

Astrometry

OUTLINE:

What is astrometry ?

Measurement principles

Scientific results and future prospects

Optical design and tricks

Astrometry: definition & units

Astrometry : measuring the position of sources on the sky

Technically challenging:

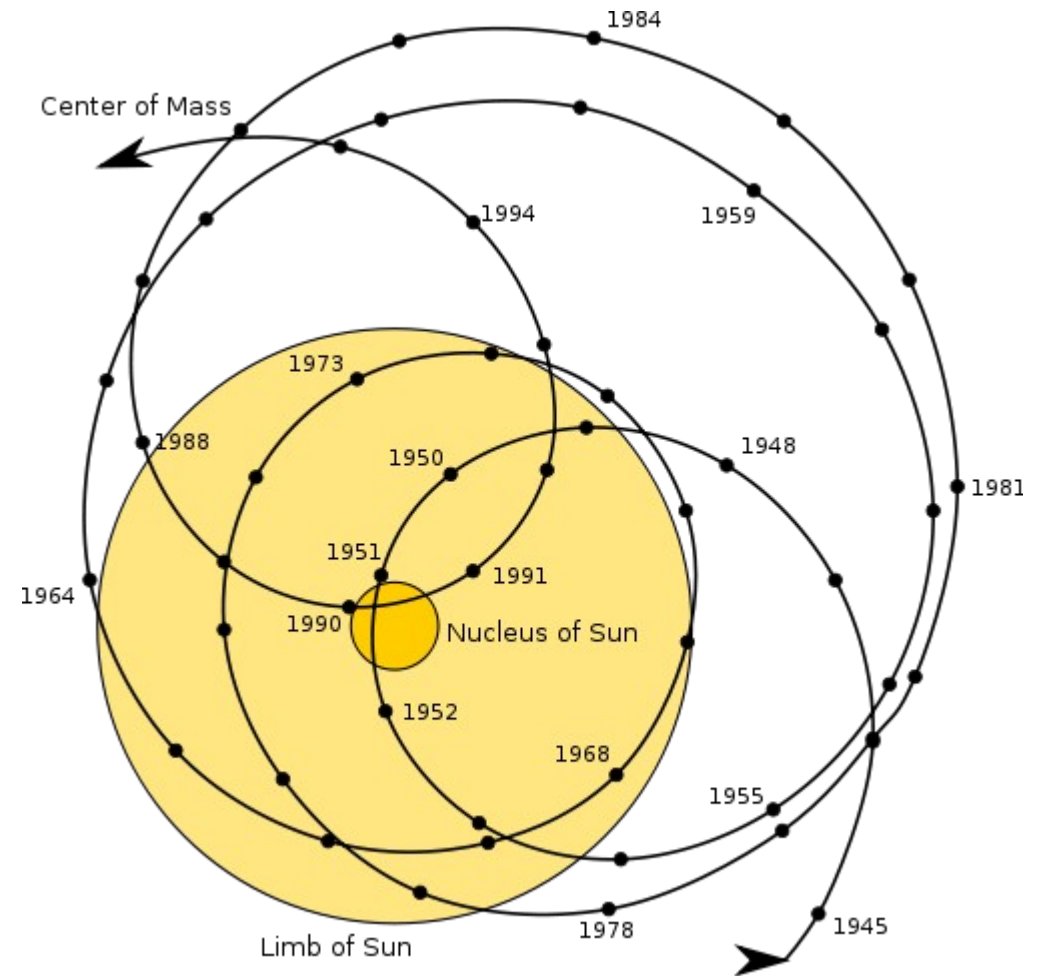
- Angles to be measured are often extremely small
- Difficult to find a proper reference frame

Units:

arcsecond ($1/3600$ deg) = 4.8×10^{-6} rad
(4.8mm at 1km distance)

milliarcsecond = 4.8×10^{-9} rad

microarcsecond = 4.8×10^{-12} rad



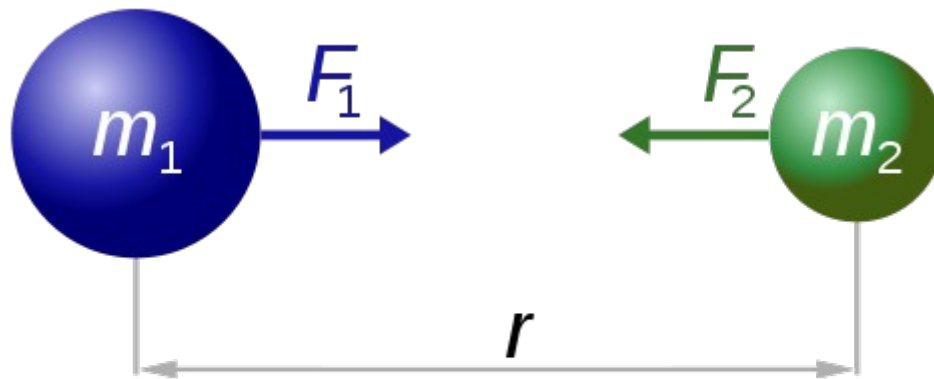
***Astrometric motion of the Sun
due to solar system planets***

Astrometry and masses

Gravity → objects do not travel in straight lines in space (mass generates acceleration)

- objects fall on Earth (parabola)
- satellites orbit planets (moons)
- planets orbit stars
- stars orbit the center of galaxies
- galaxies orbit center of galaxy clusters

astrometric measurement allows orbits/trajectories to be mapped → mass measurement



$$F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}$$

Example: Galactic center

Galactic center hosts a massive black hole

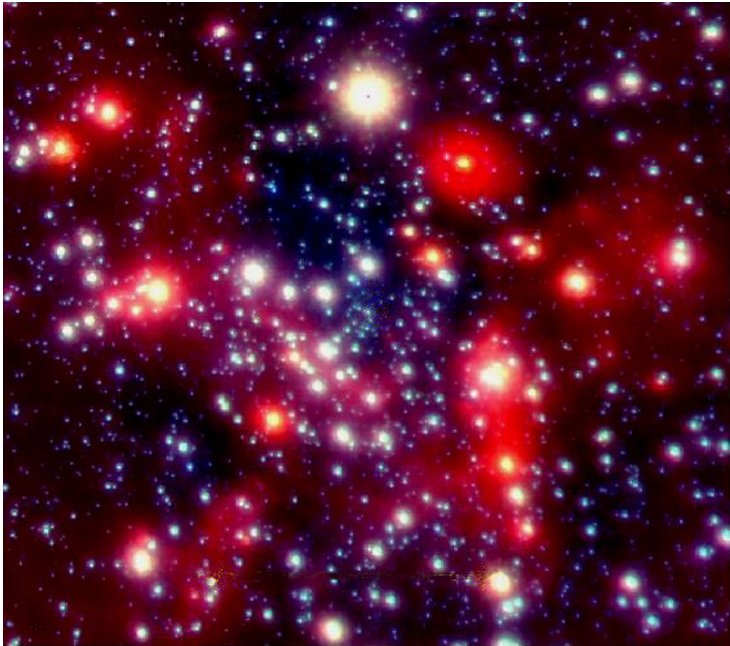
The black hole was confirmed and its mass measured by astrometry of stars orbiting it, using adaptive optics on large telescopes (VLT, Keck)

Astrometric measurement accuracy required is \ll arcsec

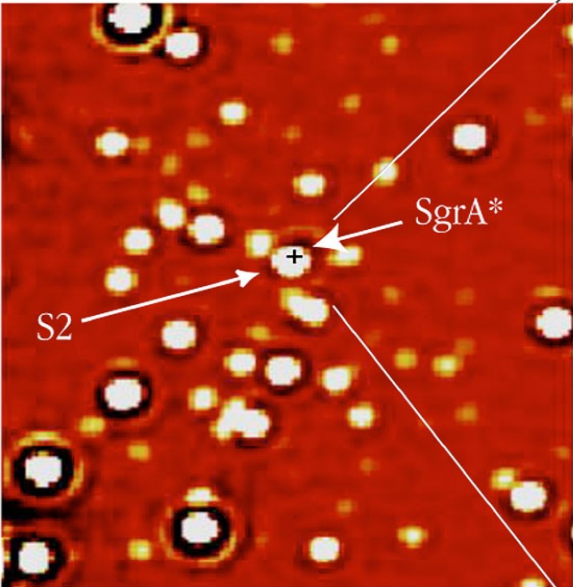
Field is particularly crowded

Black hole mass = $2.6 \times 10^6 \times \text{Sun}$

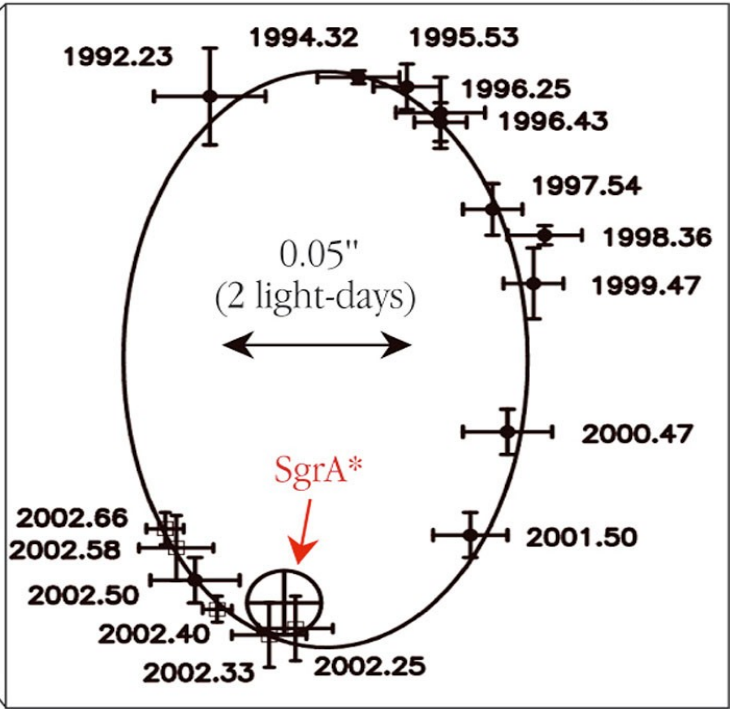
NOTE: best ground-based astrometric precision $\sim 100 \mu\text{as}$



NACO May 2002



S2 Orbit around SgrA*



Astrometric signature of planets

Star AND planet orbit the center of mass of the system

planet mass

star mass

distance to system

orbit radius

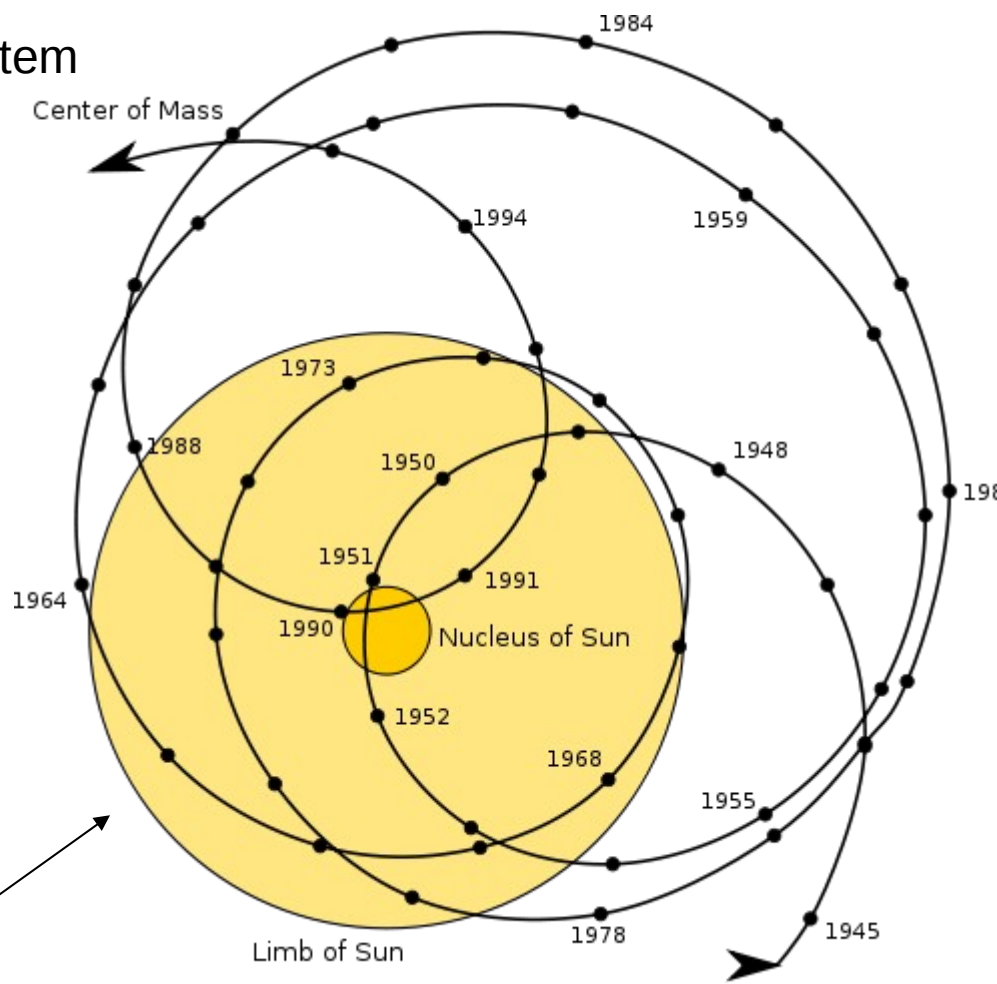
signal = $(m_{\text{planet}}/m_{\text{star}}) (a/d)$

0.3 uas for Earth around Sun at 10 pc
0.5 mas for Jupiter around Sun at 10 pc

signal period = orbital period

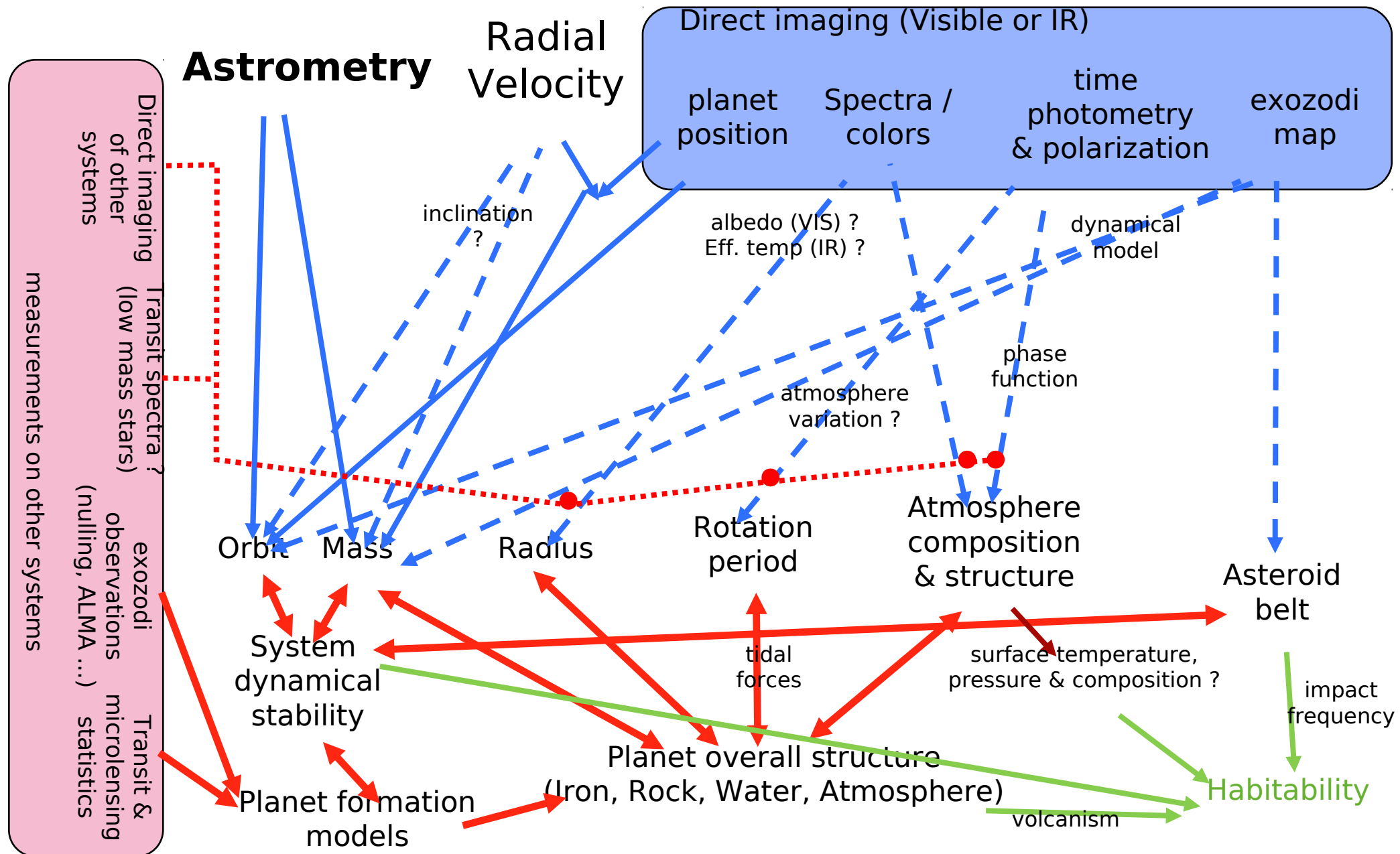
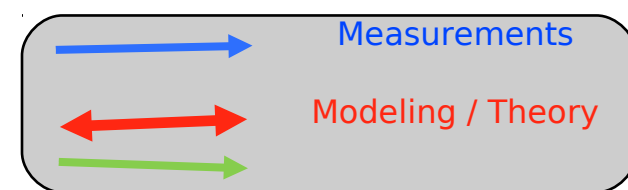
For circular orbit, an ellipse is observed
(circle only if viewed face-on)

At 10pc, sun diam = 1mas



Combined astrometric signal of solar system planets

From observations to planet habitability



Astrometric Measurement

Astrometric measurement consists of:

- **Proper motion (2 parameters):** Star motion in space, projected on sky. Typically $\sim 1''$ per year for nearby stars.
- **Proper motion acceleration (1 parameter – sometimes ignored):** At constant 3D speed, the apparent 2D proper motion accelerates as star becomes closer
- **Parallax (1 parameter: distance to system):** Apparent motion on sky due to Earth's orbit around Sun, which changes view point. Amplitude = $0.1''$ for star at 10pc, inverse proportional to distance. Shape is ellipse (circle for star at ecliptic pole)
- **Aberration of light (known):** stretch due to velocity of observer relative to viewing angle: $40''$ amplitude, 1 yr period
- **Gravitational signal (known):** Sun bends light (was first proven by observing solar eclipse)
- **Exoplanet 3D orbit and mass (7 parameters):** Period, Time of periastron, eccentricity, semi-major axis of stellar motion ellipse, longitude of ascending node, argument of periastron, orbital inclination

Period and semi-major axis of stellar motion ellipse \rightarrow planet mass (assuming star mass is known)

Note: exoplanet signal is usually much smaller than proper motion and parallax

TOTAL : $5 + 7n$

5 parameters for star (position on sky (2), distance (1), proper motion (2))

7 parameters for each planet

\rightarrow **multiple observations are required**

How well can the position of a source be measured in an image ?

Photon noise limit (rarely achieved): std deviation (1D) = $0.318 / \sqrt{N_{\text{ph}}}$

A.1. Astrometric accuracy at the diffraction limit

With an ideal telescope (no wavefront aberration, no central obstruction), the accuracy with which a monochromatic PSF (exact Airy function) can be localized in 2D is, in the photon-noise regime:

$$\sigma_{2D} [\lambda/D] = \sqrt{2}/(\pi \sqrt{N_{\text{ph}}}) = 0.450/\sqrt{N_{\text{ph}}} \quad (\text{equ A.1.1})$$

where D is the telescope diameter, λ is the wavelength and N_{ph} is the total number of photon available for the measurement.

If the PSF position is measured along one axis only, the measurement error is:

$$\sigma_{1D} [\lambda/D] = 1/(\pi \sqrt{N_{\text{ph}}}) = 0.318/\sqrt{N_{\text{ph}}} \quad (\text{equ A.1.2})$$

These measurement accuracies are obtained with an optimal matched filter which optimally weights each pixel according to SNR. Numerically these equations can be verified by starting from the PSF (noted $\text{PSF}(x,y)$) and computing:

$$(1/\sigma_x)^2 = \text{SUM}_{x,y} (1/\sigma_{\text{pixel } x,y})^2 = \text{SUM}_{x,y} ((d(\text{PSF}(x,y))/dx)^2 / \text{PSF}(x,y)) \quad (\text{equ A.1.3})$$

with $\sigma_{1D} = \sigma_x = \sigma_y$, and $\sigma_{2D} = \sqrt{2} * \sigma_{1D}$. Equation (A.1.3) shows for each pixel the signal (equal to $d(\text{PSF}(x,y))/dx$) and the noise (equal to $\sqrt{\text{PSF}(x,y)}$).

Approximate rule of thumb:

Precision \sim PSF size / $\sqrt{N_{\text{ph}}}$

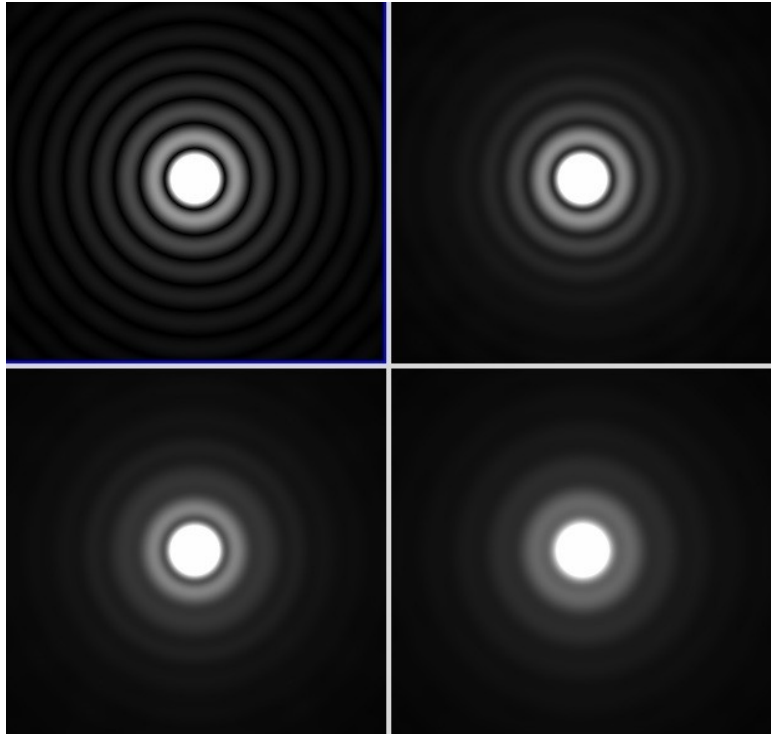
1-m diameter telescope at diffraction limit
20% wide spectral band, visible light

$m_v=5$ star, 1 hr observation

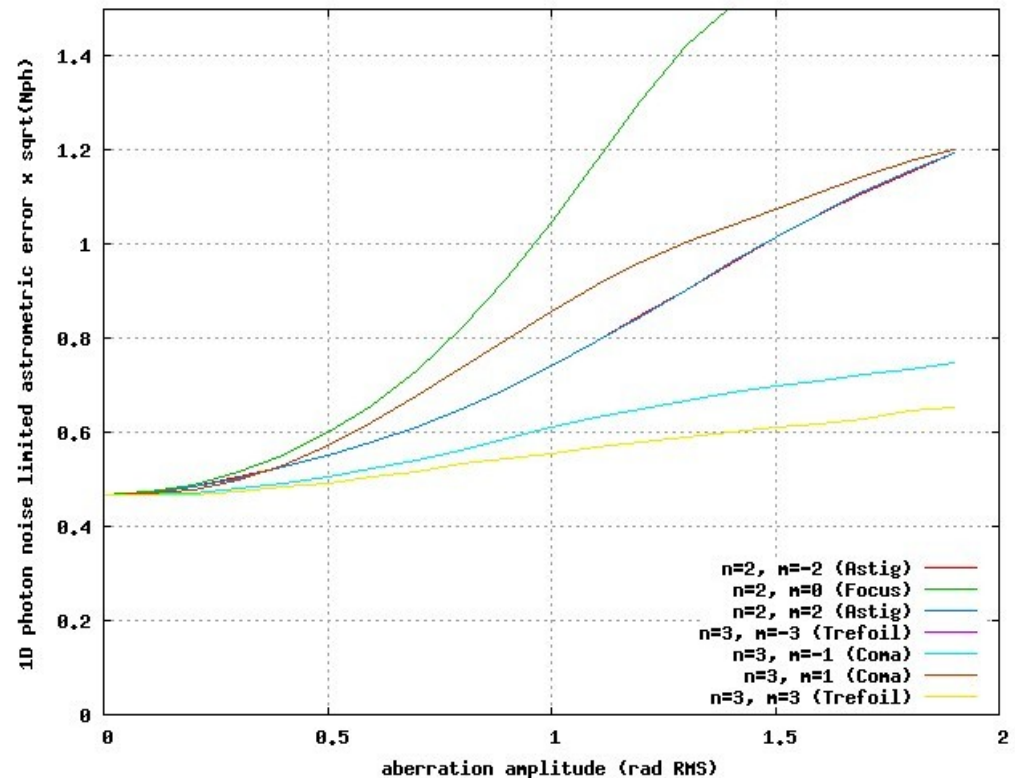
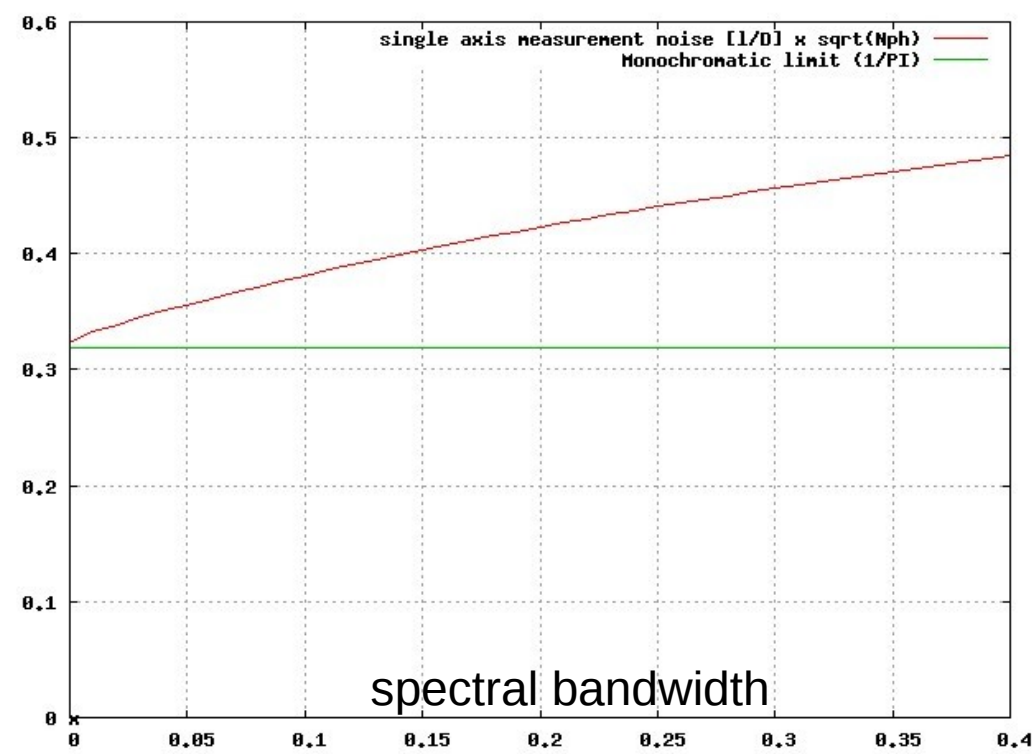
$\rightarrow 0.2 \mu\text{as}$ (sufficient to detect Earth at 10pc
with ~ 30 observations)

How well can the position of a source be measured in an image ?

Chromaticity, PSF aberrations

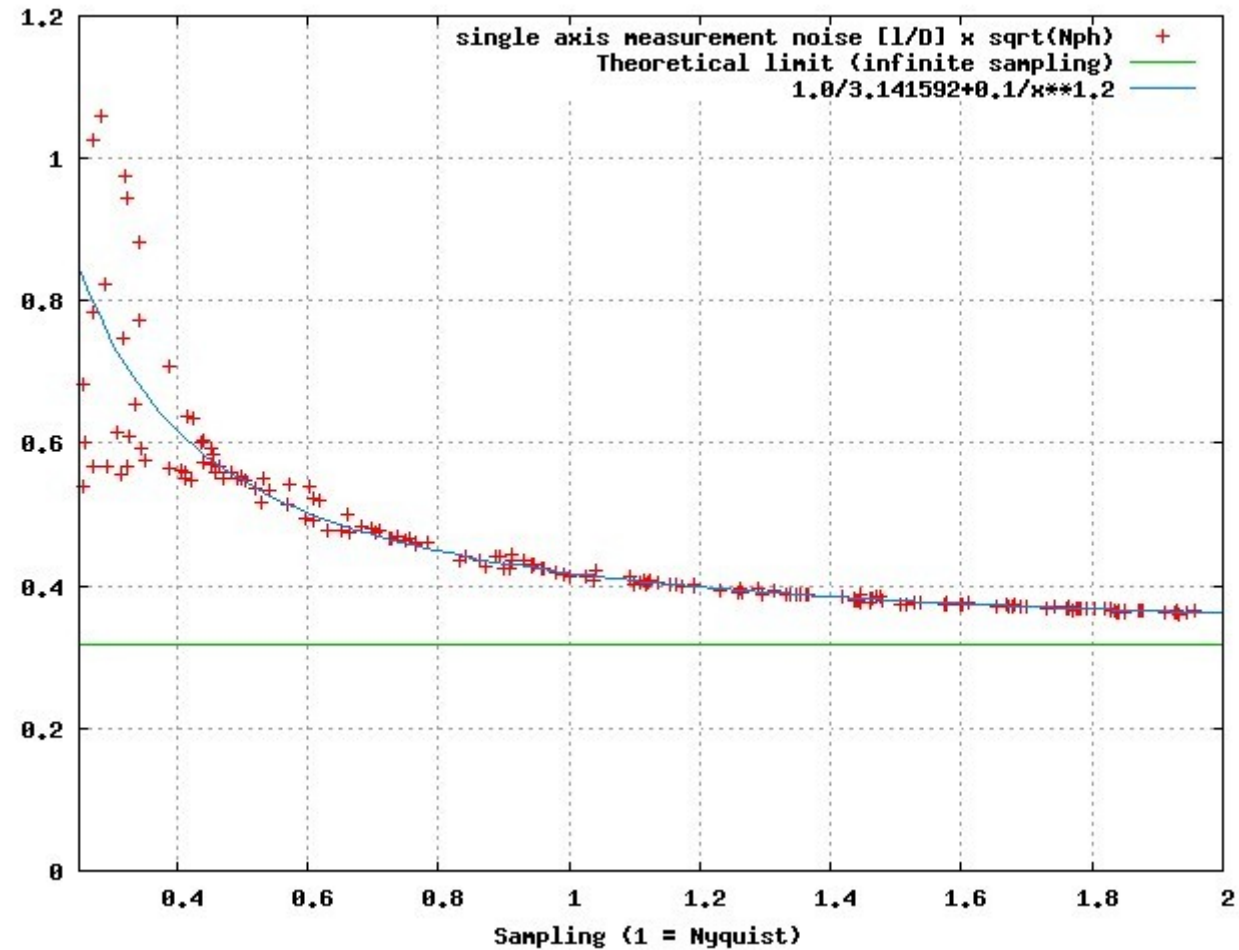
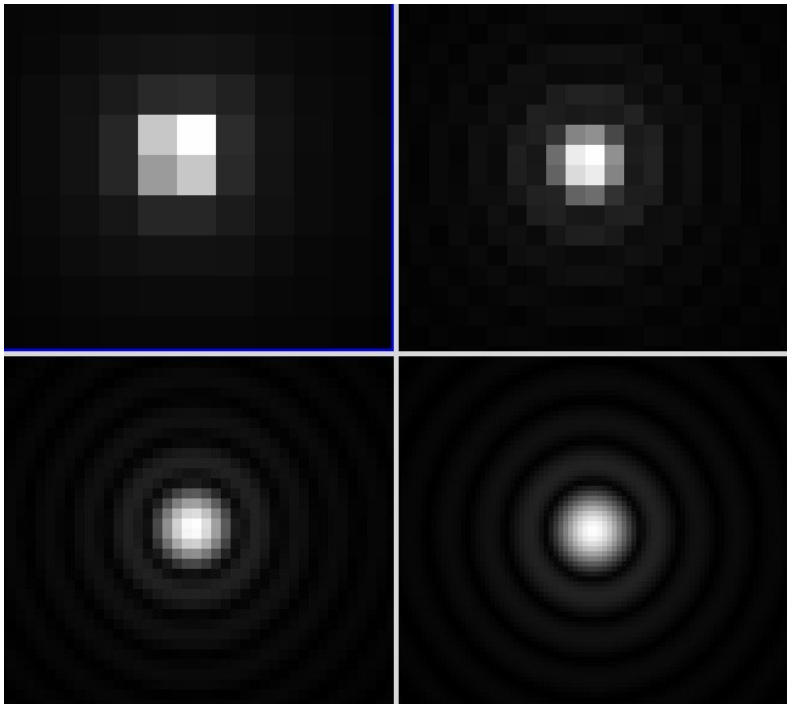


astrometric precision (normalized to 1 photon)



How well can the position of a source be measured in an image ?

Effect of pixel sampling



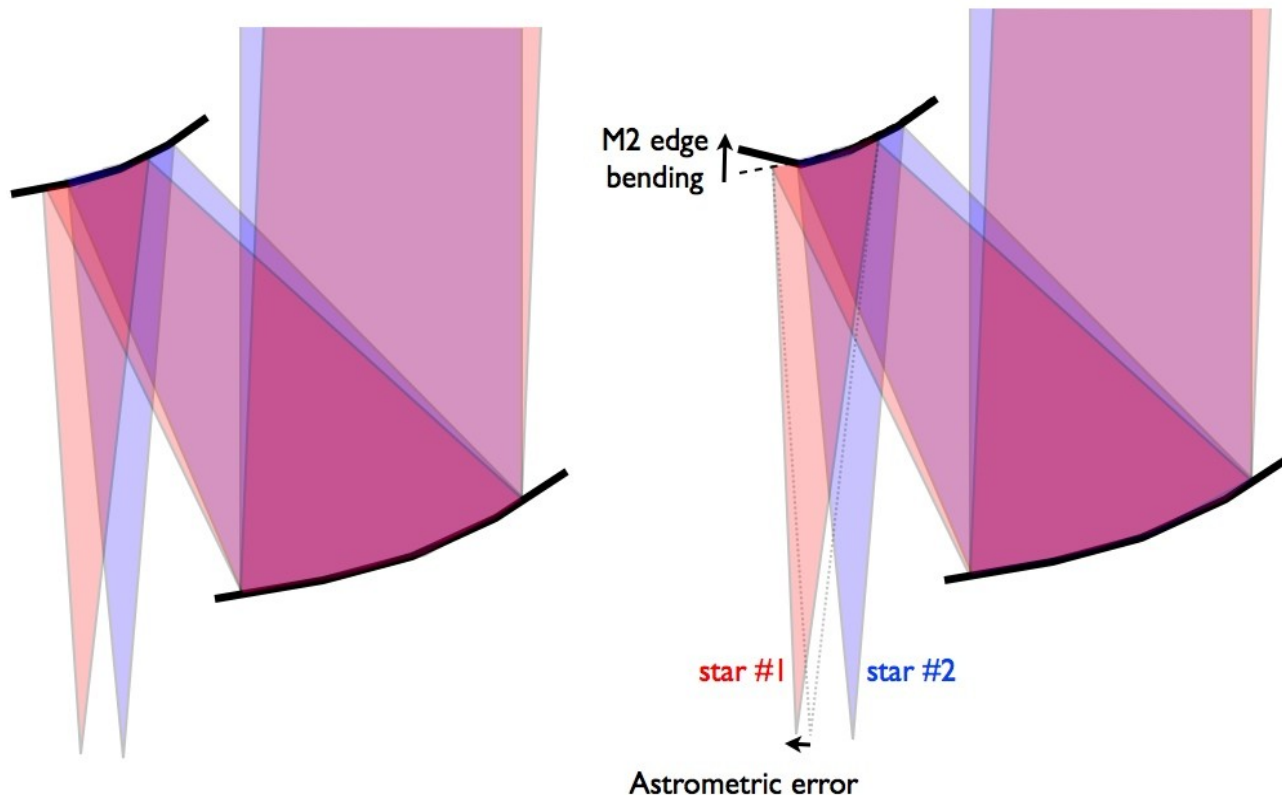
Astrometric distortions introduced by telescope and instrument: **USUALLY DOMINANT TERM**

Optics that are not in pupil plane introduce astrometric distortions: beam footprint on these optics is function of source position on sky

→ sub-uas level astrometry requires pm-level stability (or knowledge/calibration) of optical elements

→ astrometry with conventional imaging systems is extremely challenging at the uas level

On-axis and off-axis stars illuminate different (but overlapping) parts of M2.
Edge bending on M2 is seen by star #1, but not star #2.



+ detector is not sufficiently stable

Focal plane with 10um pixels,
Nyquist sampled
1-m diameter telescope in visible:
1 pixel = 0.05 arcsec

1uas = 1/50000 pixel = 0.2 nm

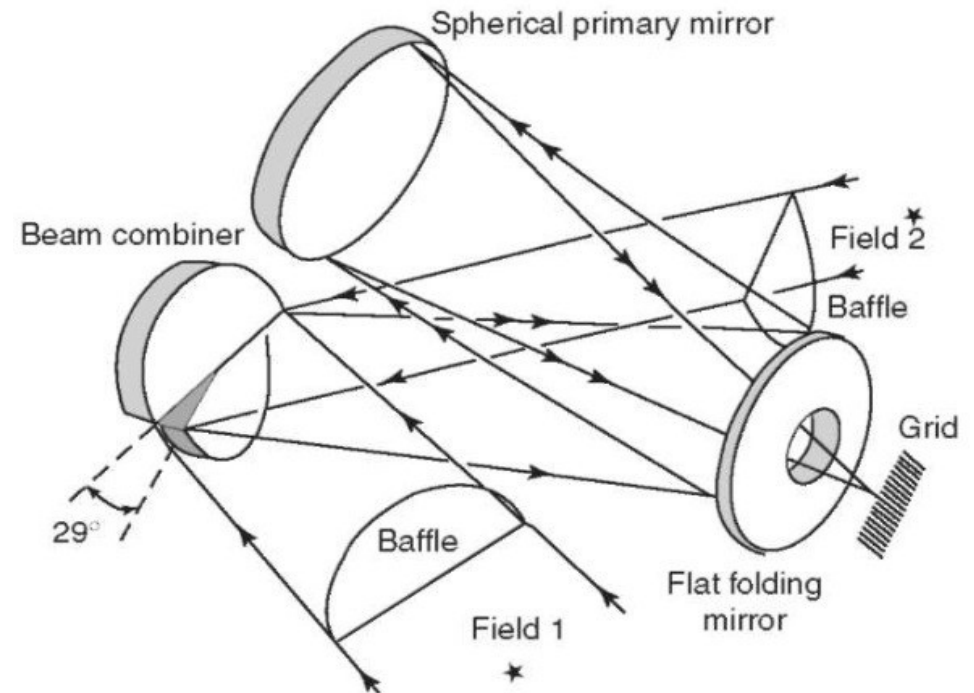
Hipparcos (ESA, 1989-1993)

milliarcsecond-level astrometry of >100000 stars

Observes 2 fields 58 deg apart with 29cm wide field telescope

Uses a fine pitch transmission grid in the focal plane to modulate signal of a star as it drifts on the grid. Star position measured by phase of the signal.

→ allows high precision photometric measurement without large pixel count detector (not available in 80s)

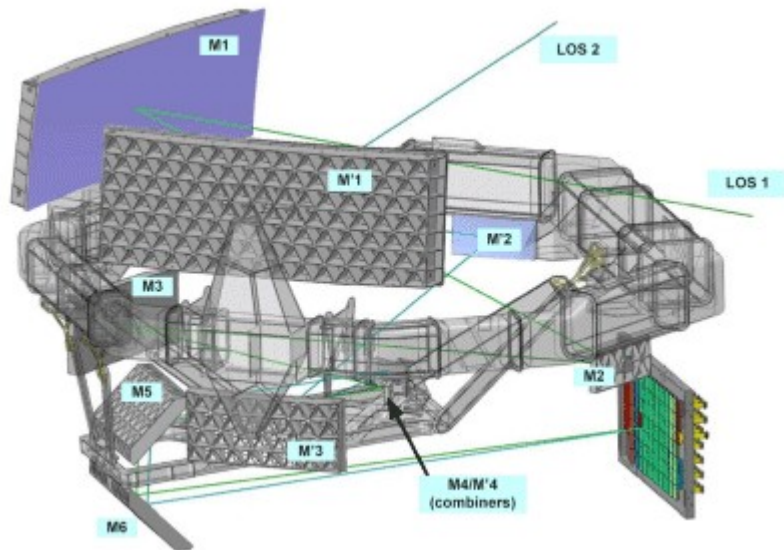


GAIA (ESA)

Simultaneously observes two lines of sight, 106.5 deg apart

Two wide field telescopes with aspheric mirrors

Continuously scans the sky as pointing rotates – each star observed 70 times



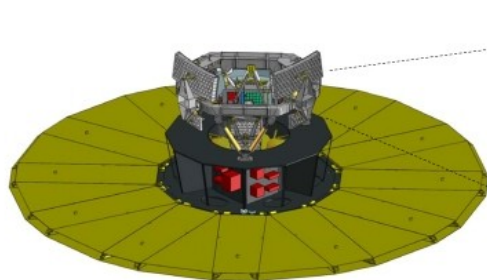
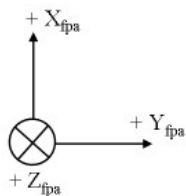
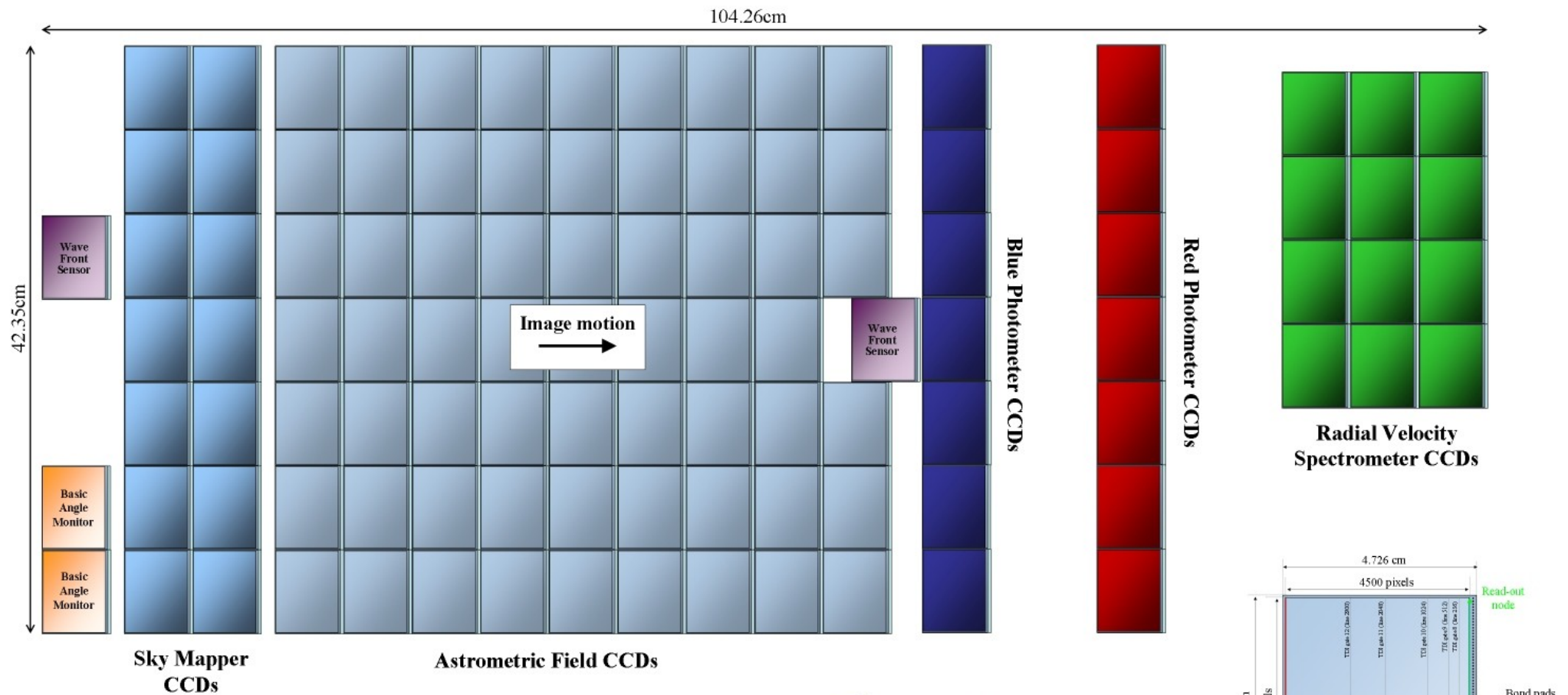
Astrometric performance (parallax error)		
Stellar type	V mag	$\sigma(\pi)$ μ as
B1V	< 10	< 7
	15	< 25
	20	< 300
G2V	< 10	< 7
	15	< 24
	20	< 300
M6V	< 10	< 7
	15	< 12
	20	< 100



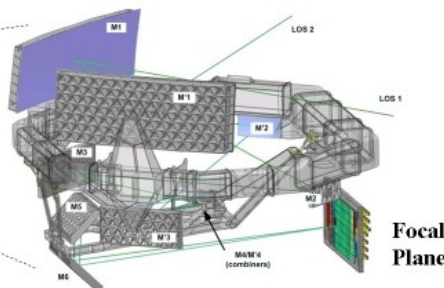


Gaia Focal Plane

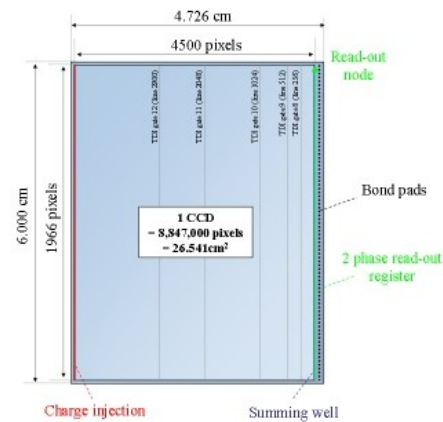
106 CCDs \approx 938 million pixels \approx 2800 cm²



Gaia (MLI removed)



Payload Module (telescopes)



NEAT concept (Shao et al.)

Telescope distortions due to out-of-pupil optics
 → use telescope with no secondary mirror
 → use interferometer fringes to calibrate detector geometry

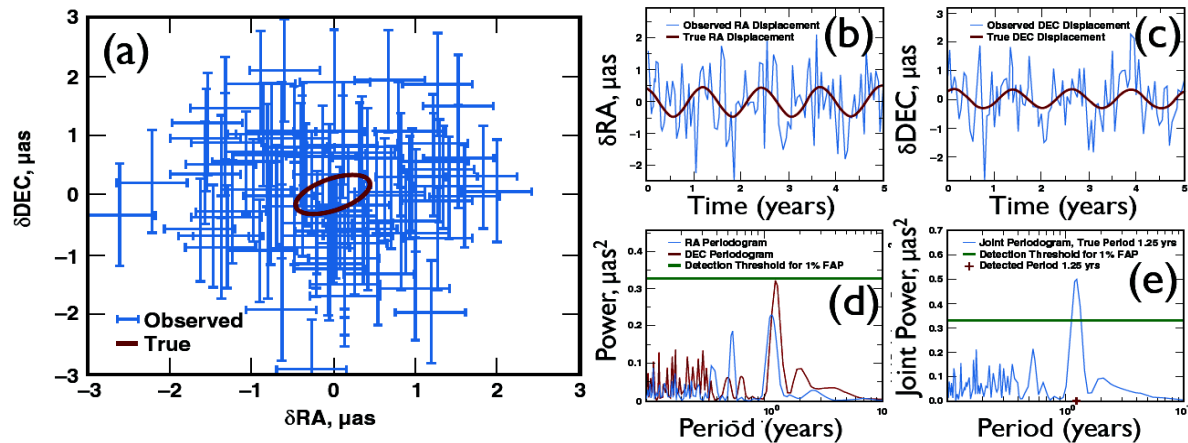


Fig. 3: Simulation of astrometric detection of a planet with 50 NEAT measurements (RA and DEC) over 5 yrs. Parameters are: $M_P = 1.5 M_{\oplus}$, $a = 1.16 \text{ AU}$, $M_* = 1 M_{\odot}$, $D = 10 \text{ pc}$, $\text{SNR} = 6$. (a) Sky plot showing the astrometric orbit (solid brown curve) and the NEAT measurements with error bars (in blue); (b) and (c) same data but shown as time series of the RA and DEC astrometric signal; (d) Separated periodogram of RA (blue line) and DEC (brown line) measurements. (e) Joint periodogram from R.A. and DEC simultaneously. Whereas the orbit cannot be determined from the astrometric signal without the time information, its period is reliably detected in the joint periodogram (1.25 yr), with a false-alarm probability below 1% (green line). Then, the planetary mass and orbit parameters can be determined by fitting the astrometric measurements.

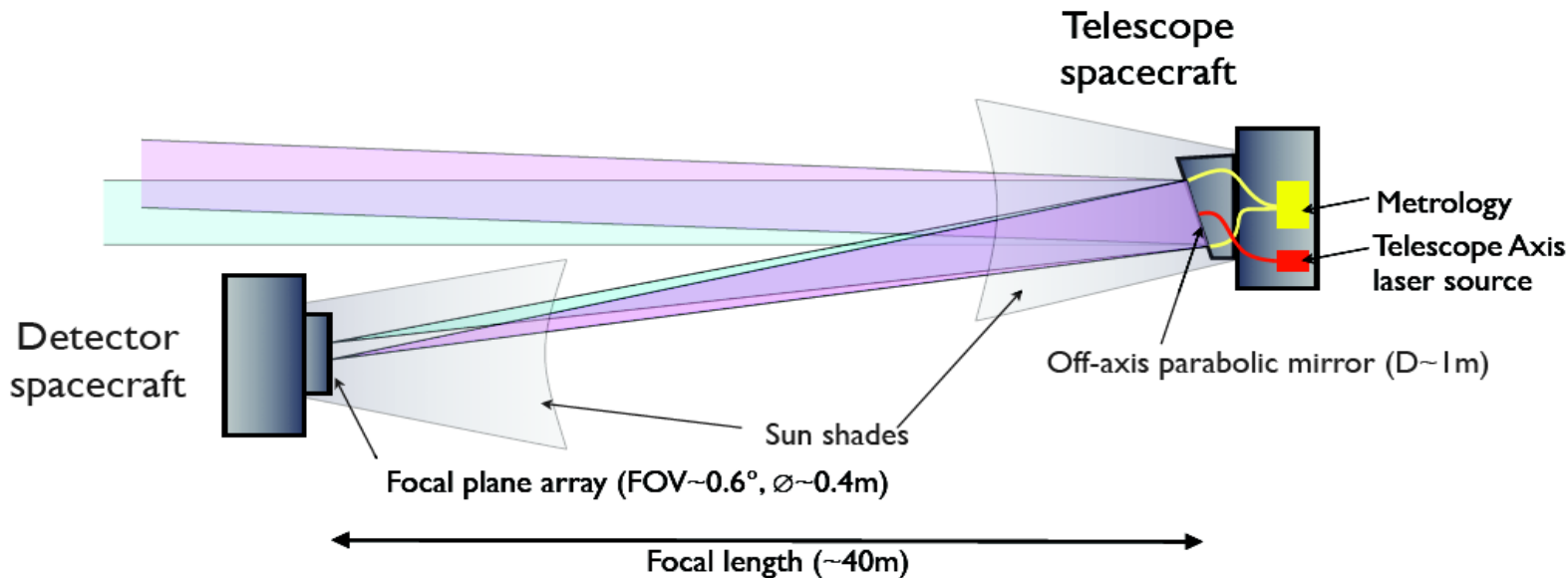
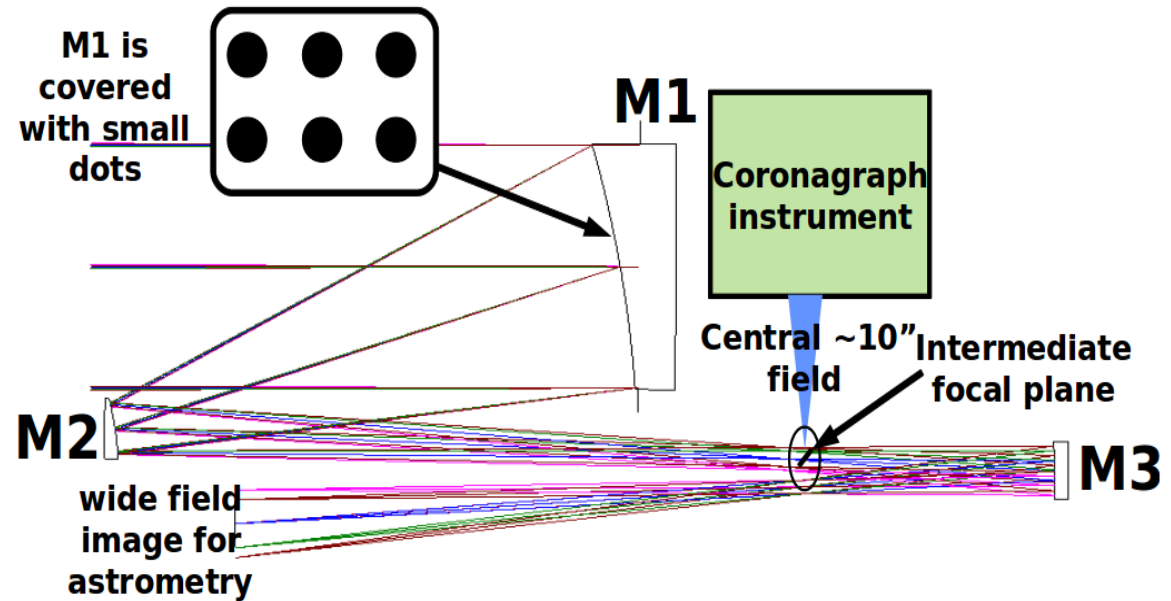


Fig. 2: Proposed concept: two separated modules, the first one carrying the primary mirror (upper right) and the second one the detector plane (bottom left).

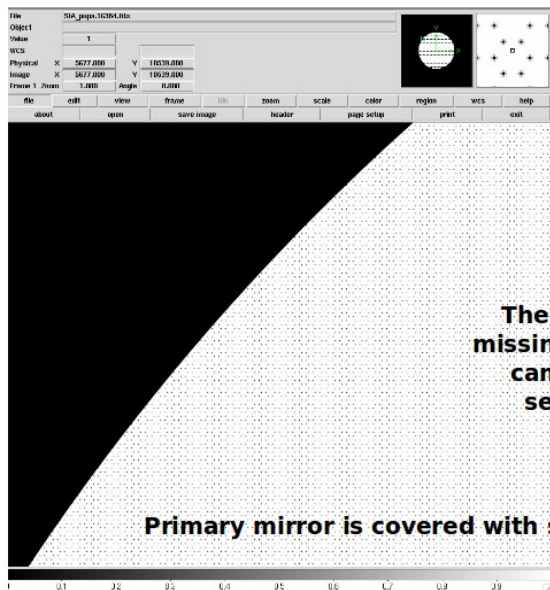
Diffractive pupil telescope (Guyon et al.)

Concept is compatible with coronagraphy
 → potential to acquire coronagraphic images simultaneously with astrometry

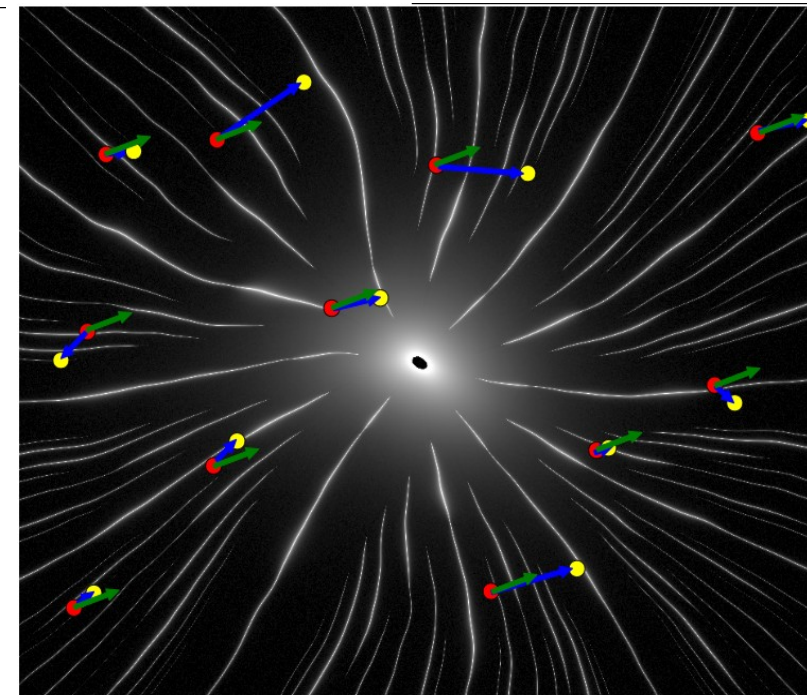
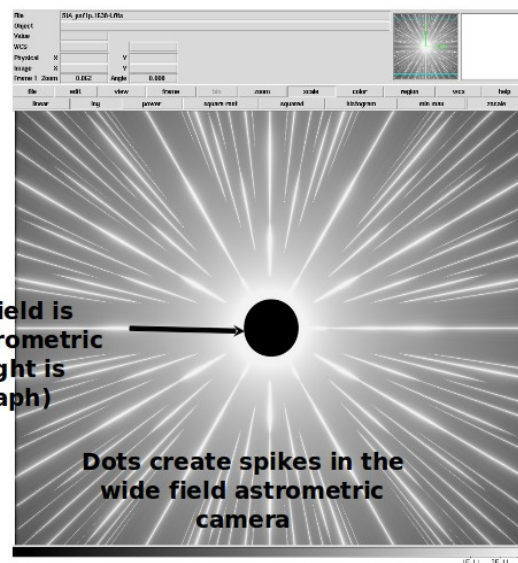
(lab at UofA)



Dots on primary mirror create a series of diffraction spikes used to calibrate astrometric distortions



The center of the field is missing from the astrometric camera (central light is sent to coronagraph)



Astrometry with interferometers

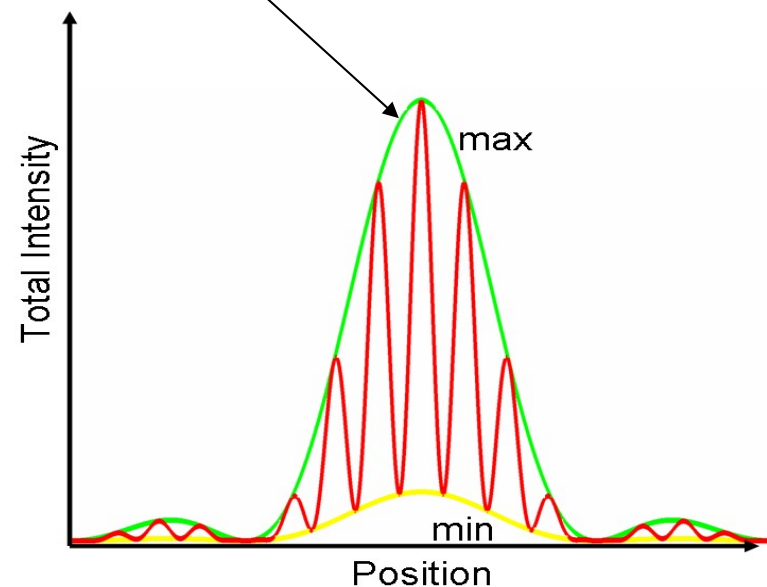
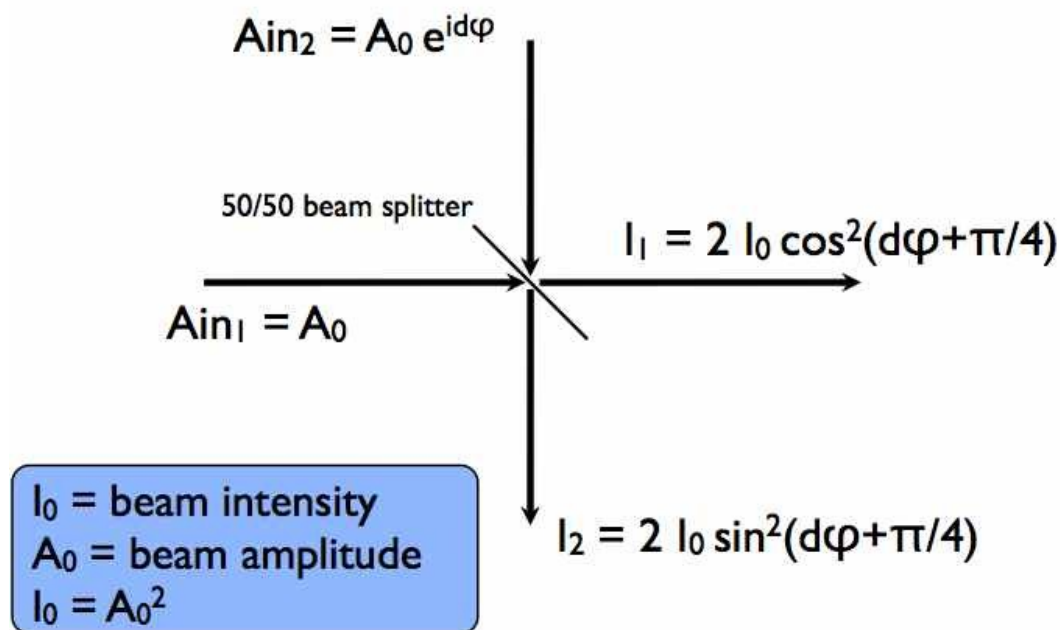
Astrometric measurement precision improves rapidly with telescope diameter

An interferometer offers significant gain for astrometry:

- good angular resolution ($\lambda/\text{Baseline}$) at moderate cost and mass
- calibration between a small number of beams can be done accurately

Astrometric signal derive from fringe phase

Michelson Interferometer



Space Interferometry Mission (NASA – cancelled)

6m baseline, 0.5m apertures
visible light

Identify Earth-like planets around nearby stars

→ feed target list to future coronagraphic imaging mission

Includes internal and external metrology to calibrate fringe phase

