Modern Astronomical Optics

1. Fundamental of Astronomical Imaging Systems

OUTLINE:

A few key fundamental concepts used in this course:
- Light detection: Photon noise
- Geometrical optics: Pupil and focal plane, Lagrange invariant
- Diffraction: Diffraction by an aperture, diffraction limit
- Spatial sampling

Earth's atmosphere: every ground-based telescope's first optical element
- Effects for imaging (transmission, emission, distortion and scattering) and quick overview of impact on optical design of telescopes and instruments

Astronomical measurements & important characteristics of astronomical imaging systems:
- Collecting area and throughput (sensitivity)
  - flux units in astronomy
- Angular resolution
- Field of View (FOV)
- Time domain astronomy
- Spectral resolution
- Polarimetric measurement
- Astrometry

Light detection: Photon noise

Poisson noise
Photon detection of a source of constant flux F. Mean # of photon in a unit dt = F dt.
Probability to detect a photon in a unit of time is independent of when last photon was detected → photon arrival times follows Poisson distribution
Probability of detecting n photon given expected number of detection x (= F dt):
$f(n,x) = x^n e^{-x}/(n!)$
x = mean value of f = variance of f

Signal to noise ration (SNR) and measurement uncertainties
SNR is a measure of how good a detection is, and can be converted into probability of detection, degree of confidence

Signal = # of photon detected

Noise (std deviation) = Poisson noise + additional instrumental noises (+ noise(s) due to unknown nature of object observed)

Simplest case (often valid in astronomy): Noise = Poisson noise = $\mathrm{sqrt}(N_{ph})$
Most of the time, we assume normal distribution (good approximation of Poisson distribution at high flux)

For example:

Telescope observes source for 5s, and detects 200 photon → measured source flux is 40 ph/s with a 3-σ measurement error of 3xsqrt(200)/5 = 8.5ph/s → 99.7% probability that actual flux is between 31.5 ph/s and 48.5 ph/s

Geometrical Optics: Lagrange Invariant

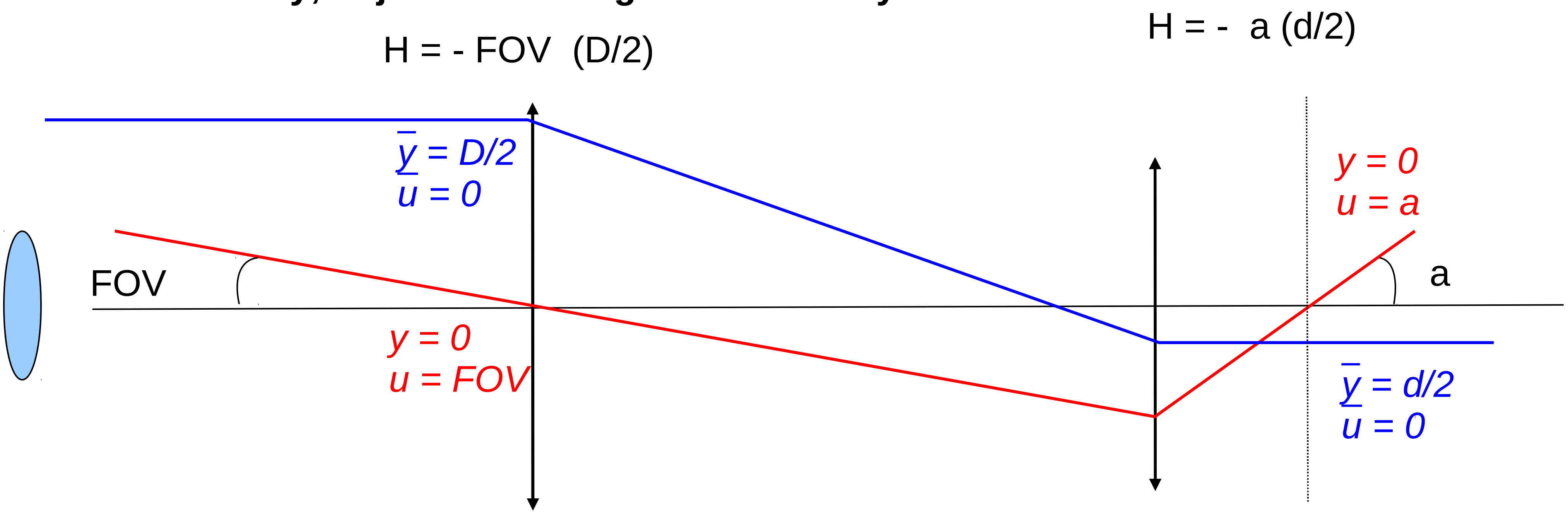
$H = n\,\bar{u}\,y - n\,u\,\bar{y}$

n = ambient refractive index (= 1 in most cases, unless H is computed inside a lens)
y = chief ray height
u = chief ray angle
$\bar{y}$ = marginal ray height
$\bar{u}$ = marginal ray angle
(see next slide for visual representation of these terms)

→ **large field of view and large collecting area requires large optics**
example: 10-m telescope, 1 deg field of view
if beam in compressed to 10cm (100x compression), angle = 100 deg → very difficult to design re-imaging optics of sufficiently high quality

→ **small beam = compressed propagation distances, lots of beam walk and diffraction effects at fixed physical distance from pupil**
Example: 10-m diameter beam compressed to 10mm (1000x lateral compression)
In this beam, lateral compression = 1e6: 10mm along the small beam is 10 km along the 10-m diameter beam

Example: afocal telescope (= beam reducer), input diameter D → output diameter d
In Astronomy, object to be imaged is at infinity



Chief ray (starts at edge of object, crosses center of aperture)

PUPIL= where chief ray intersects optical axis = conjugated to aperture stop

Marginal ray (starts at center of object, crosses aperture at its edge)

FOCAL PLANE = where marginal ray intersects optical axis = conjugated to infinity

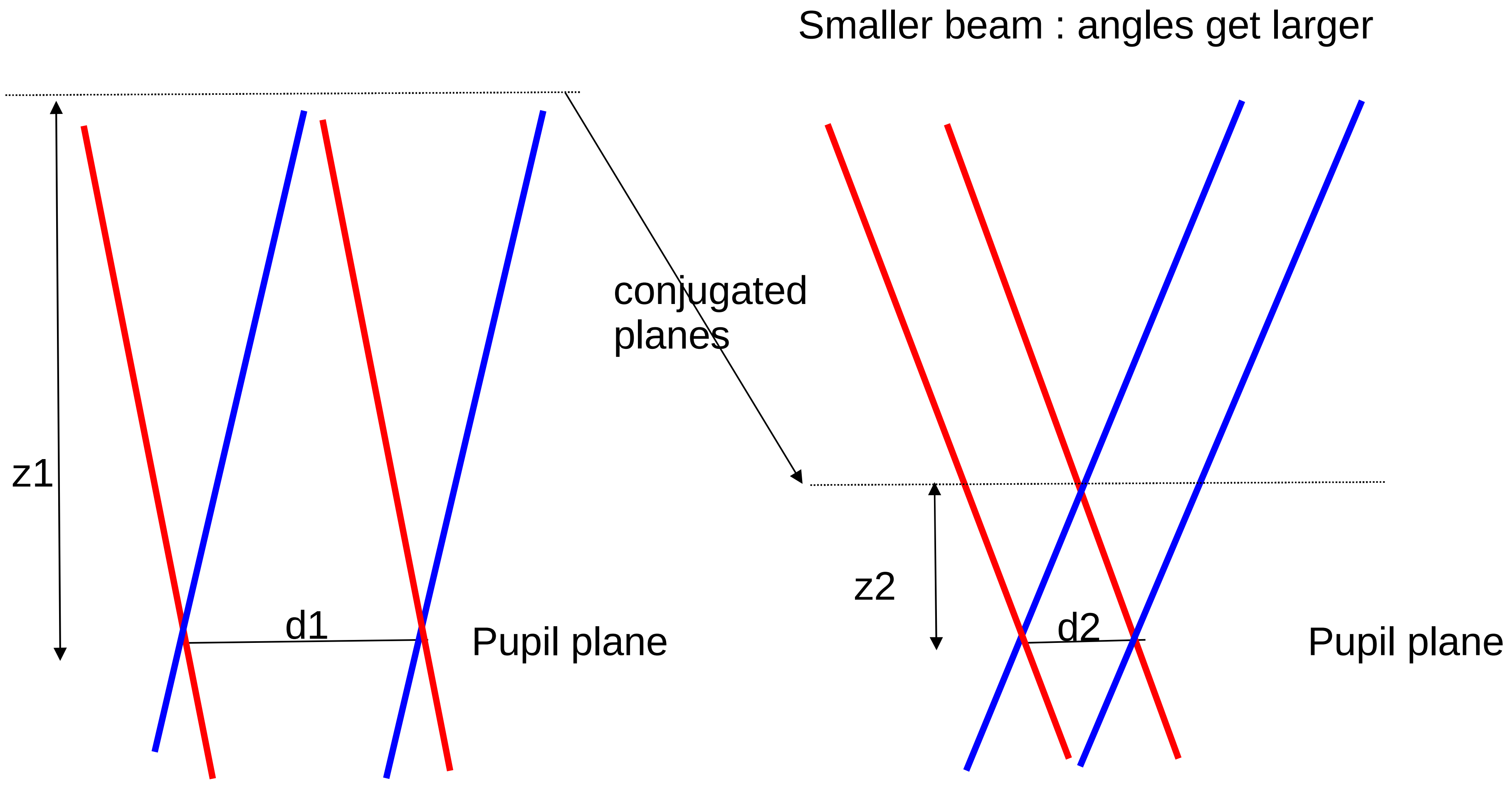
a = FOV (D/d)

Compressing the beam by factor x = multiplying angles by factor x

Note: The Lagrange invariant can also be seen as conservation of retardation (unit: waves, or m) between one size of the beam and the other: reducing the beam size conserves this retardation, and therefore amplifies angles.

→ impossible to build a wide field of view large telescope using small relay optics !!

→ large FOV & large diameter telescopes are challenging to build and have very large optics



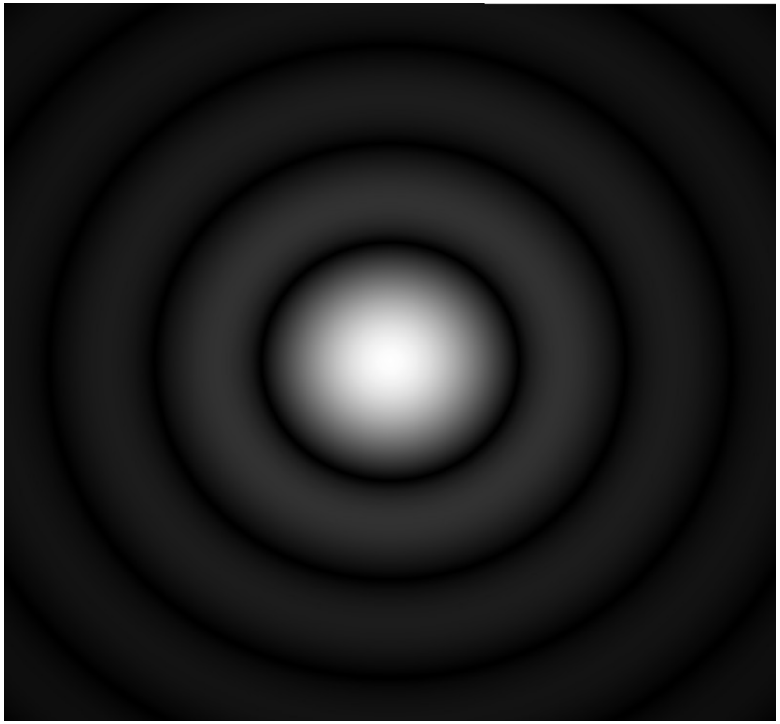
Lagrange invariant → $d1^2 / z1 = d2^2 / z2$
Reducing beam size by x compresses propagation distances by x^2
Drawing above provides physical illustration by looking at overlap between beams

Note:
Diffractive propagation equations (Talbot distance) show same beam volume compression effect: Talbot distance goes as f^{-2}, where f is the spatial frequency. If the beam is compressed by x, spatial frequencies are also multiplied by x, and the Talbot distance is divided by x^2

Diffraction by an aperture – telescope diffraction limit

Fresnel diffraction integral: $$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint E(x',y',0) e^{\frac{ik}{2z}[(x-x')^2+(y-y')^2]} dx' dy'$$

In imaging telescope, focal plane is conjugated to infinity (far field)
Fraunhofer is far field approximation of the Fresnel diffraction integral – and can easily be computed as a Fourier transform.



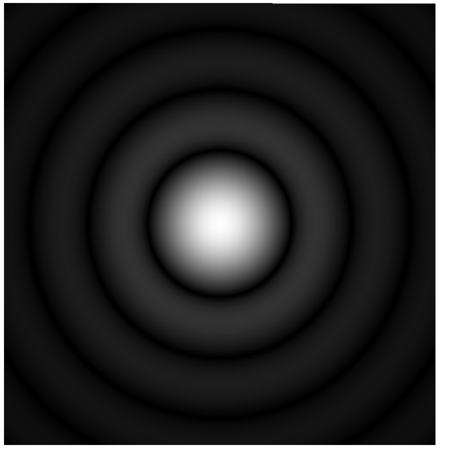
For circular aperture without obstruction : Airy pattern
First dark ring is at ~1.22 λ/D
Full width at half maximum ~ 1 λ/D
The "Diffraction limit" term = 1 λ/D

D=10m, λ=0.55 μm → λ/D = 0.011 arcsec

On large telescopes, image angular resolution is limited by atmospheric turbulence on the ground, at about 1 arcsecond
→ Adaptive optics required for < arcsecond imaging

Note: In astronomy, we use arcsecond (1/3600 deg) as unit for small angles

Spatial sampling of images



Astronomical imaging systems use arrays of pixels.
How many pixels across image to capture signal ?

Nyquist-Shannon sampling theorem:
If a function contains no spatial frequency of period smaller than P, then it is fully specified by its values at interval P/2

The Optical Transfer Function of a telescope goes to zero at λ/D: an noiseless image is band limited (telescope acts as a low pass filter in spatial frequencies)
→ Nyquist limit:
2 pixels per resolution element (= λ/D if diffraction limited)

Sampling and physical size of pixels defines F/ratio of optical beam onto the detector

Example:
Diffraction-limited telescope with Adaptive Optics
D=5m, λ=1.0 μm → λ/D = 0.04 arcsec
Nyquist limit : 20 mas (0.02 arcsec) per pixel

With 20 μm pixels, 1 mas / μm on the detector: 1 mas x f = 1 μm
f = 206m → f/D = 40

Increasing sampling beyond Nyquist limit doesn't bring new information.

Flux units in optical astronomy

At optical wavelengths, the most common unit is the astronomical magnitude scale. Historically, from 0 (brightest stars in sky) to 6 (faintest stars visible to the eye in night sky).
Large number = faint source !!!

Magnitude scale has since been defined for different colors, and extends beyond visible light to both IR/near-IR and near-UV.

Magnitude scale is logarithmic:
5 magnitudes = 100x flux (1 magn = $100^{1/5}$ ratio = 2.512 ratio in flux)

$m = -2.5 \log_{10}(F/F_0)$
$F = F_0\ 2.512^{-m}$
With F_0 given in table below

Conversion between Jy and $ph.s^{-1}.m^{-2}.\mu m^{-1}$:
1 Jy = 1e-26 $W.m^{-2}.Hz^{-1}$
(Johnson-Cousins-Glass)

Band	B	V	R	I	J	H	K
effective wavelength (μm)	0.436	0.545	0.638	0.797	1.22	1.63	2.19
zero mag flux (Jy)	4000	3600	3060	2420	1570	1020	636
zero mag flux ($ph.s^{-1}.m^{-2}.\mu m^{-1}$)	1.38E11	9.97E10	7.24E10	4.58E10	1.94E10	9.44E9	4.38E9

Flux units in optical astronomy

V magnitudes:

Sun : -26
Full moon : -13
Brightest star (Sirius) : -1.4
Faintest naked eye stars: 7
Faintest stars imaged by Hubble Space Telescope: 30

Magnitude scale also used for surface brightness: magn.arcsec^{-2}

Absolute Magnitude

Astronomical unit (AU) = Sun-Earth distance = 1.496e7 m
parallax = amplitude of apparent motion of a source on background sky due to Earth's orbit
parsec (pc) = parallax of one arcsecond = 3.0857e16 m = 3.26156 light year (ly)
Absolute magnitude (M): apparent magnitude an object would have if located 10 pc from Earth

If object is at 10pc, M=m
If object is at D_L pc, apparent flux = $(D_L/10)^{-2}$
$m = M + 5 (\log_{10}(D_L) -1)$
$M = m – 5 (\log_{10}(D_L) -1)$

Problem #1:
How big a telescope does it take to image an Earth-like planet at 10pc (32.6 lyr) in 1hr ?

Assume:
detection SNR = 5
0.1 μm bandpass filter at 0.55 μm (V band)
50% efficiency
no background
Sun V band absolute magnitude = 4.83
Earth is 1e10 fainter than Sun

<u>**Solution to problem #1**</u>

How many photons needed ?

SNR = 5 is reached with 25 photons, for which signal (S) = 25 and noise (N) = sqrt(25) = 5

Zero point of the system as a function of collecting area

According to the table of magnitude zero points, in one hour, a 0.1 μm wide filter around V band gives for a magnitude zero source :

N_0 = 0.1 μm x 3600s x 9.97E10 $ph.s^{-1}.m^{-2}.\mu m^{-1}$ = 3.59E13 $ph.m^{-2}$

With the 50% efficiency, the number gets reduced to zp = 1.79E13 $ph.m^{-2}$

Apparent magnitude of the planet

The apparent magnitude of the star is:

$m = M + 5 \times (\log_{10}(D_L)-1) = M = 4.83$

The planet is 25 mag fainter (= 1e-10 flux ratio)

→ m = 29.83

Number of photon collected per hour from the planet

$N = zp \times 2.512^{-m}$ = 21.0 $ph.m^{-2}$

Telescope diameter required

Collecting area required = 25/45.9 = 1.19 m^2

→ telescope diameter required = 1.2 m

(Note: other effects ignored here are the star halo, background, detector noise etc...)

The first optical element in every ground-based telescope: Earth's atmosphere

Transmission

Atmosphere is fairly transparent in optical when not cloudy

nearIR: windows of transparency exist, main absorber is water vapor

→ choose right wavelenght bands for observations

Emission: the sky is not fully dark

In visible light: airglow (~100km altitude)

→ optical filtering and/or calibration

In IR: blackbody emission from water vapor

→ high altitude, dry and cold sites better

Wavefront distortions

fluctuations in refractive index (temperature, humidity, pressure, water content) introduce wavefront errors

Atmospheric turbulence

typical angular distortion = 1” = diffraction limit of 10cm telescope in visible

→ ***Adaptive optics can mitigate this issue***

Atmospheric refraction

refraction is chromatic: stars turn into spectra at low elevation

→ ***Can be compensated by atmospheric dispersion compensator***

Rayleigh Scattering

Daytime sky too bright for observations

Moonlight increases sky brightness in visible light (but near-IR is OK)

→ observe in the near-IR / IR during bright time, visible during dark time

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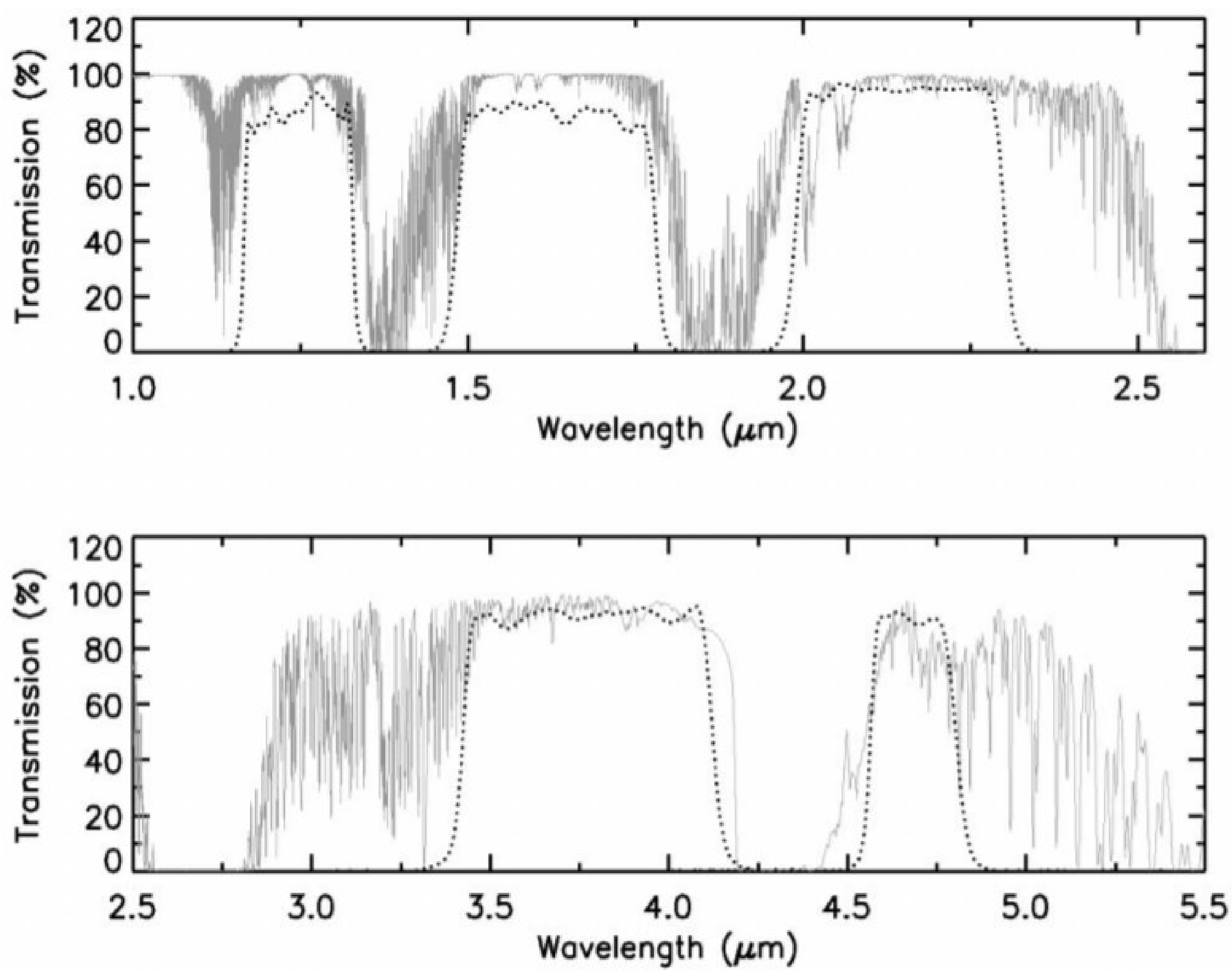
Transmission & Emission in near IR

In IR: poor transmission = high thermal emission (sky is glowing)

→ IR filters for ground-based observations chosen to match high transmission windows



J, H, Ks, L', and M' filter profiles superposed on the atmospheric transmission at Mauna Kea kindly provided by G. Milone for 1 mm precipitable water vapor and an air mass of 1.0

Tokunaga, Simons & Vacca, 2002

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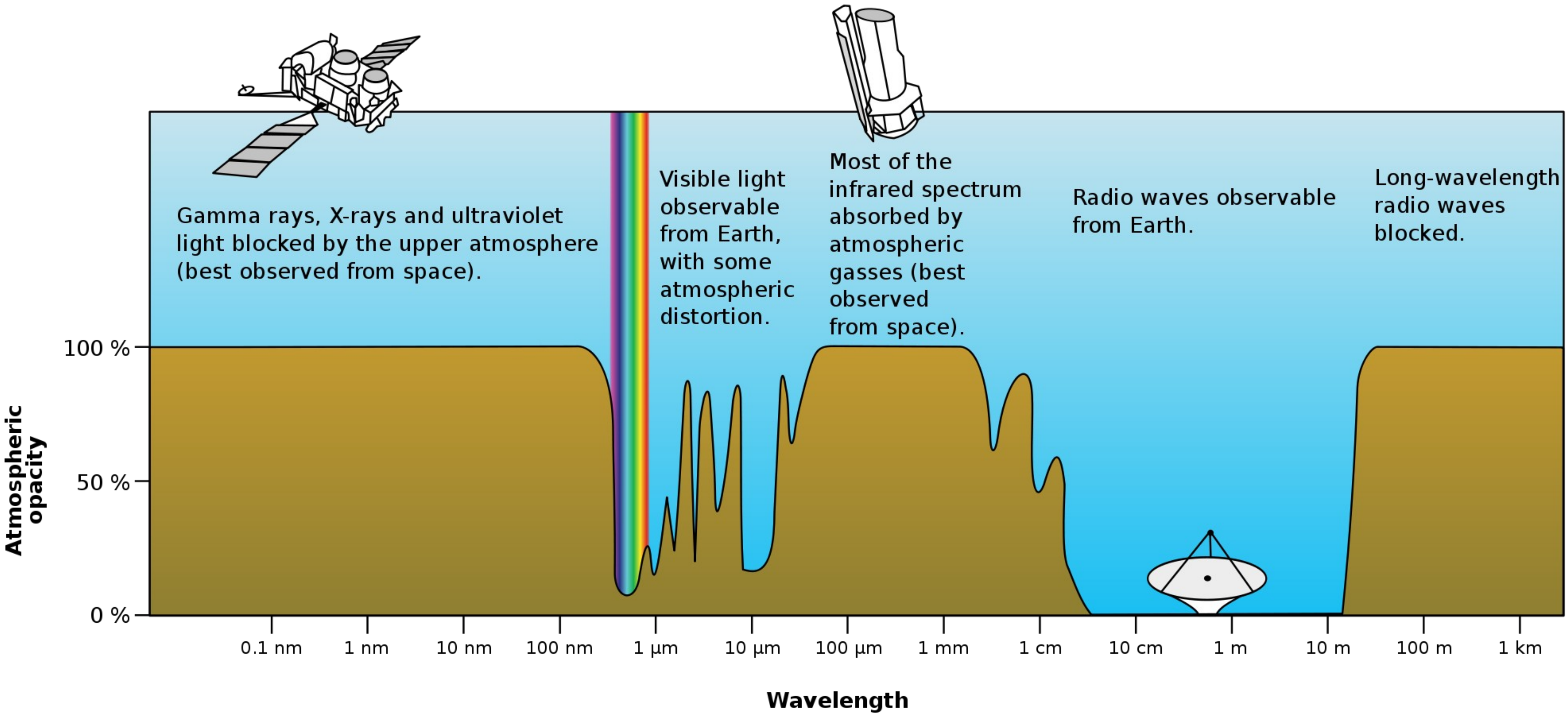
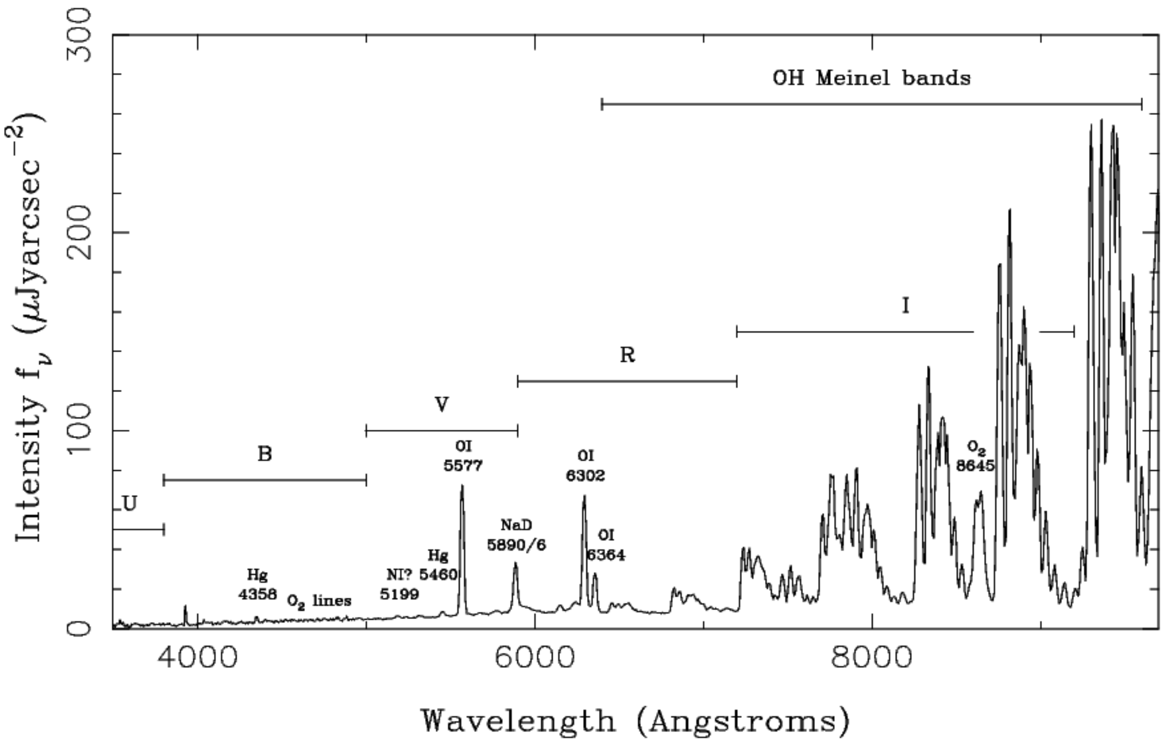


The first optical element in every ground-based telescope: Earth's atmosphere

Optical emission : airglow

Emission from OH (red & nearIR), O (visible green line) and O_2 (weak blue light) at ~90km

Airglow is time-variable, has structure over wide angles: it is very important for spectroscopy to either optically filter it out or have a good scheme to calibrate it and subtract it from the spectra



The first optical element in every ground-based telescope: Earth's atmosphere

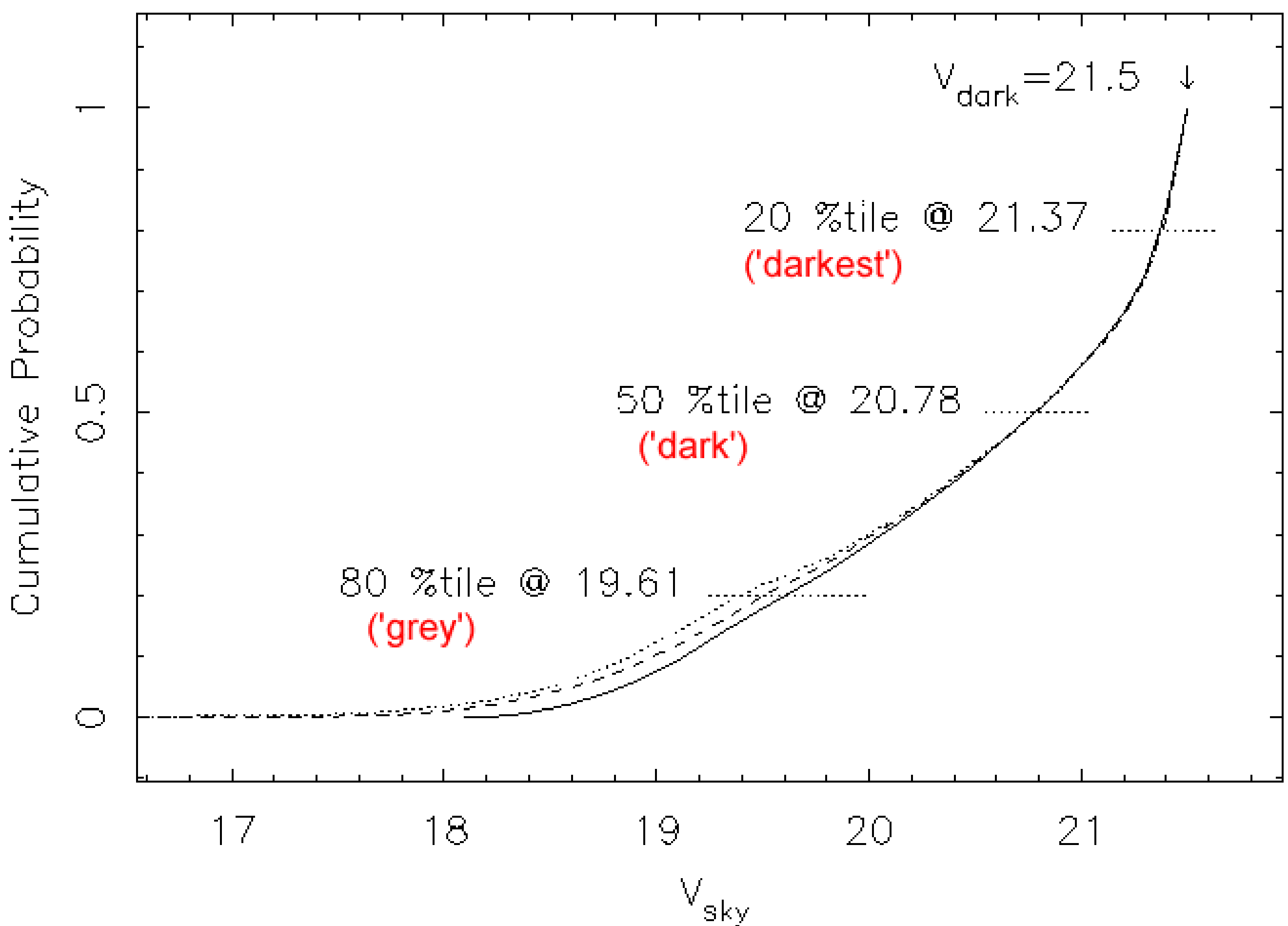
Moonless sky background in the optical (V band):

Airglow : $m_V = 22.4$ arcsec^{-2}

Zodiacal light : $m_V = 23.3$ arcsec^{-2} (brighter closer to ecliptic)

+ scattered starlight (much smaller)

Total darkest sky background ~ $m_V = 21.9$ arcsec^{-2} (rarely achieved from ground)



Cumulative probability distributions of V-band sky brightness at an arbitrary phase in the solar cycle for three model observation scenarios
Gemini North Telescope

The first optical element in every ground-based telescope: Earth's atmosphere

This image shows bands of airglow :

Credit: D. Duriscoe, C. Duriscoe, R. Pilewski, & L. Pilewski, U.S. NPS Night Sky Program
Full resolution image on Astronomy Picture of the Day (APOD), 2009 Aug 27