PIAA mirrors shapes

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1 Purpose of test

The PIAA technique is described extensively in the paper "Phase-Induced Amplitude Apodization of Telescope Pupils for Extrasolar Planet Imaging" (available on http://www.naoj.org/staff/guyon/publications/PIAA.ps). The minimum goal of the experiment that will use the optics described in this specification is to achieve a PSF contrast of 10-5 at $2\lambda/D$ in the visible (desired goal contrast of 10-6 at $2\lambda/D$ in visible). The radial profile of the apodized beam is chosen to meet this requirement.

2 Specifications

2.1 Overall description

The layout of the test is shown in Figure 1. The source is collimated by the First PIAA Mirror (M1) and reimaged by the Second PIAA Mirror (M2). The figure of each of these mirrors is a base paraboloid of revolution modified precisely to apply the desired apodization in the test set-up. The spacing between the mirrors, as well as the focal length of the parabolic mirrors is 15 times the beam diameter, thus it is 1125 mm with a 75 mm clear aperture diameter.

2.1.1 PIAA first corrector mirror, M1

The front surface of the PIAA first corrector mirror is nominally parabolic, but with a prescribed spatially dependent general asphere departure. The approximate functional dependence of surface sag versus position on the surface is shown in Figure 2.**The sharp bend in the outer part of mirror M1 is critical to the success of the experiment, and the specifications given in table 2 have to be followed up the edge of the mirror.** The maximum departure from a sphere in M1 is 8 μm. In §3, the exact shape of M1 is described.

2.1.2 PIAA second corrector mirror, M2

The front surface of the PIAA second corrector mirror is nominally parabolic, but with a prescribed spatially dependent general asphere departure. The approximate functional dependence of surface sag versus position on the surface is given in Figure 2, and will be precisely defined in a format agreeable with the vendor. The maximum departure from a sphere in M1 is 60 μm. In §3, the exact shape of M2 is described.

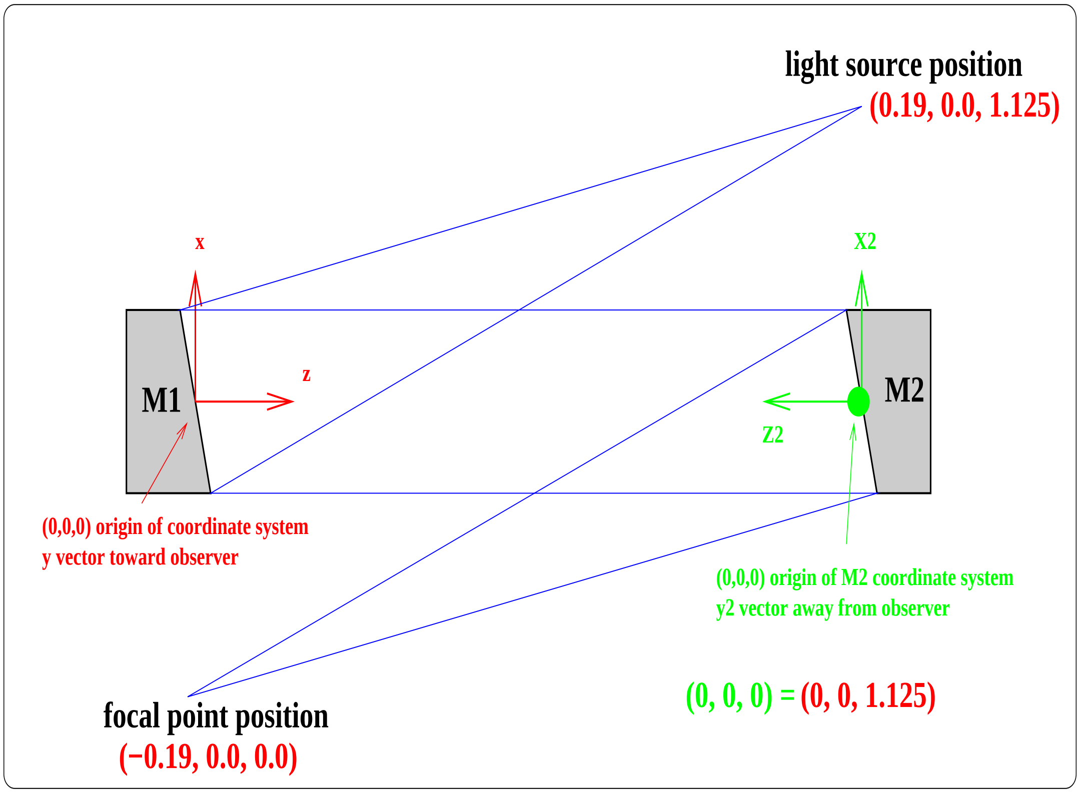

Figure 1: Layout of a PIAA system unit (all units are in m).

2.2 Mirror figure specifications

2.2.1 Surface quality effect on demonstration

Errors in the surface of the two mirrors will affect the PIAA system in two ways.

EFFECT 1: Phase errors in the output beam. Phase errors in the PIAA optics will produce phase errors in the output beam, which might make the PSF unsuitable for high-contrast imaging. If these errors are less than 500nm, the deformable mirror (DM) which we plan to use in the experiment should be able to correct for them.

EFFECT 2: Modification of the apodization function. The distribution of light intensity in the output beam of the PIAA system is modified, and might become unsuitable for high-contrast imaging.

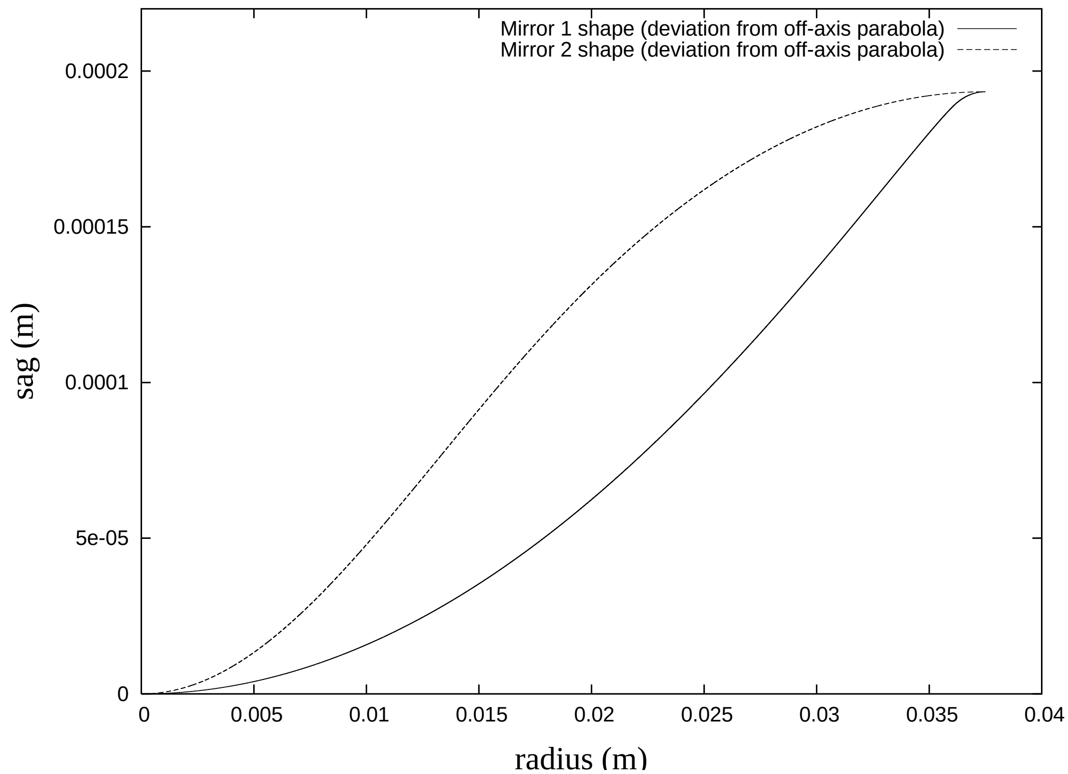

Figure 2: Term $f_i(r)$ for Mirrors 1 and 2 (units are in m for both axis) within the beam diameter. Outside of the beam radius r_0, $f_i(r_0 + r) = f_i(r_0 - r)$.

2.2.2 Polishing and testing the PIAA optics

The first PIAA optic (M1) is the most challenging to polish and test because of the sharp bend at its outer edge. The second PIAA optic (M2), however, is relatively smooth and does not have such sharp features. Since the two PIAA optics act as a phase null (the phase of the first optic is nulled by the second so that the entire system does not introduce phase errors), one optic can be used to test the other. We therefore suggest the following two-step procedure to polish/test the PIAA optics. This procedure avoids the difficulty of polishing accurate null optics to separately test the two PIAA mirrors, and uses the fact that EFFECT 2 is negligible compared to EFFECT 1. The corresponding wavefront specifications are also given. As a goal, the values apply after the reflective coatings have been applied.

CPA	maximum surface RMS error
0 – 5	500 nm
5 – 10	300 nm
10 – 20	100 nm
20 – 40	70 nm
$>$ 40	50 nm

Table 1: M2 optic surface quality (RMS) required to obtain a flux distribution compatible with 10-7 contrast at 2λ/D and 600 nm.

CPA	maximum wavefront RMS error
0 – 5	100 nm
5 – 10	100 nm
10 – 20	50 nm
20 – 40	35 nm
$>$ 40	25 nm

Table 2: Output beam wavefront quality (RMS) required to reach a 10-6 contrast at 2λ/D and 600 nm with the use of a DM.

Step 1: Figure/Test Mirror M2 relative to an absolute standard The M2 PIAA mirror shall be figured, polished, and tested to satisfy surface tolerances in Table 1. These are surface error tolerances relative to the specified "perfect" profile of M2.

Step 2: Figure/Test Mirror M1 to null Mirror M2

The M1 PIAA mirror is figured and polished using the M2 PIAA mirror for a null test, so that the output beam of the PIAA two-mirror system shown in Figure 1 meets the requirements given in Table 2. In the aligned system, the wavefront error should not exceed 0.5 micron peak (to stay well within the stroke of the DM).

2.3 Test

Mirror M2 will be figured and tested to the requirements of Table 1. The vendor shall choose the appropriate verification technique, perhaps testing against a phase hologram. Mirror M1 shall be figured and tested to null the wavefront of the aligned pair of mirrors to the requirements of Table 2. The vendor shall choose the appropriate verification technique.

3 Exact shape of the mirrors

As shown in fig. 1, the coordinate system adopted to describe the shape of the optics is as follows:

- **z** is pointed from the center of M1 to the center of M2. **z** is horizontal in fig. 1, pointing to the right.
- **x** is vertical in fig. 1, pointing upwards.
- **y** is perpendicular to the plane of fig. 1.
- The line **x=y=0** joins the center of M1 and M2.

In this coordinate system, the z vector is pointing from M2's surface to the back of M2.

The surface of M1 and M2 in this coordinate system can be written :

$$z_i(x,y) = OAP_i(x,y) + profile_i(r) + f_i(x,y) \tag{1}$$

where $r = \sqrt{x \times x + y \times y}$ and i is the mirror number (1 or 2). The diameter of the optics is 82.5mm for a useful beam diameter of 75mm.

- $OAP_i(x,y)$ is the off-axis parabola shape if no apodization was performed. It is a 1133 mm focal length OAP with a 190 mm off-axis distance from the center of the optical elemement. This shape is identical for the 2 mirrors although the orientation is different.
- $profile_i(r)$ is given for both mirrors in fig. 2. It can be supplied to the vendor as a numerical table. It is also well fitted by a sum of cosines.
- $f_i(x,y)$ is a corrective term to account for the fact that the system is off-axis, and tends to 0 for an on-axis system.

Alternatively, we may supply the vendor with a (x,y,z) table of points including all 3 components of equation 1.

3.1 Exact shape of M1.

$$OAP_1(x,y) = \frac{(x-0.19)^2 + y^2 - 0.0361}{4.53186327376}. \tag{2}$$

$profile_1(r)$ can be supplied as a numerical table or fitted by

$$profile_1(r) = \sum_{k=0}^{k=99} \alpha_k \cos\left(\pi k \times \frac{r}{0.0375}\right) \tag{3}$$

with $0 < r < 0.04125$. The numerical values of α_k is provided in a ASCII text file.

$f_1(x,y)$ is a sum of 3 terms:

$$f_1(x,y) = -0.0000162 \times x + 0.00099 \times r^2 - 0.0000001 \times Z_3^1(x,y) \tag{4}$$

where Z_3^1 is the zernike polynomial (m=1,l=3) of RMS amplitude 1m within the beam size (75mm diameter). To avoid confusion, graphical representation of a few Zernike polynomials will be given at the end of this document.

C code used to generate the optics shapes from these expressions will be provided.

3.2 Exact shape of M2 in the (x,y,z) coordinate system.

In the same coordinate system,

$$OAP_2(x,y) = -\frac{(x+0.19)^2 + y^2 - 0.0361}{4.53186327376}. \tag{5}$$

$profile_2(r)$ can be supplied as a numerical table or can also be fitted by

$$profile_2(r) = \sum_{k=0}^{k=39} \beta_k \cos\left(\pi k \times \frac{r}{0.0375}\right). \tag{6}$$

The number of terms is smaller than for M1, as the shape does not have the bend at the outer edge of the mirror. It should be noted that $\beta_0 \approx 1.125$, as the M2 is about 1.125m away from (0,0,0). For M2, the term $f_2(x,y)$

contains 17 terms, all of them Zernikes. Tip and focus are written explicitely in this equation:

$$f_2(x,y) = 0.0000027284 \times x + 0.000896954 \times r^2 \tag{7}$$
$$-0.0000000004714 \times Z_2^2(x,y) + 0.0000000361255 \times Z_3^1(x,y) \tag{8}$$
$$-0.00000013074775 \times Z_4^0(x,y) - 0.0000000049253 \times Z_5^1(x,y) \tag{9}$$
$$+0.00000003064 \times Z_6^0(x,y) - 0.00000000681846 \times Z_7^1(x,y) \tag{10}$$
$$-0.00000000320335 \times Z_8^0(x,y) + 0.000000003663385 \times Z_9^1(x,y) \tag{11}$$
$$-0.000000003399 \times Z_{10}^0(x,y) - 0.00000000082 \times Z_{11}^1(x,y) \tag{12}$$
$$+0.000000003431488 \times Z_{12}^0(x,y) - 0.0000000010967 \times Z_{13}^1(x,y) \tag{13}$$
$$-0.000000001706 \times Z_{14}^0(x,y) + 0.0000000014381 \times Z_{15}^1(x,y) \tag{14}$$
$$-0.000000001078 \times Z_{17}^1(x,y) \tag{15}$$

where $Z_n^m(x,y)$ is the Zernike polynomial (radial order n, azimuth order m) of RMS amplitude 1m within the beam diameter (75mm diameter). **All Zernikes are computed for the beam diameter (75mm) but they are used up to the optics surface diameter (82.5mm).** Examples of Zernikes are shown in fig. 4.

3.3 Exact shape of M2 in the (x2,y2,z2) coordinate system.

In the (x2,y2,z2) coordinate system :

$$OAP_2(x2,y2) = \frac{(x2+0.19)^2 + y2^2 - 0.0361}{4.53186327376}. \tag{16}$$

$profile_2(r)$ can be supplied as a numerical table or can also be fitted by

$$profile_2(r) = 1.125 - \sum_{k=0}^{k=39} \beta_k \cos\left(\pi k \times \frac{r}{0.0375}\right). \tag{17}$$

For M2, the term $f_2(x,y)$ contains 17 terms, all of them Zernikes. Tip and focus are written explicitely in this equation:

$$f_2(x2,y2) = -0.0000027284 \times x2 - 0.000896954 \times r^2 \tag{18}$$
$$+0.0000000004714 \times Z_2^2(x2,y2) - 0.0000000361255 \times Z_3^1(x2,y2) \tag{19}$$
$$+0.00000013074775 \times Z_4^0(x2,y2) + 0.0000000049253 \times Z_5^1(x2,y2) \tag{20}$$

$$-0.00000003064 \times Z_6^0(x2,y2) + 0.00000000681846 \times Z_7^1(x2,y2) \quad (21)$$
$$+0.00000000320335 \times Z_8^0(x2,y2) - 0.000000003663385 \times Z_9^1(x2,y2) \quad (22)$$
$$+0.000000003399 \times Z_{10}^0(x2,y2) + 0.00000000082 \times Z_{11}^1(x2,y2) \quad (23)$$
$$-0.000000003431488 \times Z_{12}^0(x2,y2) + 0.0000000010967 \times Z_{13}^1(x2,y2) \quad (24)$$
$$+0.000000001706 \times Z_{14}^0(x2,y2) - 0.0000000014381 \times Z_{15}^1(x2,y2) \quad (25)$$
$$+0.000000001078 \times Z_{17}^1(x2,y2) \quad (26)$$

where $Z_n^m(x2,y2)$ is the Zernike polynomial (radial order n, azimuth order m) of RMS amplitude 1m within the beam diameter (75mm diameter). **All Zernikes are computed for the beam diameter (75mm) but they are used up to the optics surface diameter (82.5mm).**

4 Effect of alignment errors on the wavefront

The effect of alignment errors on the wavefront is shown in fig. 7 to 12. The wavefronts are shown for a single-pass through the system, and need to be multiplied by 2 for a double-pass system.

If M2 is polished first, it can be used to measure the residual during the polishing of M1 (null test). An example test setup is shown in fig. 6. In this example, if the light source, M2 and the sphere are considered perfectly fixed, only alignment errors of M1 (and polishing errors on M1) contribute to the return wavefront. In this special case, M1 tilt errors produce a pure tilt on the return wavefront, and translations along the x,y and z axis produce modes close to pure tilts.

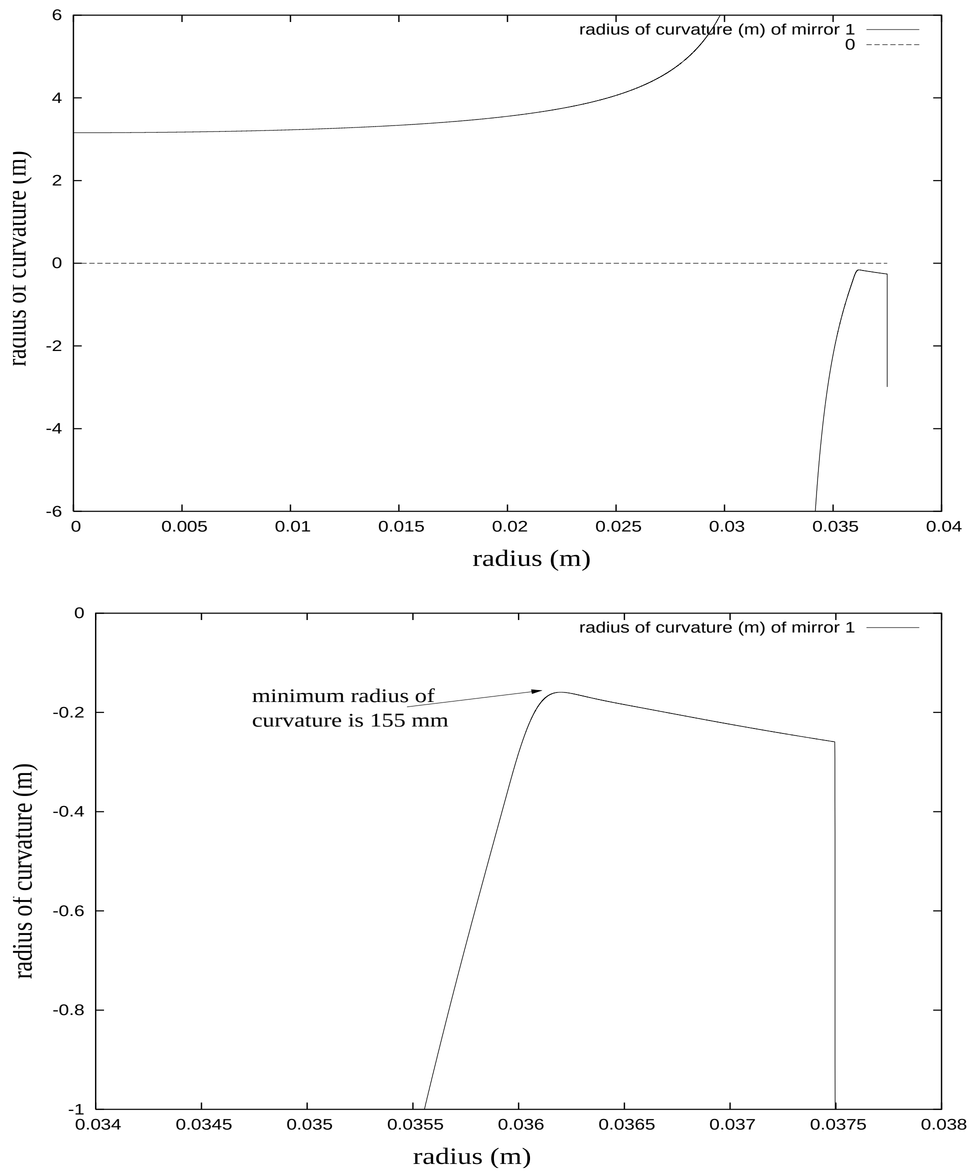

Figure 3: Radius of curvature of the radial component $f(r)$ of PIAA Mirror 1.

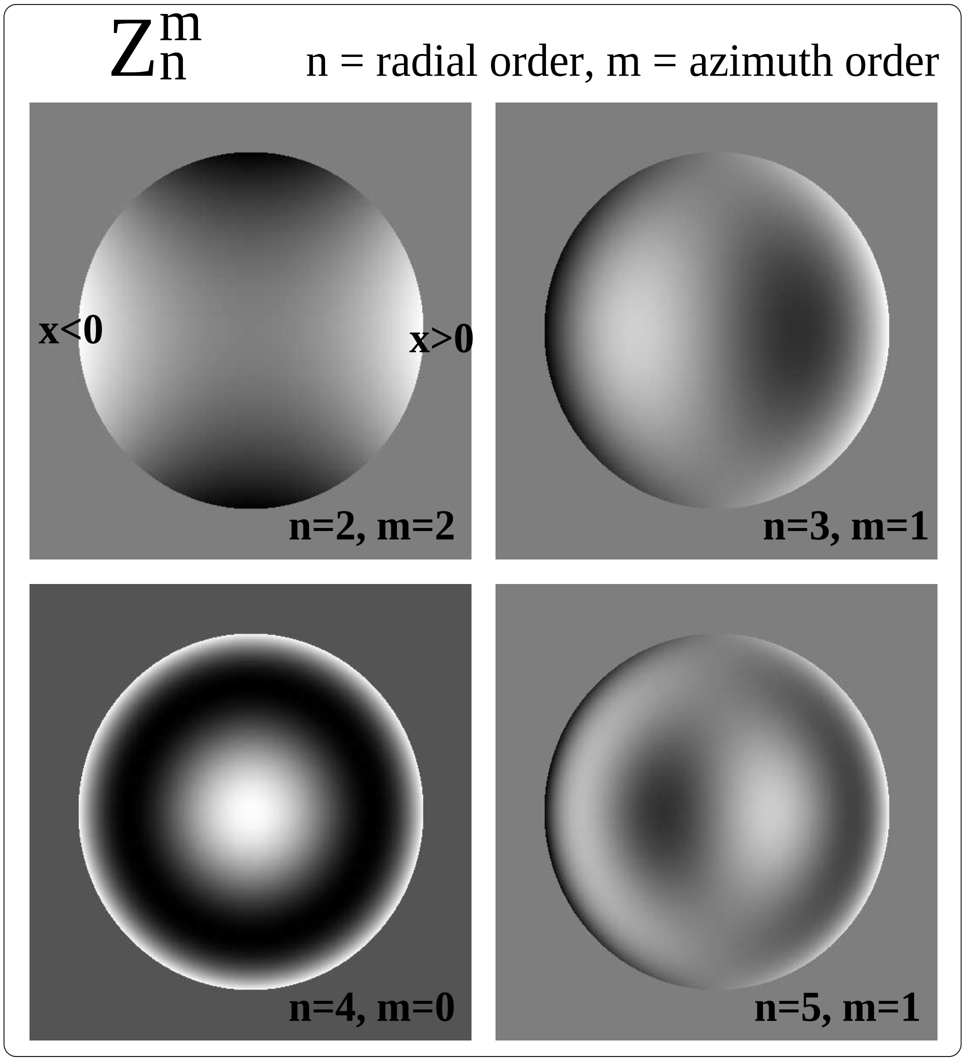

Figure 4: Examples of Zernike polynomials used in this document.

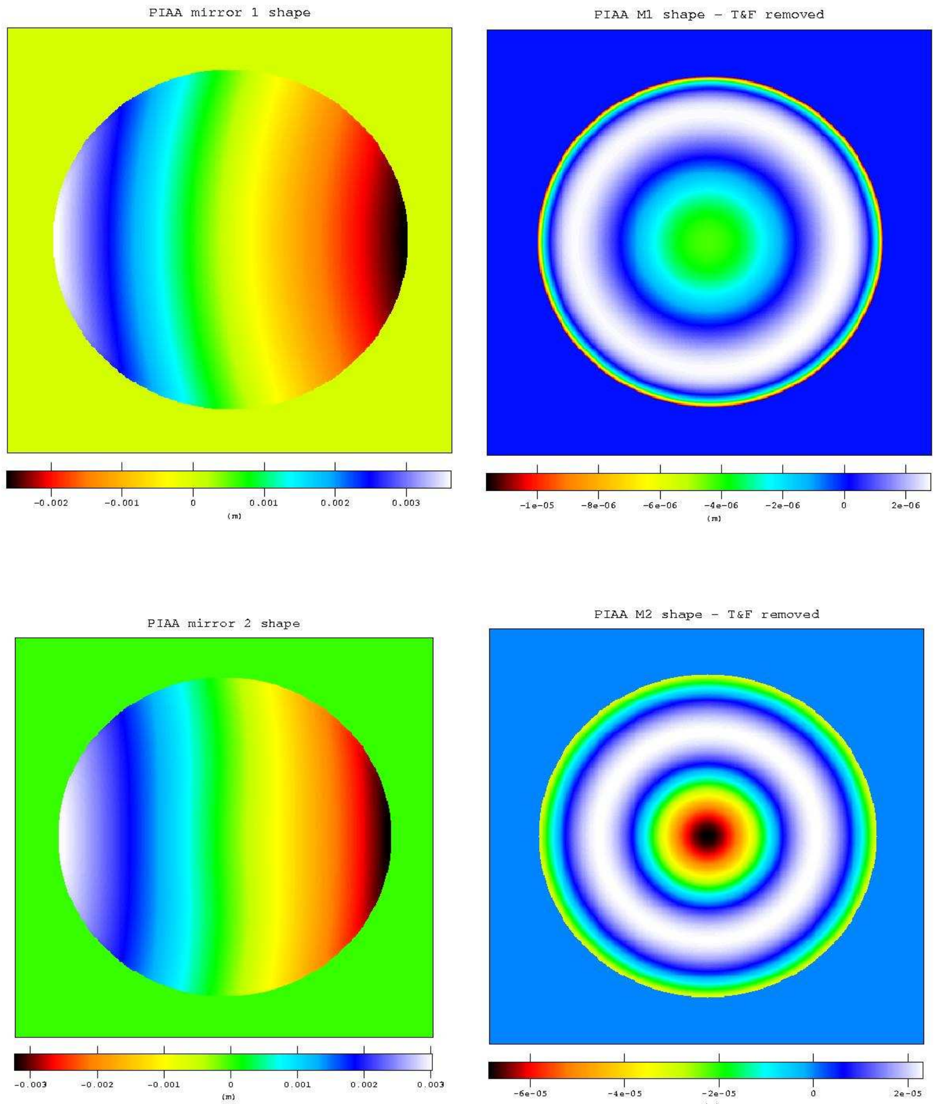

Figure 5: Shapes of M1 (top) and M2 (bottom) within the beam diameter. On the left, Tip/Tilt and Focus have been removed.

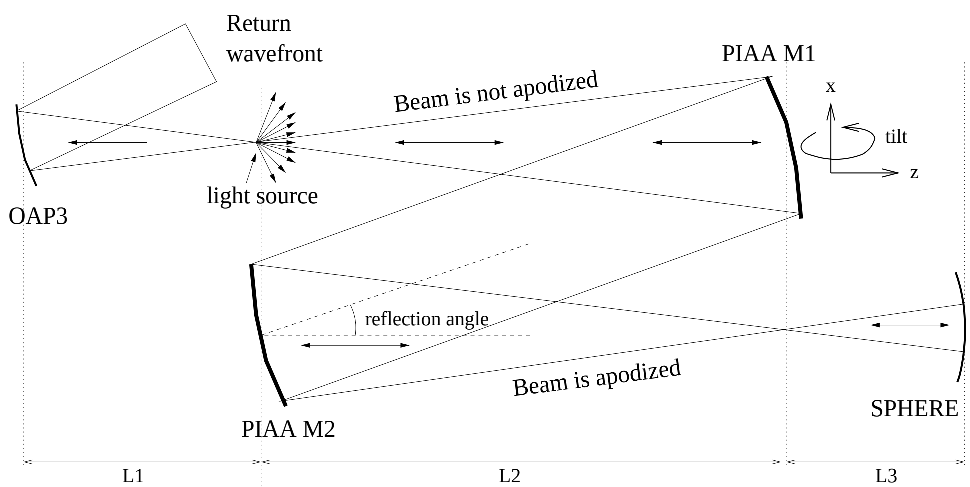

Figure 6: Example of null test setup for the polishing of M1.

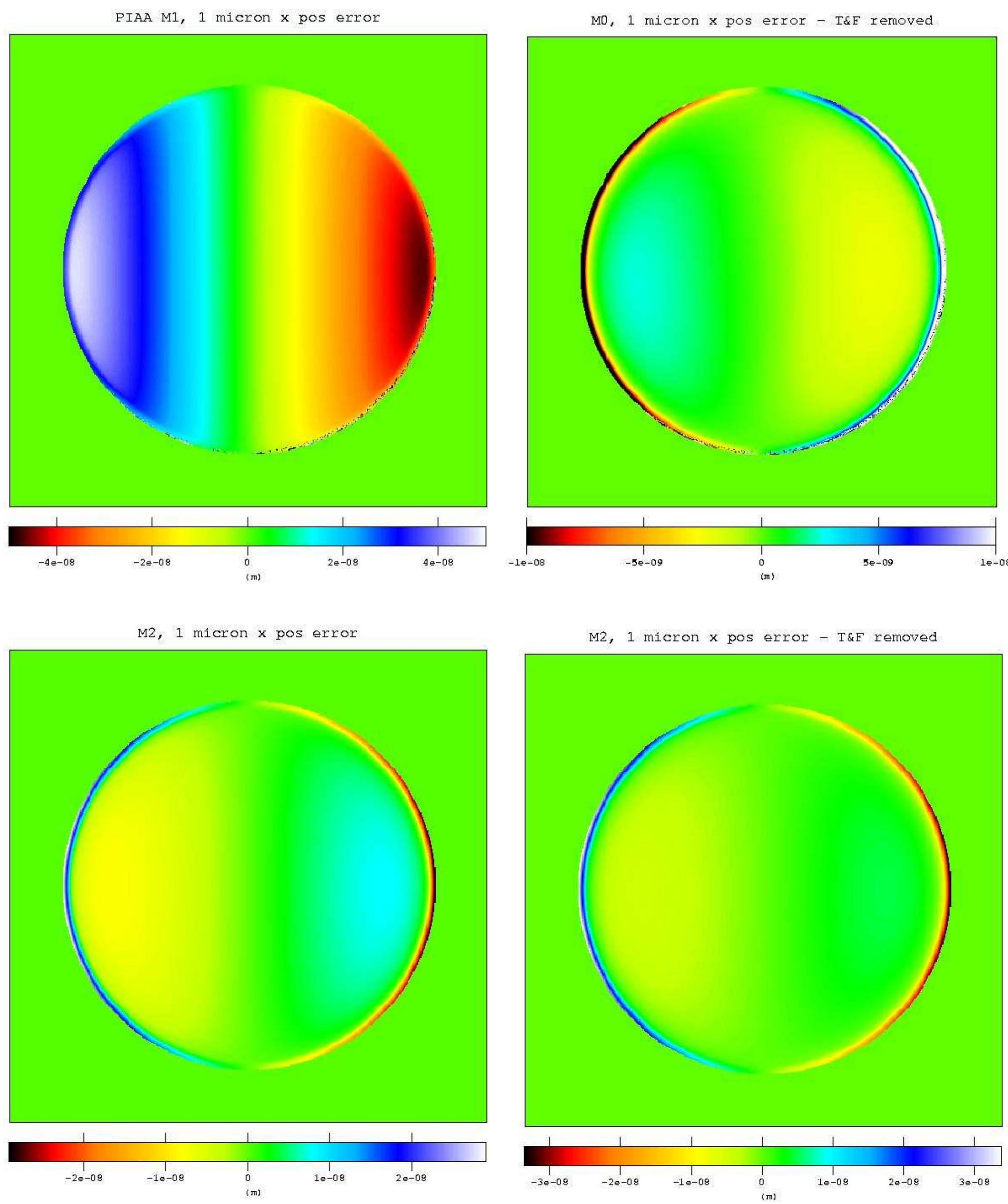

Figure 7: Effect of an alignment error of M1 and M2, translation along the x axis.

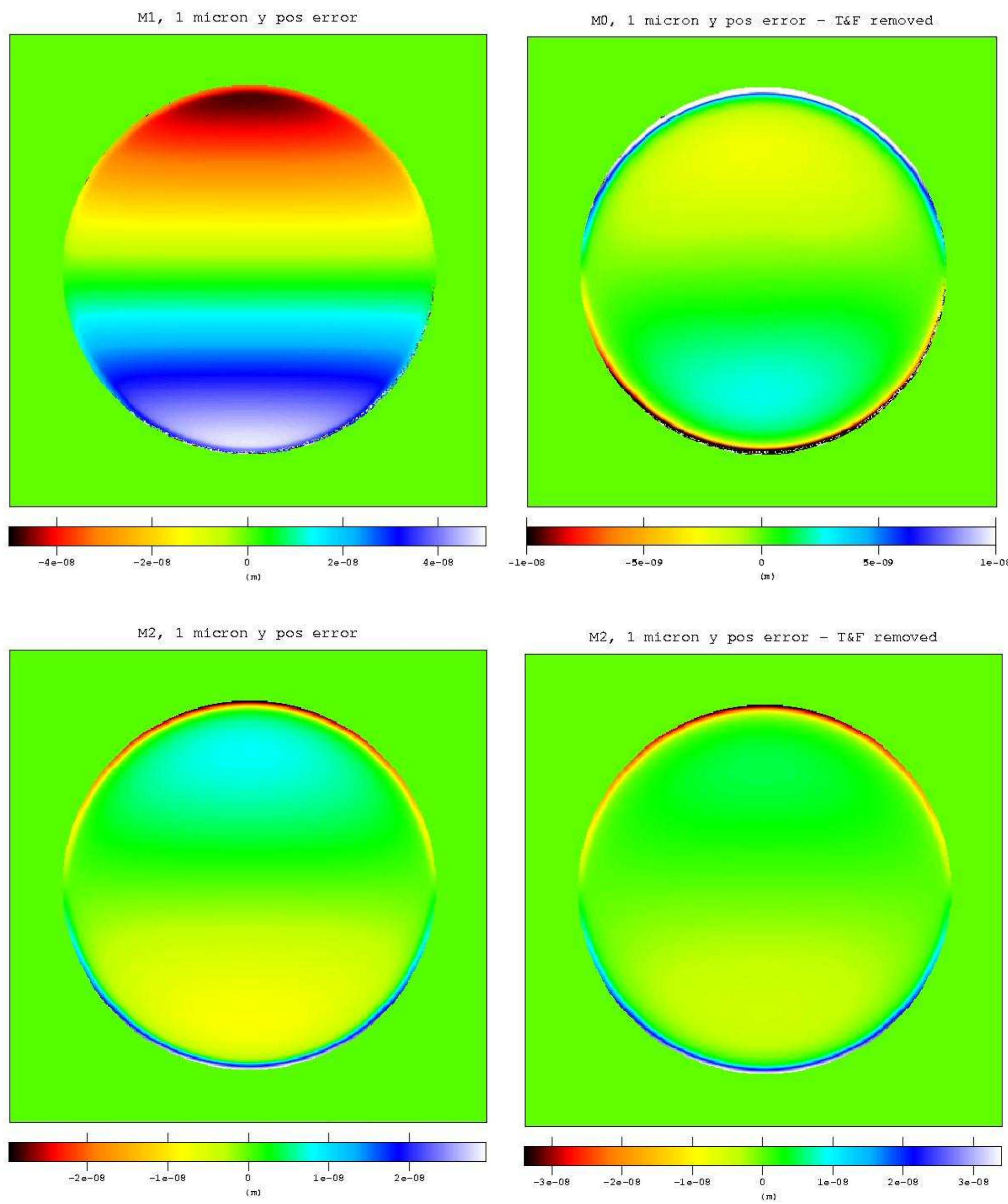

Figure 8: Effect of an alignment error of M1 and M2, translation along the y axis.

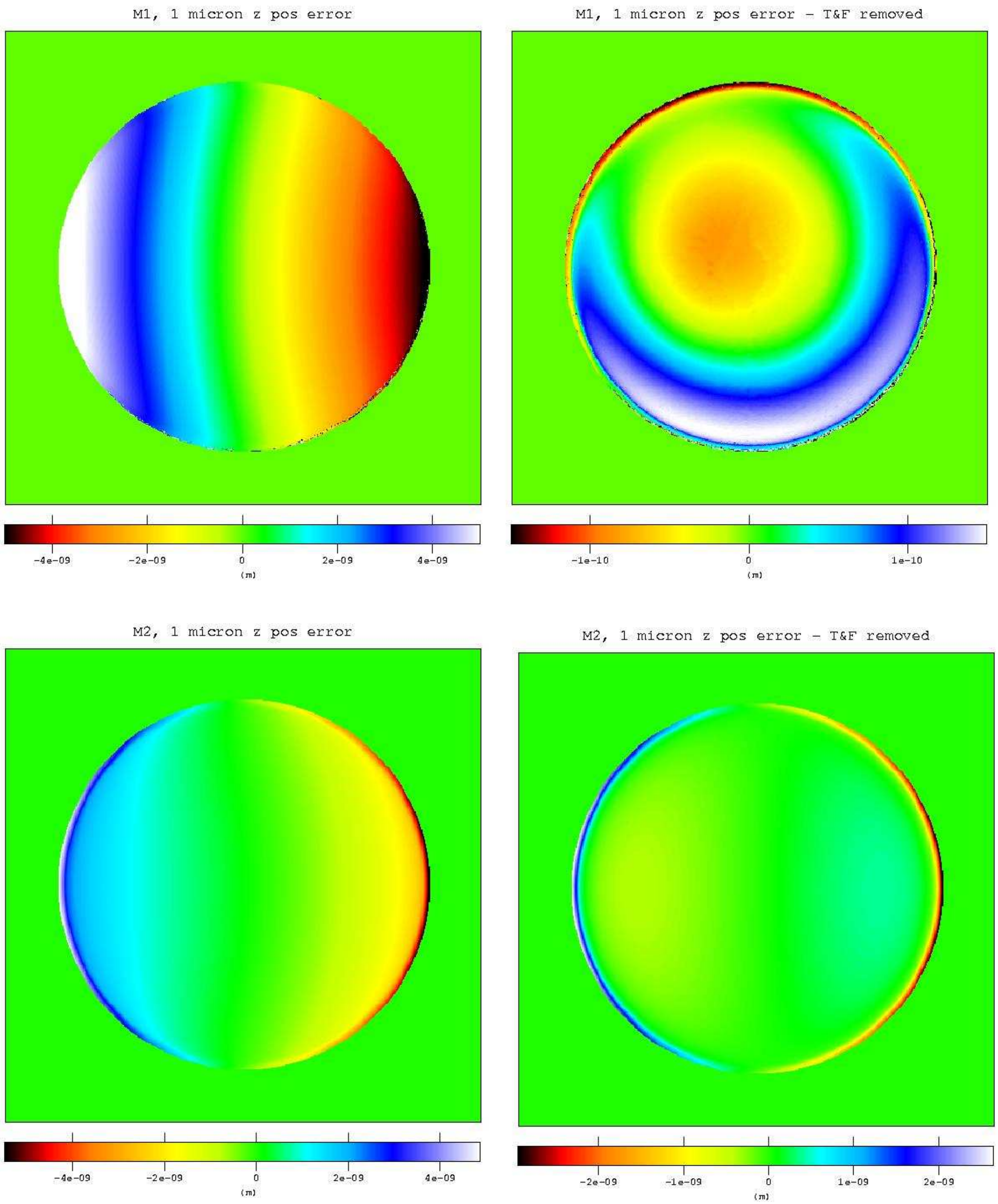

Figure 9: Effect of an alignment error of M1 and M2, translation along the z axis.

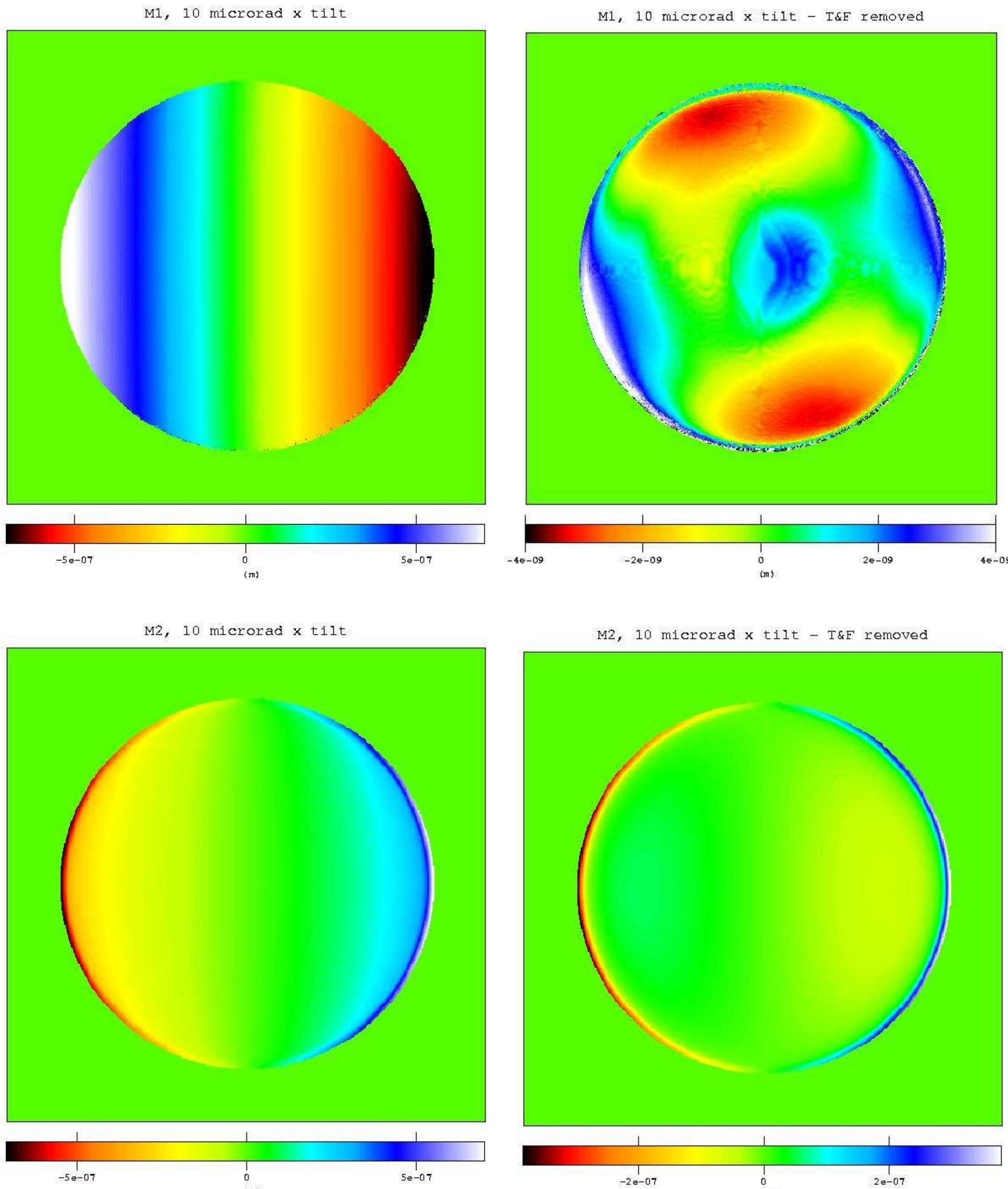

Figure 10: Effect of an alignment error of M1 and M2, rotation around the x axis.

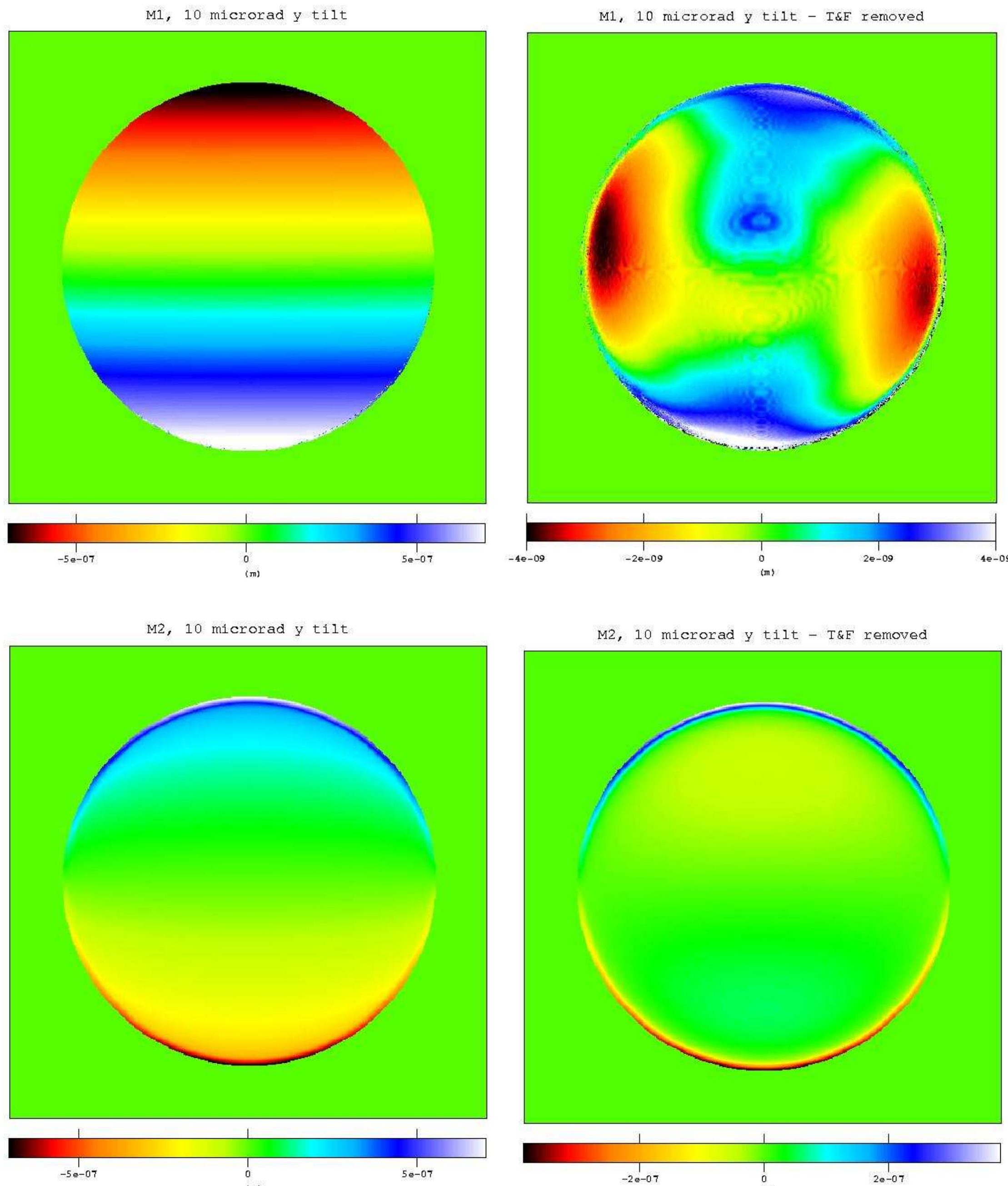

Figure 11: Effect of an alignment error of M1 and M2, rotation around the y axis.

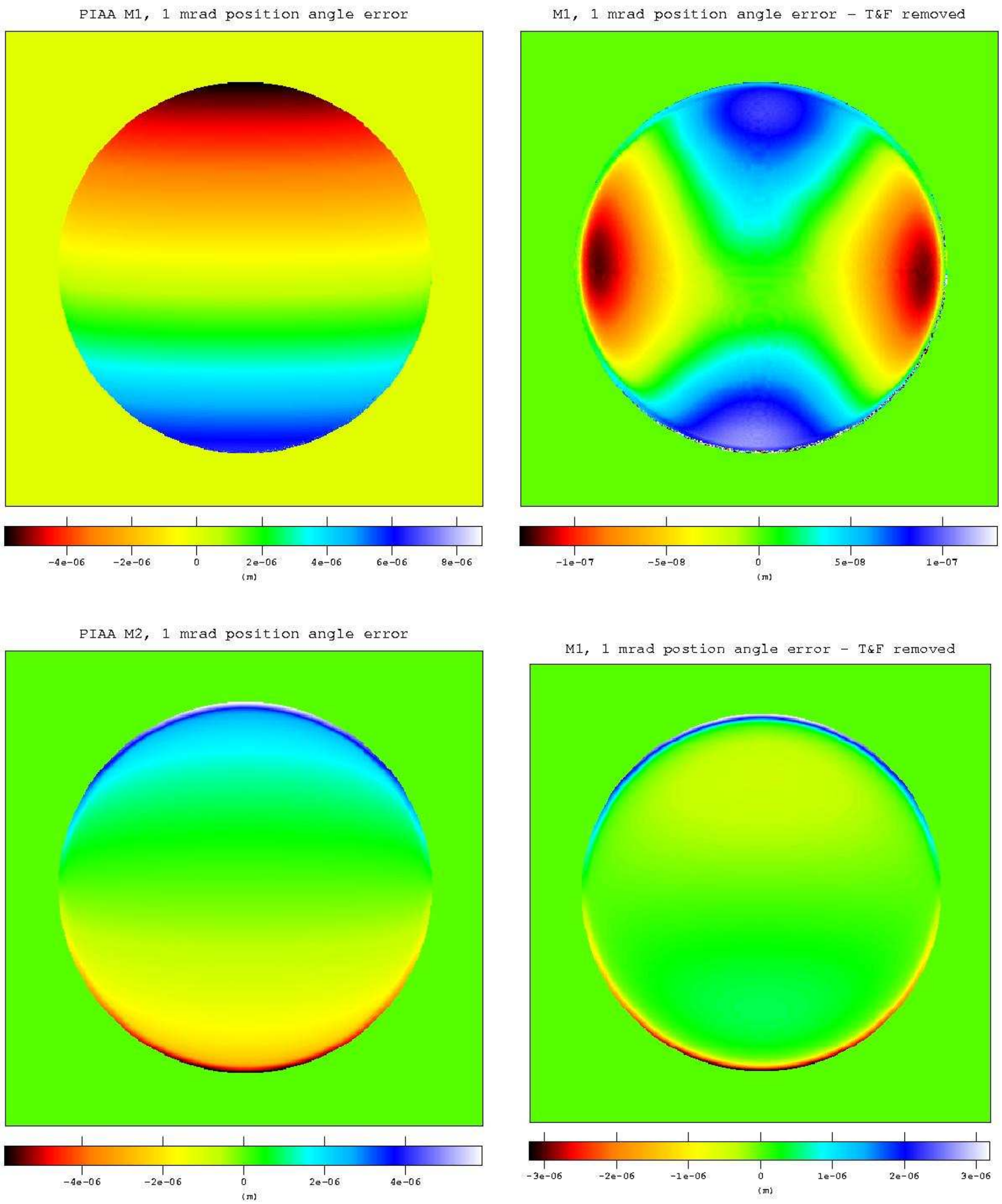

Figure 12: Effect of an alignment error of M1 and M2, rotation around the z axis.