

# High contrast imaging

## Coronagraphy

Coronagraph performance metrics  
(Inner Working Angle, Throughput, contrast, etc...)

Coronagraph fundamental limits

- linear model (in complex amplitude)
- throughput vs angular separation
- importance of stellar angular size

# Lyot Coronagraph explained by Fourier transforms

Pupil plane complex amplitude  $\leftrightarrow$  focal plane complex amplitude

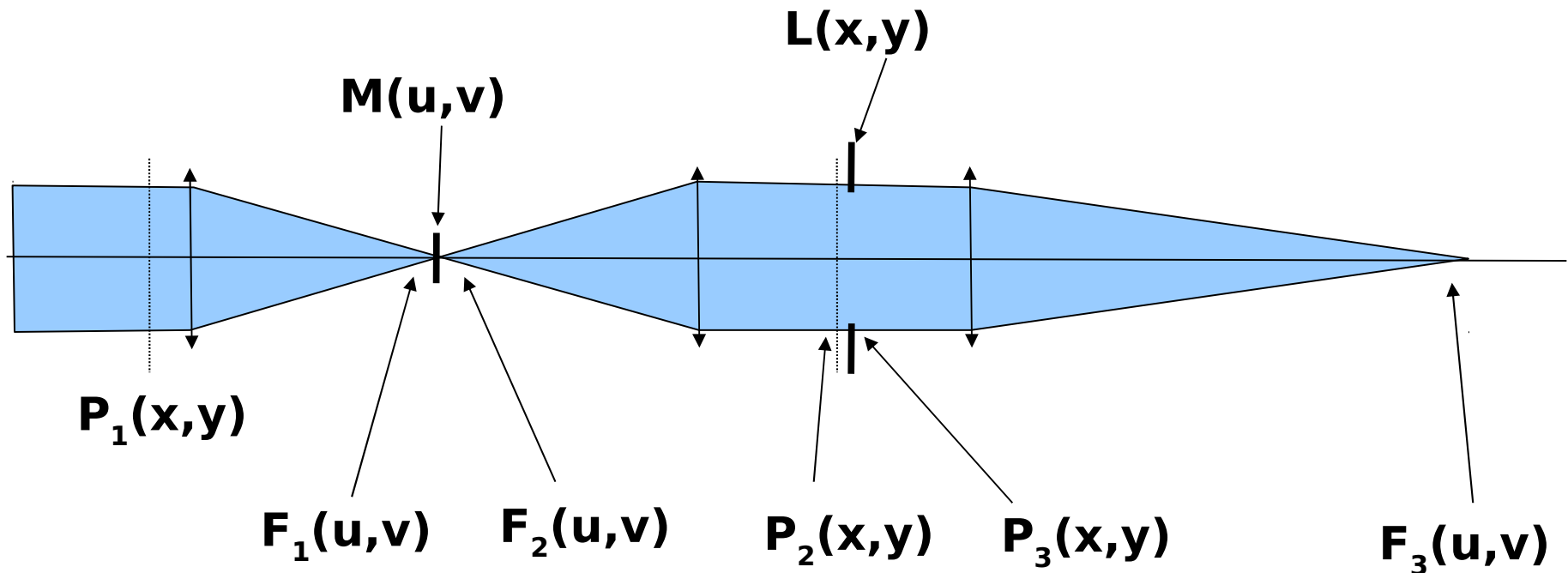
$\rightarrow$  Fourier transform

$\leftarrow$  Inverse Fourier transform

Coordinates in pupil plane:  $x, y$

Coordinates in focal plane :  $u, v$

\* denoting convolution (product = convolution in Fourier transform)



# Lyot Coronagraph explained by Fourier transforms

Entrance pupil of telescope:  $P_1(x,y)$

Focal plane complex amplitude (before focal plane mask):  $F_1(u,v)$

$$F_1(u,v) = \text{FT} ( P_1(x,y) )$$

Focal plane mask complex amplitude transmission:  $M(u,v)$

Focal plane complex amplitude (after focal plane mask):  $F_2(u,v)$

$$F_2(u,v) = F_1(u,v) \times M(u,v) = \text{FT}(P_1(x,y)) \times M(u,v)$$

Exit pupil plane:

$$P_2(x,y) = \text{FT}^{-1}( F_2(u,v) ) = \text{FT}^{-1} ( \text{FT}(P_1(x,y)) \times M(u,v) ) = P_1(x,y) * \text{FT}^{-1}(M(u,v))$$

With \* denoting convolution

$$P_3(x,y) = L(x,y) \times P_2(x,y)$$

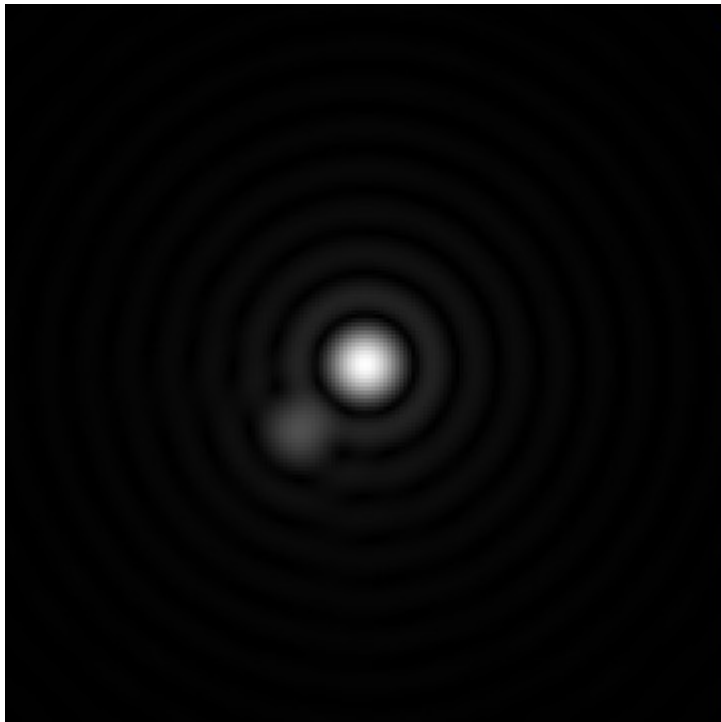
$$\mathbf{P_3(x,y) = L(x,y) \times (P_1(x,y) * FT^{-1}(M(u,v)))}$$

$$F_3(u,v) = \text{FT}(L(x,y)) * (F_1(u,v) \times M(u,v))$$

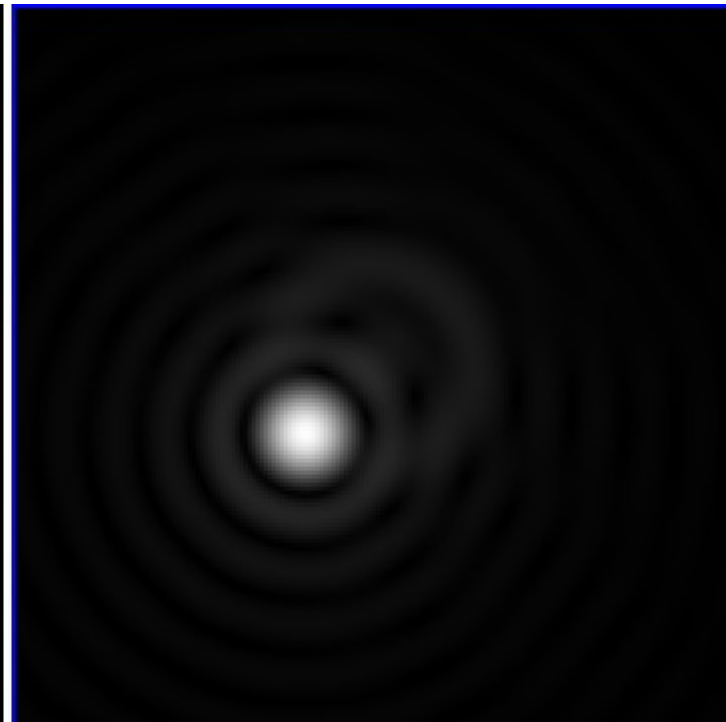
Coronagraphy problem: minimize  $P_3(x,y)$  for on-axis point source

# Numerical simulation of final image for 10:1 contrast

No coronagraph



With Lyot  
Coronagraph



# Coronagraphy performance metrics

Inner Working Angle (IWA), unit =  $\lambda/D$

How close can a faint source be imaged from the star. IWA is usually defined as the 50% peak throughput point (where the source throughput is 50% of the maximum throughput)

Contrast (example:  $1e10$  contrast, or  $1e-10$  contrast)

What is usually quoted is the raw contrast (not the detection contrast). Raw contrast = ratio of local surface brightness to peak PSF surface brightness

Null order (example: 4th order null coronagraph)

Quantifies coronagraph throughput as a function of angular separation close to the optical axis. Higher order = more immune to residual pointing error and stellar angular size (but larger IWA)

# Coronagraphy performance metrics

## Throughput

Coronagraph transmission as a function of angular separation.  
Can be a 1D function or a 2D map.

## Discovery space

Fraction of the field over which a planet can be identified.  
(Excludes zones of the image where significant starlight is present)

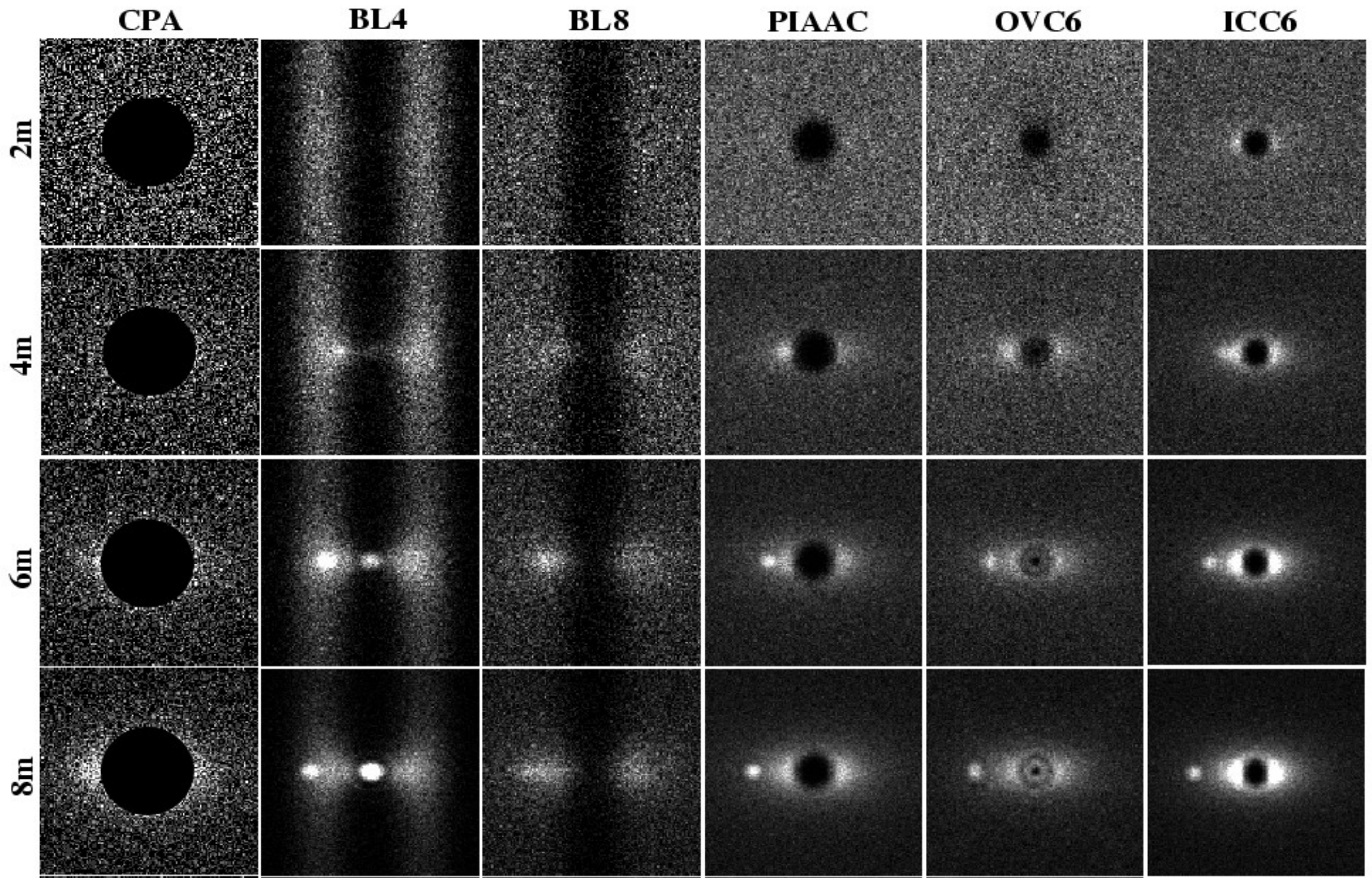
## Angular resolution

How large is the planet image (can be different from IWA !). This is important for background-limited detections and astrometry (testing proper motion and orbital motion).

## Useful throughput

Fraction of the planet light that can be used toward a discovery.  
Excludes planet light falling on top of brighter starlight.

Coronagraphs with different throughput, IWA, contrast, discovery space, angular resolution  
Simulated observations of a Sun/Earth system without wavefront errors  
Telescope diameter shown on the left

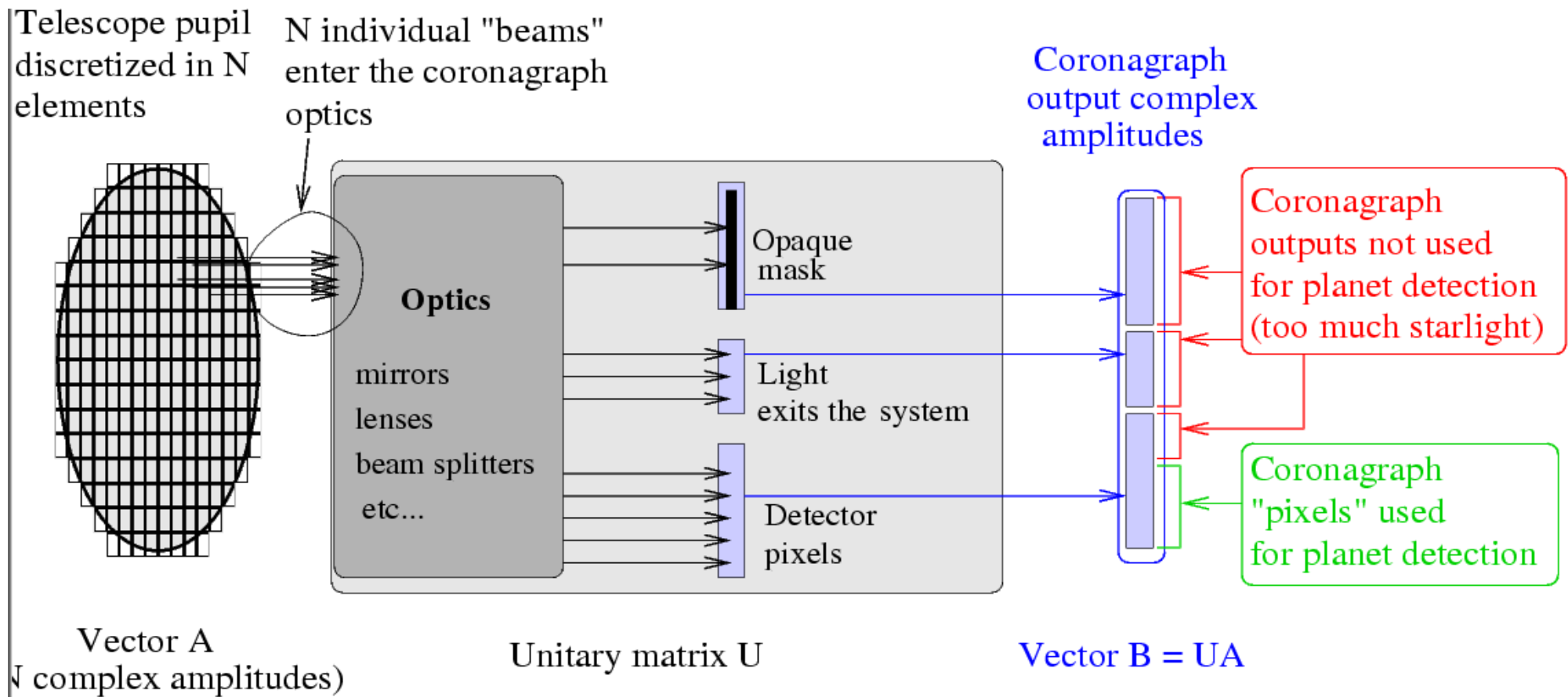


# Theoretical limits

## What can coronagraphs do ?



Coronagraphs are Linear system in complex amplitude  
 Fourier transforms, Fresnel propagation, interferences,  
 every wavefront control scheme: all are linear in complex  
 amplitude



# What is the theoretical performance limit of coronagraphy ?

Coronagraph is a linear filter (in complex amplitude) which removes starlight.

If :

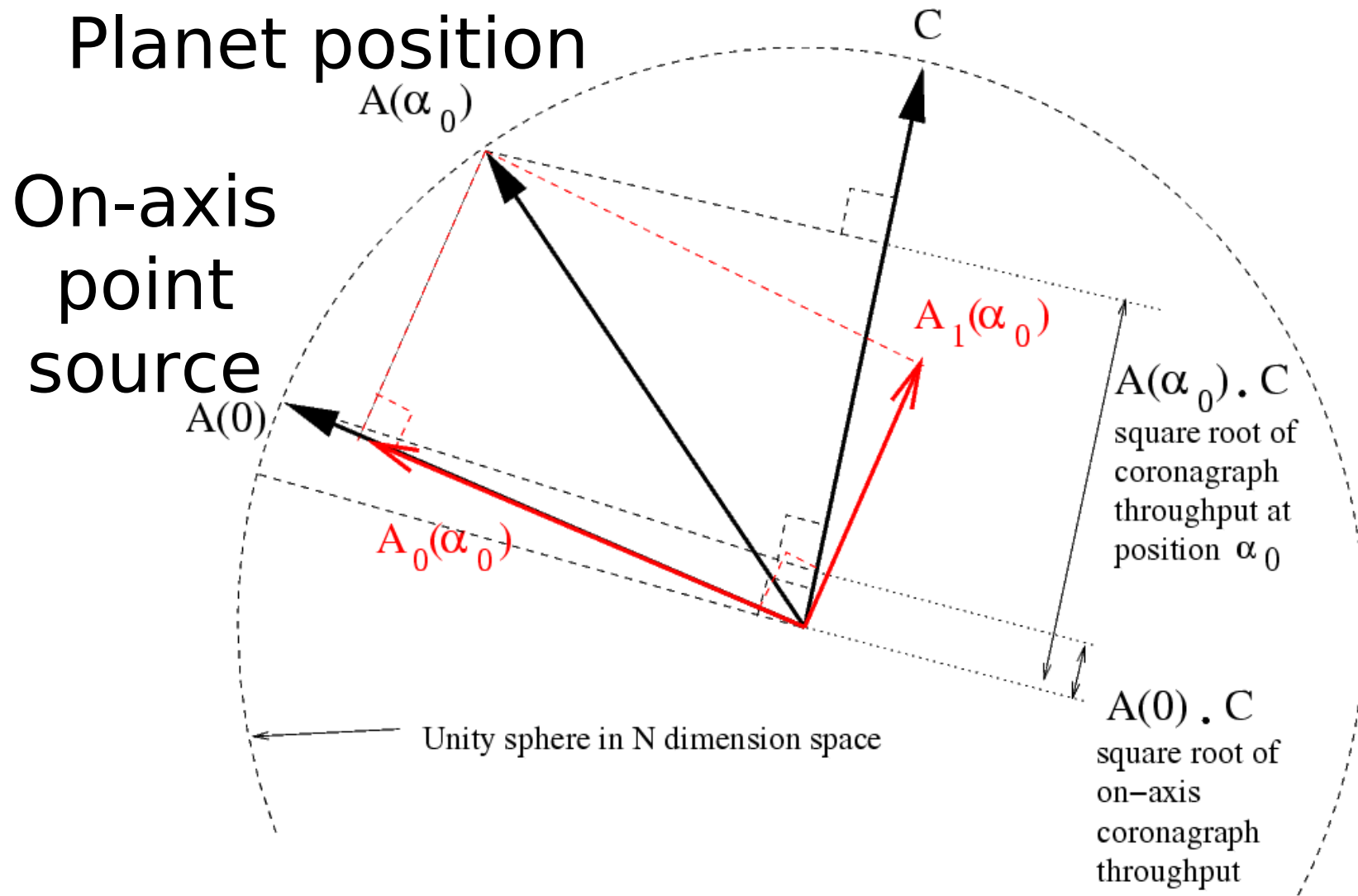
planet = 0.2 x starlight wavefront + 0.8 x something else  
then:

coronagraph throughput for planet < 0.8

If planet is close to star, planet wavefront becomes very close to star wavefront → the coronagraph has to remove most of the planet light when it removes the starlight

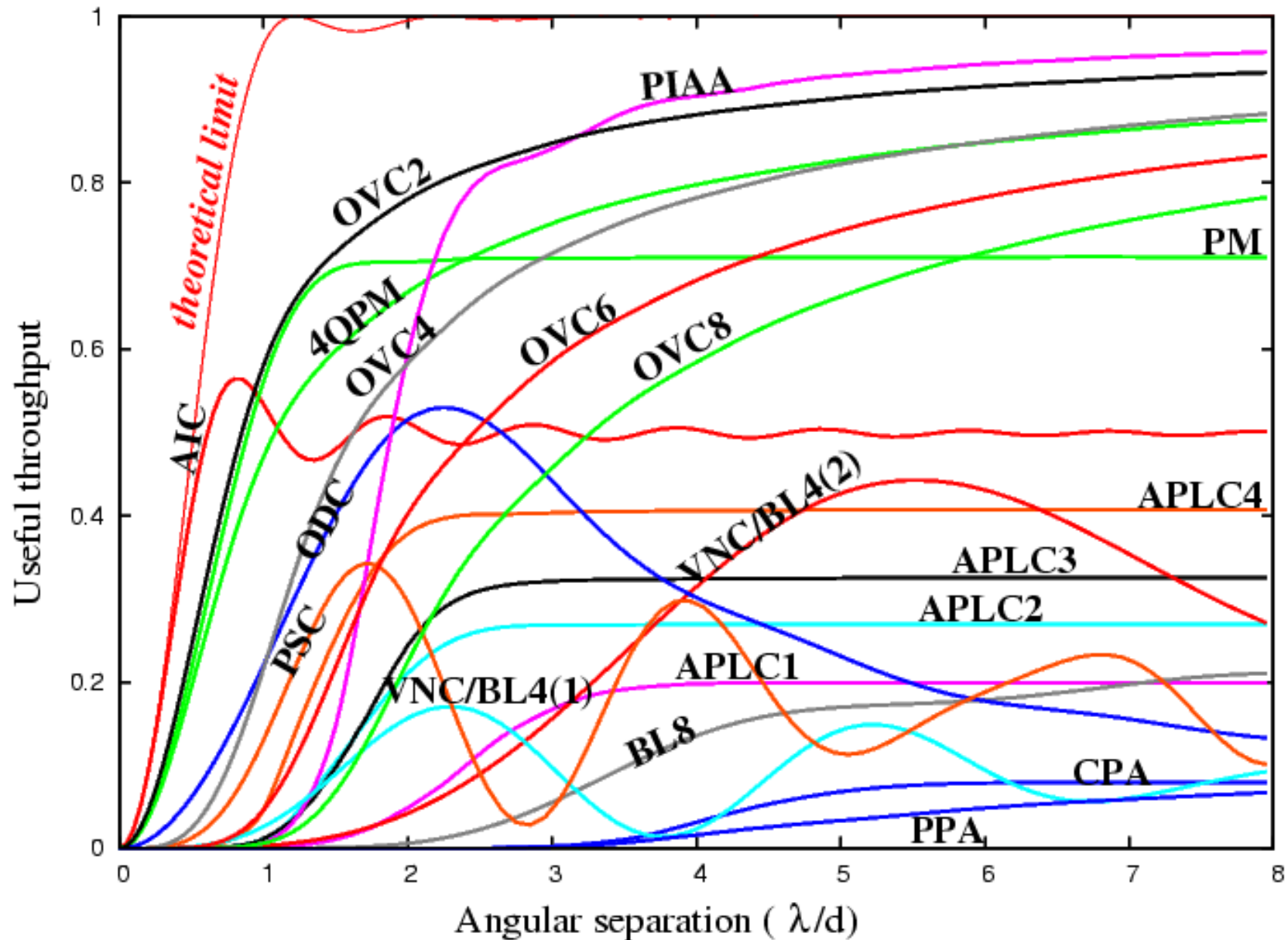
(Guyon, Pluzhnik, Kuchner, Collins & Ridgway 2006, *ApJS* 167, 81)

# Graphical representation of the coronagraph throughput



# Useful throughput of existing coronagraphs concepts

**Point source / Radially averaged throughput (1e10 contrast)**



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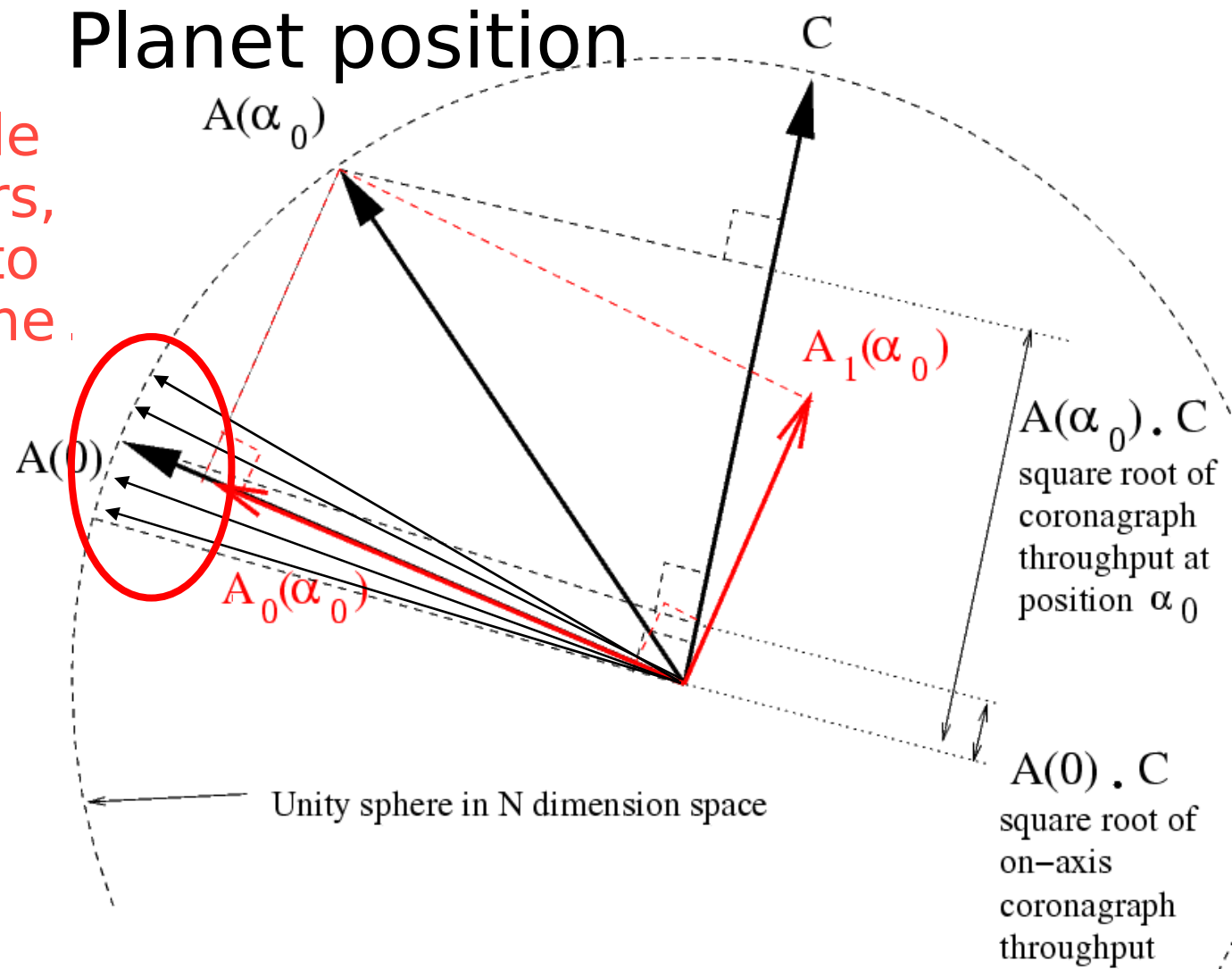
Theoretical limit would offer high contrast with little loss in throughput and inner working angle close to 1  $\lambda/D$ .

BUT: at high contrast, stars are not points ->  
performance limit is severely affected by stellar angular size

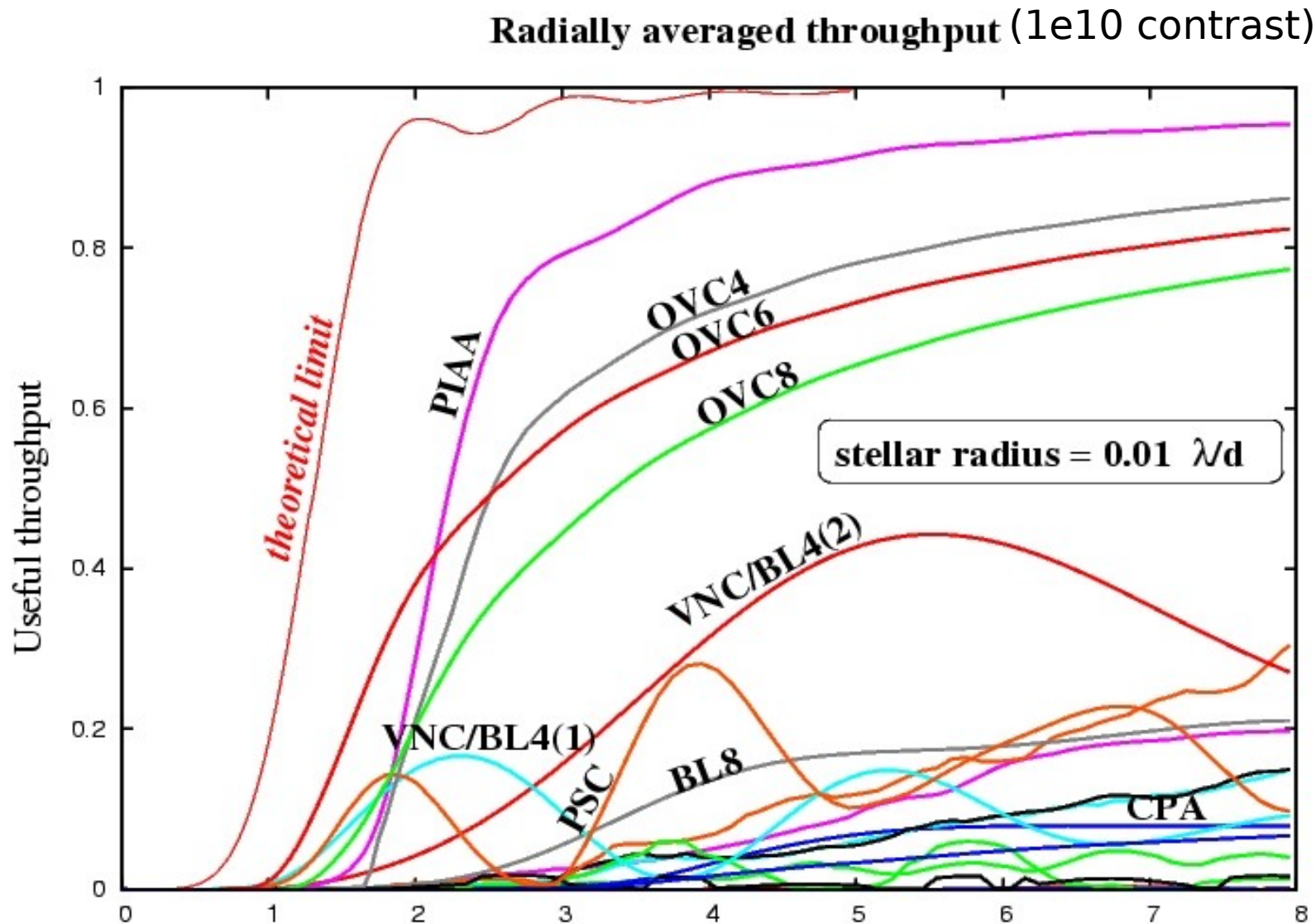
(Guyon, Pluzhnik, Kuchner, Collins & Ridgway 2006, *ApJS* 167, 81)

# Graphical representation of coronagraph throughput

Central star is made of a group of vectors, ALL of which need to be cancelled to some degree.



Problem: stars are not points !  
Sun diameter  $\sim 1\%$  of 1 AU  
If  $1\text{AU} = 2 \text{ l/d}$ , Stellar radius  $\sim 0.01 \text{ l/d}$   
Wavefront control cannot solve it



# Why is it so serious ?

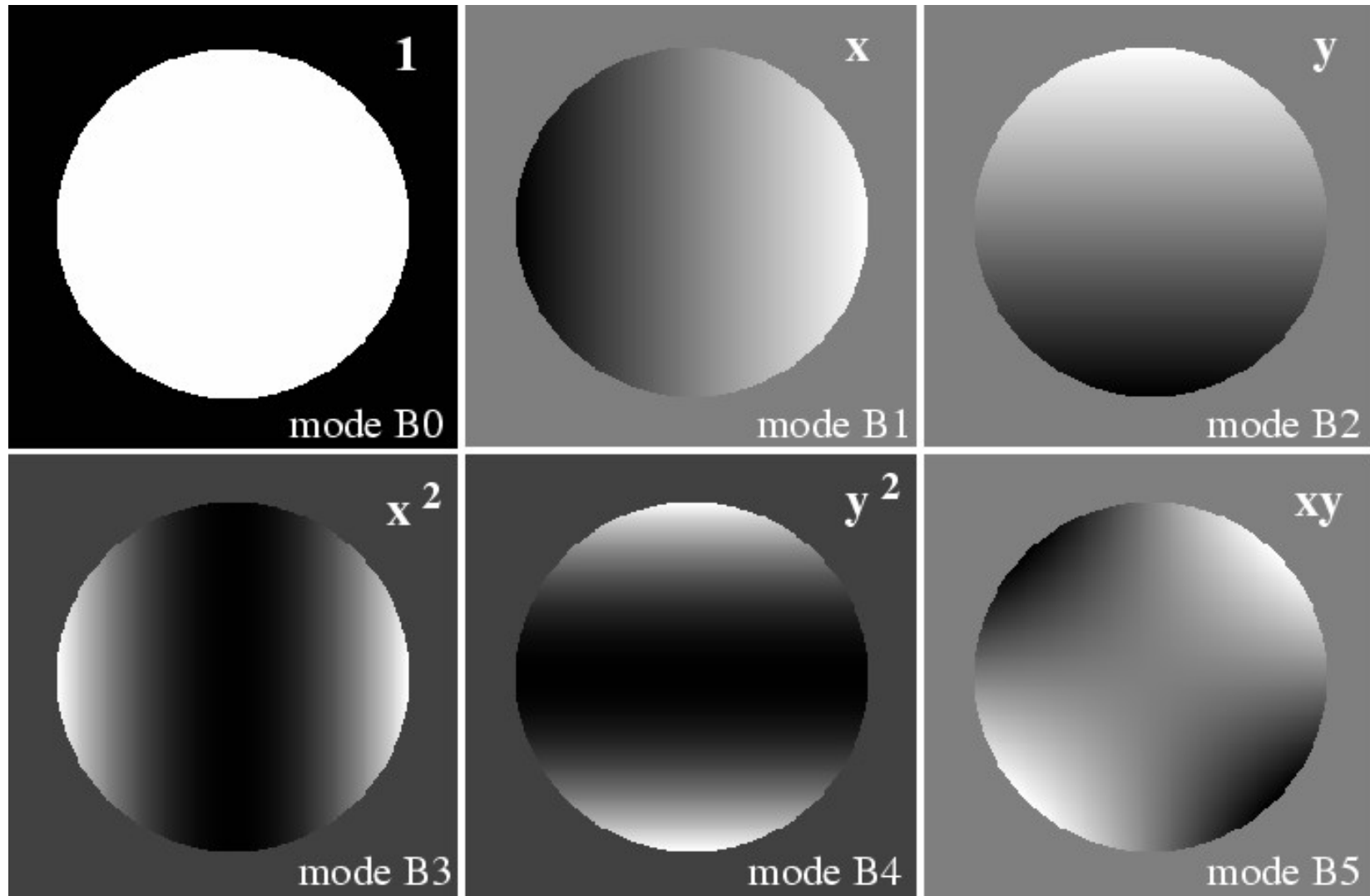
Stellar size makes light incoherent  
Sun diam = 1% of Sun-Earth distance

No hope of fixing this by wavefront control, the coronagraph has to deal with it !

In a stellar size limited coronagraph, remaining speckles have opposite complex amplitude from one side of the star to the other. Adding complex amplitude can only increase intensity.



Need to remove more than 1 mode from the incoming wavefront (how many and how well depends on the star size and desired contrast)



# Theoretical limit with increasing stellar radius (monochromatic light, $1e10$ contrast)

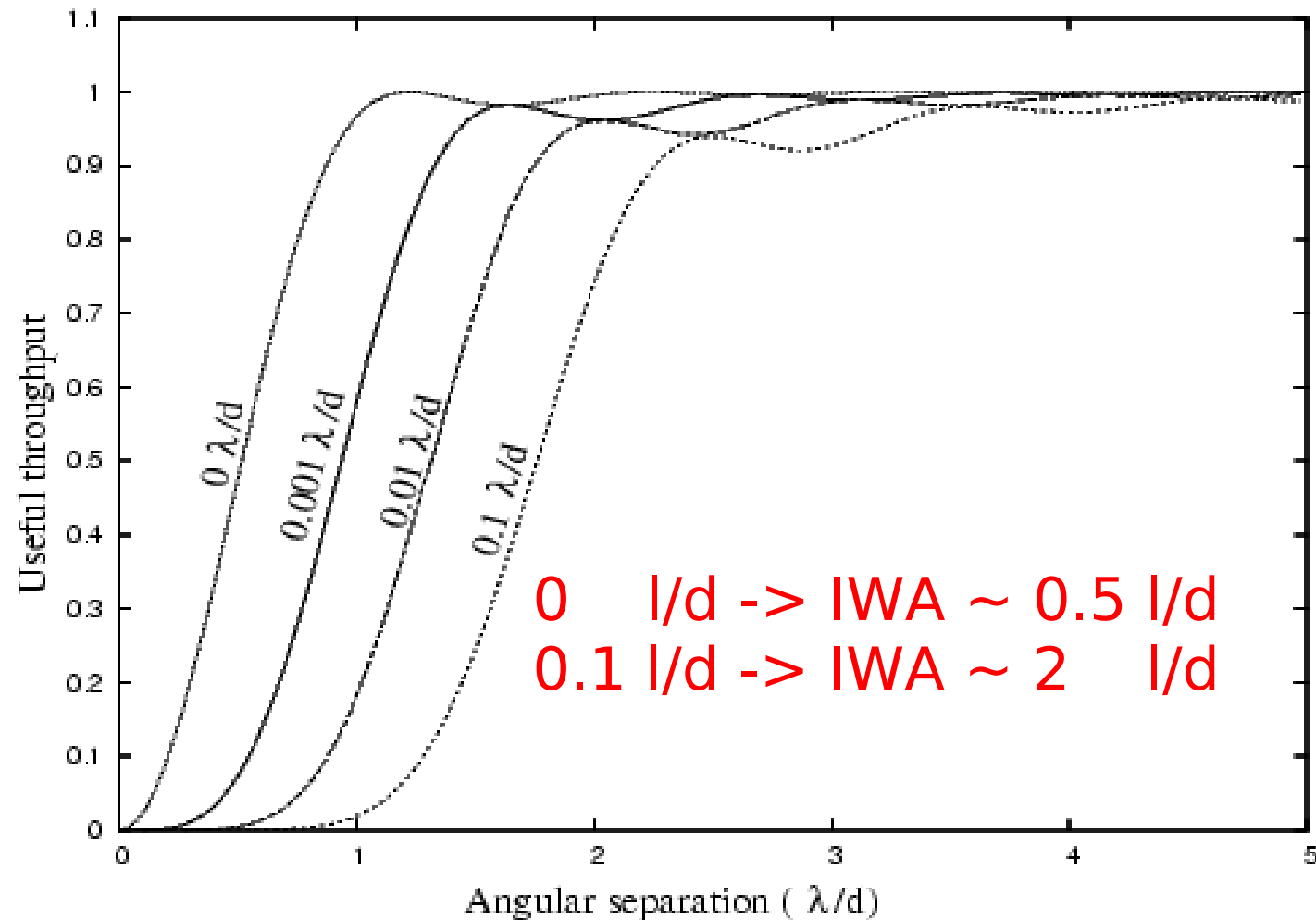


Fig. 5.— Upper limit on the off-axis throughput of a coronagraph for different stellar radii.