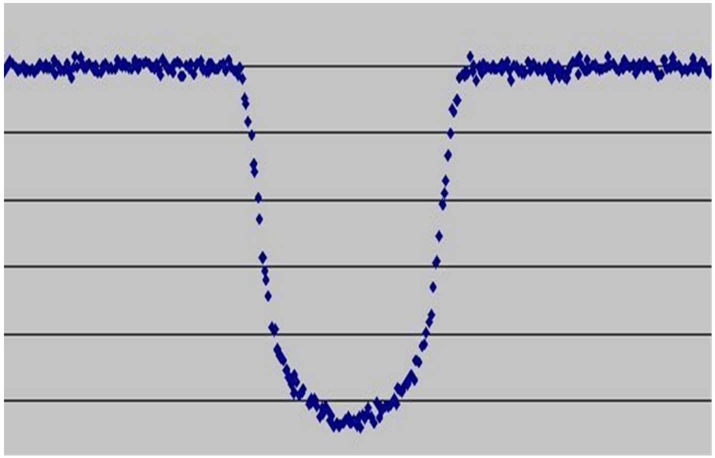
Why time domain astronomy ?
Examples :

- Asteroids, comets discovery and orbit determination
- Variable stars → understanding stellar physics
- Cepheids, Type Ia supernovae → measuring distances over large scales
- Transits of exoplanets → discovery and characterization (radius, orbit)
- Understanding Gamma Ray Bursts
- Variable accretion on young stars

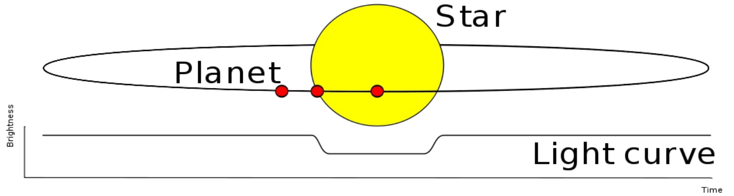
Some time-domain astronomy applications require very high accuracy:

exoplanet transit for Earth like planet is 1e-5 drop in apparent flux
→ very stable telescope system required, with stable pointing (space preferred)

Other applications require high cadence over multiple points in the sky: fast repointing, robotic telescopes



Kepler 6b transit light curve



Astrometry – measuring stars positions as a function of time

Why is astrometry important ?

Astrometry can directly measure distances (parallax)

Earth's orbit around the Sun gives us a 300 million km baseline: our viewing point changes by 300 million km in 6 months → closeby objects appear to move against the background of more distant objects

For nearby stars, the parallax ~ 1”, easily measured with seeing-limited ground based telescopes. Measurement gets more difficult with distance

distance to object [pc] = 1 / parallax angle [arcsec]

Astrometry of asteroids can measure their orbits

This is especially valuable for potentially dangerous Near Earth Orbit asteroids (NEOs)

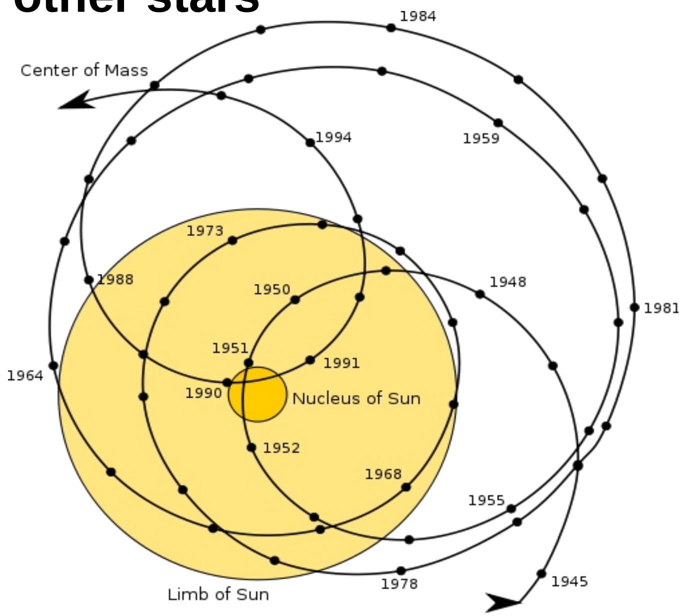
Astrometry can measure masses, and detect planets around other stars

Orbital motion directly links positions to masses

Astrometry has been used to confirm the presence of a black hole at the center of our galaxy and measure its mass

Masses of companions around stars measured by orbital period

High precision astrometry of nearby stars can reveal exoplanets: for a nearby star, Earth-like planet = ~1 micro arcsecond astrometric motion of the star

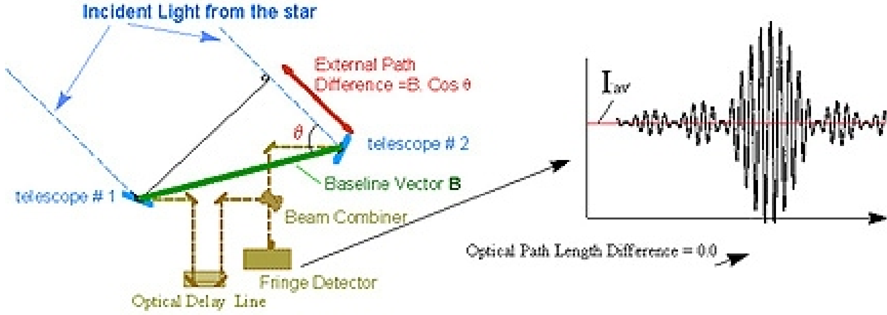


Astrometry – measuring stars positions as a function of time

Two approaches to astrometry:

Interferometry

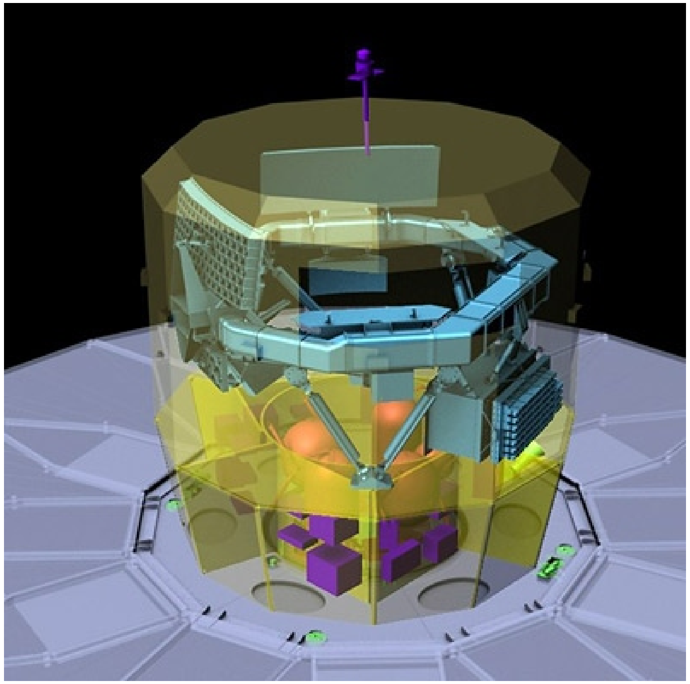
Astrometric signal mesured as an optical pathlength difference between two telescopes (= phase of interference fringes). Interferometers have an angular resolution advantage, and are the prefered solution for high precision astrometry, both on the ground and in space. Precise metrology within the interferometer is required for high precision.



Courtesy NASA/JPL-Caltech

Imaging

Astrometry is mesured from the position of the star images on the focal plane detector.
In theory, photon noise limited single axis measurement error (1-σ) = $1/(\pi \sqrt{N_{ph}})$
(but also: pixel sampling, calibration errors etc...)
This is the prefered solution when astrometric measurement of a large number of sources is required over a moderate field of view (for example, measuring the orbits of several stars around the black hole at the center of our galaxy).
Good understanding of distortions in the optical train are required.



GAIA mission (ESA)
The two primary mirrors are visible at the top of the image

Spectroscopy – measuring Intensity vs. Wavelength

Why is spectroscopy important ?

Spectroscopy is used to measure physical properties of objects.

Colors give temperature estimates

Absorption or Emission Lines can identify composition and physical conditions

Spectroscopy measures accurate velocities

Velocity measurements from shifts of lines can tell us about movement.

dl/l = v/c

Measure only radial component of velocity (astrometry only in plane of sky).

Example: Radial velocities of Exosolar Planets are measured by their influence on their star.

Jupiter creates a radial velocity variation of ~1 ms/ of the sun.

"Typical" intrinsic width of a stellar absorption line is ~0.005 nm