High contrast imaging

Lyot Coronagraph

Stellar coronagraphy is a diffractive problem (not geometrical optics)

Lyot Coronagraph was first developed to observe the solar corona



Lyot Coronagraph architecture

Relies on focal plane mask AND pupil mask (Lyot stop) to augment contrast

Why a Lyot pupil mask ?

Focal plane occulter blocks central part of the image = low spatial frequencies in pupil plane
What is left after focal plane mask are high spatial frequencies in pupil plane = light around edges

•This light can be masked by an undersized pupil plane stop



figure from Lyot project website

Pupil plane complex amplitude ↔ focal plane complex amplitude

→ Fourier transform

← Inverse Fourier transform

Coordinates in pupil plane: x,y Coordinates in focal plane : u,v * denoting convolution (product = convolution in Fourier transform)



Entrance pupil of telescope: $P_1(x,y)$

Focal plane complex amplitude (before focal plane mask): $F_1(u,v)$

 $F_1(u,v) = FT (P_1(x,y))$

Focal plane mask complex amplitude transmission: M(u,v)Focal plane complex amplifude (after focal plane mask): $F_2(u,v)$

$$F_2(u,v) = F_1(u,v) \times M(u,v) = FT(P_1(x,y)) \times M(u,v)$$

Exit pupil plane:

 $P_{2}(x,y) = FT^{-1}(F_{2}(u,v)) = FT^{-1}(FT(P_{1}(x,y) \times M(u,v)) = P_{1}(x,y) * FT^{-1}(M(u,v))$ With * denoting convolution $P_{3}(x,y) = L(x,y) \times P_{2}(x,y)$ $P_{3}(x,y) = L(x,y) \times (P_{1}(x,y) * FT^{-1}(M(u,v)))$

$$F_{3}(u,v) = FT(L(x,y)) * (F_{1}(u,v) \times M(u,v))$$

Coronagraphy problem: minimize $P_3(x,y)$ for on-axis point source

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Focal plane complex amplitude (before focal plane mask): $F_1(u,v)$ $F_1(u,v) = FT (P_1(x,y))$

> |F₁(u,v)| $P_1(x,y)$ FT **FT**-1

Focal plane mask complex amplitude transmission: M(u,v) Focal plane complex amplifude (after focal plane mask): $F_2(u,v)$ $F_2(u,v) = F_1(u,v) \times M(u,v) = FT(P_1(x,y)) \times M(u,v)$



Exit pupil plane: $P_2(x,y) = FT^{-1}(F_2(u,v))$ $= FT^{-1}(FT(P_1(x,y) \times M(u,v)) = P_1(x,y) * FT^{-1}(M(u,v))$





$$\begin{split} \mathsf{P}_{3}(x,y) &= \mathsf{L}(x,y) \times \mathsf{P}_{2}(x,y) \\ \mathbf{P}_{3}(x,y) &= \mathsf{L}(x,y) \times (\mathsf{P}_{1}(x,y) * \mathsf{FT}^{-1}(\mathsf{M}(u,v))) \\ \mathsf{F}_{3}(u,v) &= \mathsf{FT}(\mathsf{L}(x,y)) * (\mathsf{F}_{1}(u,v) \times \mathsf{M}(u,v)) \end{split}$$



Numerical simulation of final image for 10:1 contrast

No coronagraph





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