

Design of the second generation PIAA system

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1 Introduction

This document describes in detail the design of the second generation PIAA system. This system will be used in the $0.5\text{ }\mu\text{m}$ to $0.8\text{ }\mu\text{m}$ band in vacuum and will be fed by a large mirror (1m).

1.1 Overall description

The PIAA unit's function is to apodize a beam. It is designed to accept a non-apodized beam at the input, and delivers an apodized beam at its output. Most of the apodization is performed by 2 aspheric mirrors, and a

small fraction of the apodization is performed by mask(s). This document describes both the aspheric mirrors (to be manufactured by Tinsley) and the conventional apodizers (vendor not identified yet).

The layout of the PIAA system is shown in Figure 1. The source is collimated by the First PIAA Mirror (M1) and reimaged by the Second PIAA Mirror (M2). The figure of each of these mirrors is a base paraboloid of revolution modified precisely to apply the desired apodization in the test set-up. The spacing between the mirrors, as well as the focal length of the parabolic mirrors is 10 times the beam diameter, thus it is 900 mm with a 90 mm clear aperture diameter. Two apodizers (shown in blue) are also contributing to the apodization.

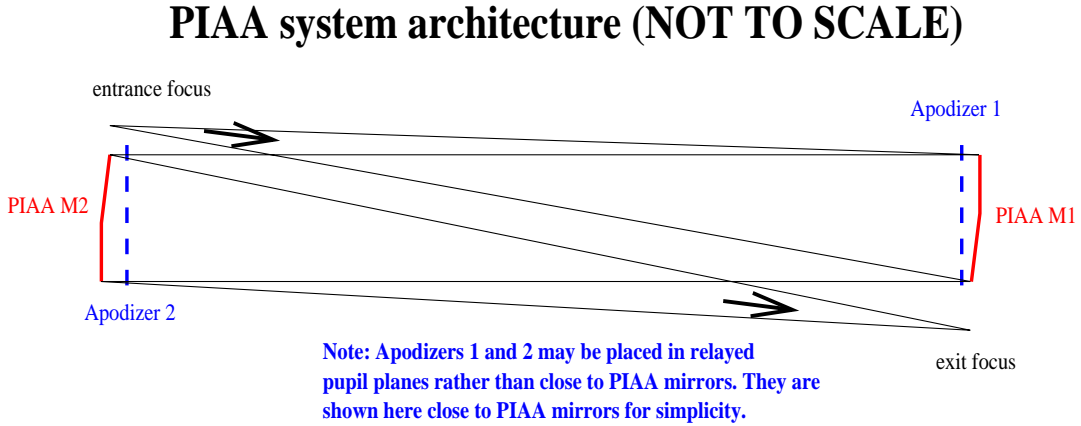


Figure 1: Layout of a PIAA system unit (all units are in m).

2 Choice of apodization functions

2.1 Choice of final apodization function

The final apodization function is the beam surface brightness after apodization by both the PIAA mirrors and the conventional apodizers.

The final apodized beam is to be radially apodized: surface brightness is only a function of distance to the center of the beam. The adopted radial profile amplitude $f(r)$ (square root of surface brightness) was obtained by optimizing the distance from the optical axis at which the contrast exceeds

1e-12. For convenience, a simple analytical expression was chosen for $f(r)$:

$$f(r) = \exp(a_1 \times r^{b_1} + a_2 \times r^{b_2}) \quad (1)$$

with $0 < r < 1$. An optimization of parameters a_1 , a_2 , b_1 and b_2 led to the following values, which minimized in the focal plane the separation beyond which the contrast is better than 1e-12:

$$a_1 = -14.414531; b_1 = 2.003950; a_2 = 1.660601; b_2 = 2.036393; \quad (2)$$

The total throughput of this profile is 3.9214% (this would be the throughput of a conventional apodization system using this profile). The profile $f(r)$ obtained is very close to a gaussian.

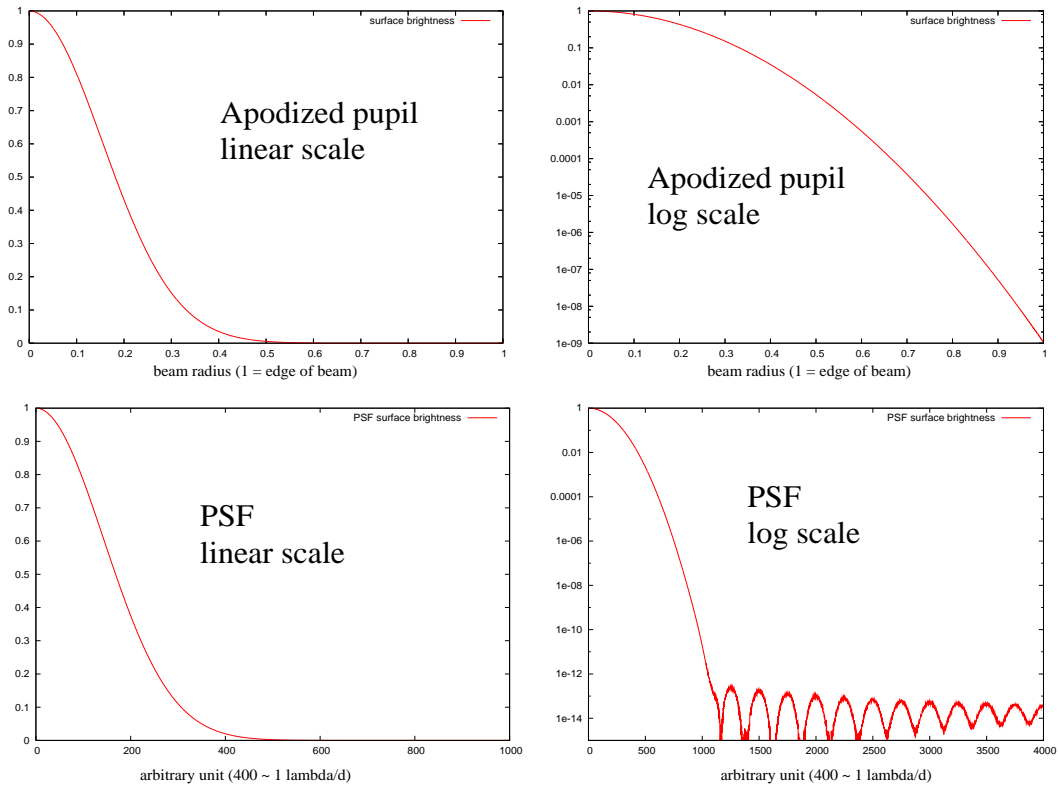


Figure 2: Apodization profile adopted for the design of the high contrast PIAA system (top) and corresponding high contrast PSF (bottom).

Figure 2 shows the apodization profile and the corresponding high profile PSF. The inner working angle (IWA) for this profile has not been accurately

measured yet but is likely $2\ 1/d$ or slightly more at the $1e12$ contrast. Our experience is that systems at $1e10$ contrast have inner working angle slightly less than $2\ 1/d$ and IWA increases as contrast is pushed higher. The radial profile adopted in this work is however also compatible with lower contrast / smaller IWA, but this may require redesign of the conventional apodizer(s) and focal plane mask.

2.2 Apodization sharing between PIAA mirrors and conventional apodizers

2.3 Roles of Apodizers

The PIAA mirrors share the apodization with two apodizers, as shown on figure 1. There are two reasons to have such apodizers:

- **PIAA mirrors manufacturing.** If the PIAA mirrors were to do all of the apodization, the outer edge of PIAA M1 would be very curved in order to spread light and project onto PIAA M2 the very faint outer wings of the apodized profile. Such a strongly curved edge would be very challenging to manufacture and test. The PIAA mirrors are therefore designed to do most, but not all, of the apodization, and **Apodizer 2** completes the apodization.
- **Chromaticity of the PIAA system.** If PIAA M1 had a very sharp outer edge, this sharp feature would be seen slightly differently by different wavelengths: diffraction propagation would then create a chromatic variation of the output OPD and amplitude of the apodized beam. It is therefore important to use **Apodizer 2** to mitigate this problem. In addition to this effect, the edge of the beam entering the PIAA unit needs to be apodized to avoid chromatic propagation effects created by the sharp edge of a beam: this is done by **Apodizer 1**, which is “softening” the edge of the beam, outside the clear aperture of the PIAA unit.

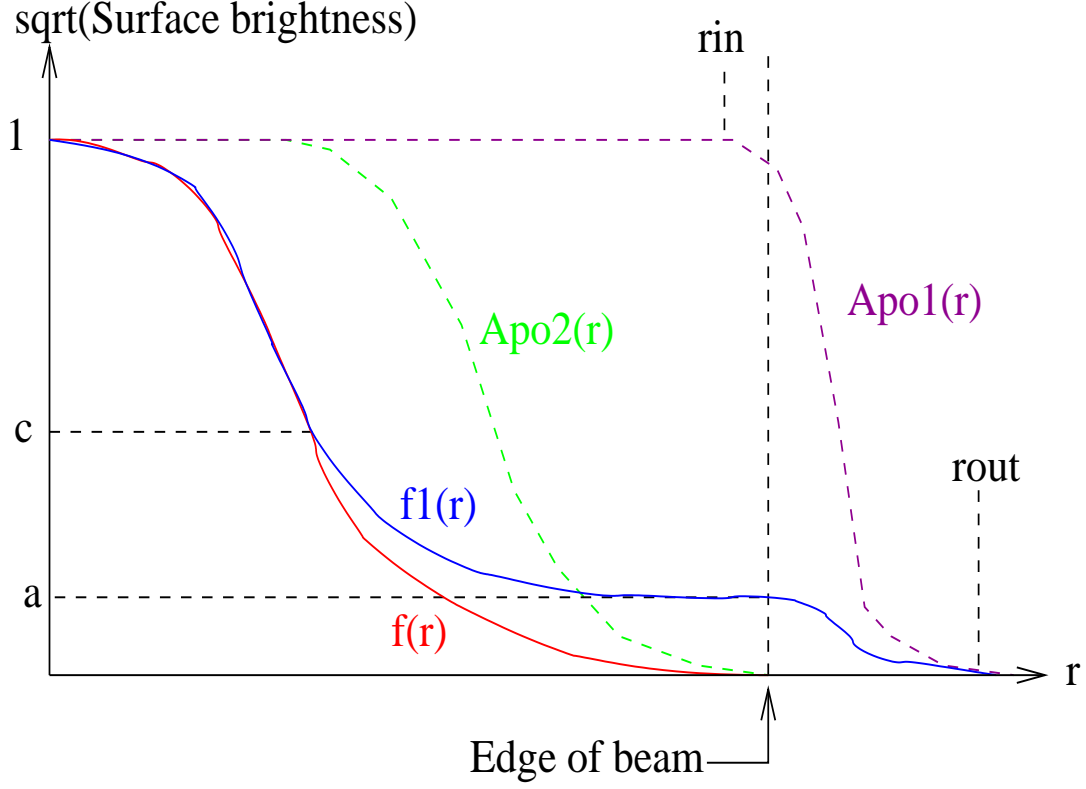


Figure 3: Roles of apodizers 1 and 2. The PIAA mirrors deliver the beam profile $f_1(r)$ which is multiplied by $Apo_2(r)$ (apodizer 2) to yield the final apodized profile $f(r)$. Beyond the edge of the beam, multiplication by $Apo_1(r)$ (apodizer 1 on PIAA M1) mitigates edge diffraction effects.

2.4 Achromaticity vs. throughput vs. manufacturability optimization

2.4.1 Parameters used for the optimization

The apodization sharing between PIAA and the apodizers is defined here by the following parameters:

For **Apodizer 2**:

- **Parameter a**: This is the minimal value of $f_1(r)$ within the clear aperture. If $f(r) = 0$, then $f_1(r) = a$
- **Parameter c**: This is the value of $f(r)$ above which the apodizer will

not remove any light. If $f(r) > c$, then $f_1(r) = f(r)$

- **Parameter b:** This quantifies the transition speed between the 2 regimes outlined above.

The exact equations for the relationship between $f(r)$ and $f_1(r)$ are: If $f(r) > c$, $f_1(r) = f(r)$ (no apodization), otherwise,

$$f_1(r) = f(r) + a \times \left(0.5 + 0.5 \times \cos(\pi \times f(r)/c)^b\right). \quad (3)$$

In addition, two parameters are used for **Apodizer 1** on PIAA M1:

- **Parameter r_{in} :** Inner edge of the apodizer 1. If $r < r_{in}$, apodizer 1 throughput is 1.
- **Parameter r_{out} :** Outer edge of the apodizer 1. If $r > r_{out}$, apodizer 1 throughput is 0.

Between r_{in} and r_{out} , apodizer 1 throughput is :

$$Apo_1(r) = (0.5 + 0.5 \times \cos(\pi \times (r - r_{in})/(r_{out} - r_{in})))^2 \quad (4)$$

2.4.2 Goal of the optimization

The optimization looks at many possible designs, each described by the values of parameters a , b , c , r_{in} and r_{out} . For each design, a diffraction propagation simulation is used to measure the contrast in a 20% wide band centered at $0.7\mu\text{m}$. The system throughput is also measured, taking into account the losses due to beam oversizing (value of r_{out}) and apodizers. The goal of the optimization is to find a tradeoff between :

- **PIAA mirrors manufacturability:** This is mostly a function of parameter a , which quantifies how dark the edges of the PIAA-apodized beam should be.
- **Science performance:** This is quantified by both system throughput and achromaticity.

All simulations assume a 90mm clear aperture beam and a 900mm separation between PIAA M1 and PIAA M2.

	a	b	c	r_{in}	r_{out}
Design 1 (Easy)	0.100364	1.413834	0.345524	0.962062	1.016005
Design 2 (Medium)	0.070506	1.642965	0.411239	1.004930	1.011599
Design 3 (Challenging)	0.056029	1.505329	0.333700	0.986906	1.011236
	System Throughput		Worst case contrast in 20% band		
Design 1 (Easy)	0.762825		1.321975e-11		
Design 2 (Medium)	0.848389		1.227975e-11		
Design 3 (Challenging)	0.891744		1.195350e-11		

Table 1: Parameters describing the 3 PIAA designs. The system throughput assumes 1e-12 contrast goal; for 1e-10 contrast, a smaller portion of PIAA M1 may be illuminated and the throughput is larger.

2.4.3 Results of the optimization

Table 1 gives the values of the parameters and the performance for 3 specific designs. As shown in figure 4, these 3 designs were picked out of more than 6000 designs which were evaluated for throughput and chromaticity.

2.5 Apodizer design

Apodizers can be made either continuous or binary. Binary apodizers (“shaped pupils”) look more attractive since they are relatively achromatic and easier to manufacture. A binary apodizer can be a thin plate in which an opening is cut. Both apodizers 1 and 2 have been designed to not extend near the center of the beam, where most of the light is. Apodizer 1 is less critical since it only operates on the very edge of the beam. Apodizer 2 is more challenging as it acts within the clear aperture of the PIAA system.

Figure 5 shows a binary design for Apodizer 2, along with the corresponding high contrast PSF. The binary design is simply obtained by ensuring for each value of the radius r that the fraction of the light left between the spokes (equal to one minus the number of spokes times the spokes width) is equal to the throughput of the ideal continuous apodizer. The only limitations from using a binary apodizer are an outer working angle and additional scattering of background light into the dark hole - both of these get better as the number of spokes is increased.

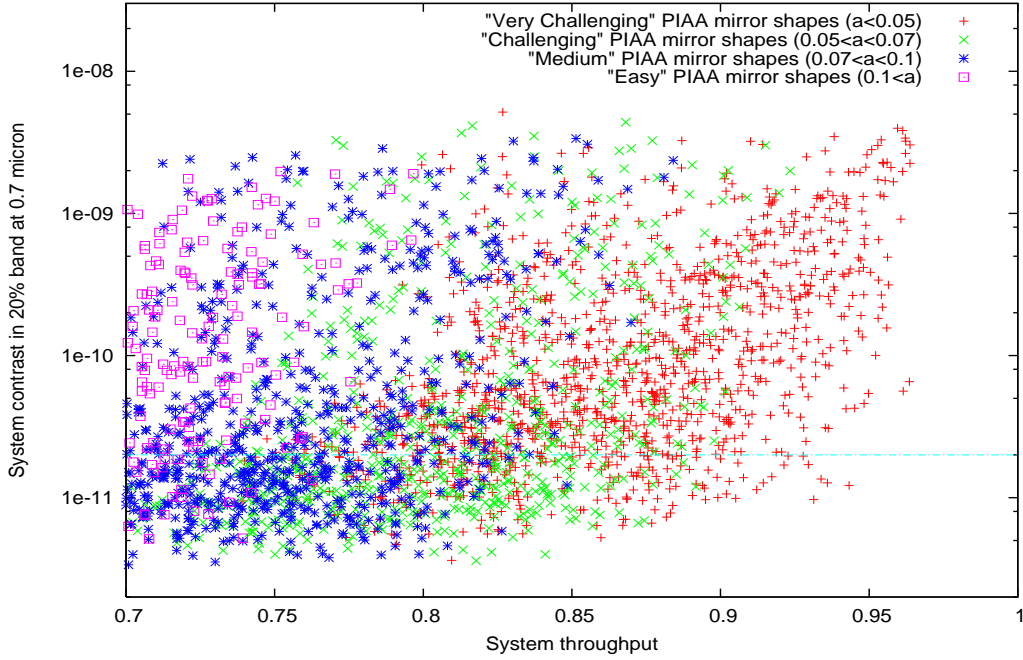


Figure 4: Chromaticity vs. throughput for a large number of PIAA system designs. The horizontal line is drawn at the 2×10^{-11} contrast level. All contrasts values are “worst case” (highest contrast value beyond the IWA of the coronagraph).

3 PIAA Mirrors Specifications for 3 on-axis designs

3.1 Mirror shapes for on-axis configuration

The mirror shapes have been computed for an on-axis system, where the entrance focus of the PIAA system would be on the surface of PIAA M2, and the output focus would be on the surface of PIAA M1. This configuration is different from the true configuration, but it can be more easily used to estimate how challenging each design is, as the curvature on the mirrors is only very slightly affected by moving the input and output foci off-axis.

The PIAA system configuration and exact shapes of the mirrors for PIAA design 1 is in the following files:

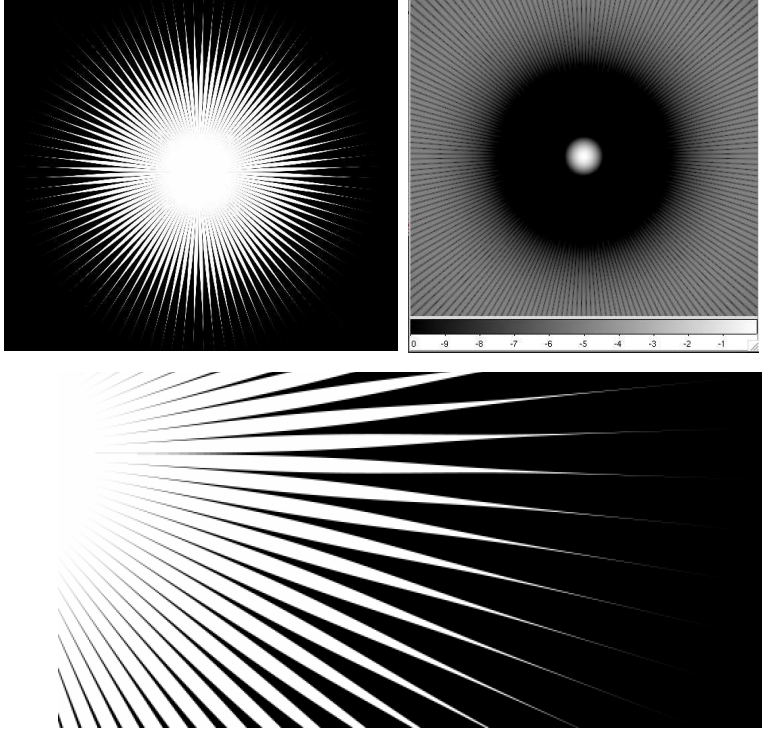


Figure 5: Example design for Apodizer 2. Views of a binary version of apodizer 2: full pupil (top left) and detail (bottom). The central part of apodizer 2 is totally clear. The corresponding PSF (top right) yields better than $1e10$ contrast over a field of view set by the number of spokes in the apodizer.

- **File “pup.prof.des1”:** This is the amplitude (square root of intensity) which the PIAA mirrors create, without the apodizer. column 1 = radius (from 1 to 10000, unitless), column 2 = amplitude (from 0.0 to 1.0).
- **File “Mshape_800000.dat.des1”:** This is the radial sag of mirrors PIAA M1 (column 2) and PIAA M2 (column 1). Column 1 is the radius in meter. Please note that the coordinate system is the same for both mirror: the zero point of the coordinate system is at the center of PIAA M1. The $+z$ direction points to PIAA M2, so PIAA M2 is sitting at $z \approx 0.9\text{m}$ looking down.

- **File “optics_shape.dat.des1”:** This describes the optics shapes by following, for each line of the file, a geometrical ray. column 1 = radius of the ray (m) on PIAA M1, column 2 = radius of the ray (m) on PIAA M2, column 3 = sag of PIAA M1, column 4 = sag of PIAA M2, column 5 = measured system OPD for the light ray, column 6 = slope on PIAA M1, column 7 = slope derivative on PIAA M1, column 8 = curvature on PIAA M1, column 9 = rate of change of curvature on PIAA M1.

For Designs 2 and 3, the file names end by “.des2” and “.des3” respectively.

4 Off-axis optics shapes for design 2

4.1 Introduction

For an on-axis PIAA configuration, the optics shapes are circular-symmetric, and a single 1-D differential equation can be used to solve for the exact shape of both optics, whether the system design adopts collimated or focused input and output beams. Circular symmetry allows for the 1-D solution of the differential equation to be rotated around the optical axis to yield exact 2-D optics shapes.

If the off-axis configuration is designed for collimated beam input and output, the 2-D optics shapes can be obtained by adding a tilted plane function to the on-axis shapes. However, in the general 2-D off-axis case (for example for the focus input/output system configuration), no simple analytical relationship from the on-axis to the off-axis optics shapes is known.

4.1.1 Existence of a solution

While a single exact PIAA optics shapes solution exists for both the 1-D and off-axis circular-symmetric 2-D configurations, arbitrary 2-D remapping are overconstrained and usually have no solution: they cannot be produced by continuous (note: in this section, we adopt the term continuous function to describe a C^2 function and rightly assume that all realistic optical surfaces for PIAA mirrors are C^2 functions) mirror shapes, even if the remapping is continuous.

A good example of a remapping for which no solution exists is a collimated beam system with $x_2 = f(x_1)$, $y_2 = y_1$, where (x_1, y_1) and (x_2, y_2) are the positions of the same light ray in the input and output collimated beams

respectively, and f is a continuous remapping function with $df(x_1, y_1)/dy_1 \neq 0$ in at least part of the beam footprint. With $M_1(x_1, y_1)$ and $M_2(x_2, y_2)$ the mirror shapes of respectively mirror M1 and mirror M2, $y_2 = y_1$ imposes that $dM_1(x_1, y_1)/dy_1$ and $dM_2(x_2, y_2)/dy_2$ are both equal to 0 (the PIAA system must not move the light rays in the y coordinate), and are therefore simply functions of x_1 and x_2 respectively. Since $d^2 M_i/dx_i dy_i = d^2 M_i/dy_i dx_i$, dM_i/dx_i must be independant of y, which is inconsistent with the fact that $df(x_1, y_1)/dy_1 \neq 0$: no solution exists for this remapping geometry.

When designing 2-D off-axis PIAA mirror shapes, it is important to keep in mind that if the remapping is imposed, there may not be a solution to the optics shapes. The algorithm developed to solve for the optics shapes therefore:

- Is an converging iterative algorithm rather than a direct derivation of a solution. A direct derivation of the solution would fail if no exact solution exists, while a properly designed iterative solution will find the solution the closest to the problem.
- Does not impose a remapping function. The constraint imposed in the iterative algorithm is the output beam intensity distribution rather than the remapping function.

A method based upon these two principles is given in the next section.

4.2 Method to solve for 2-D off-axis shapes

4.2.1 Starting point

A first approximation of the optics shapes for an off-axis configuration can be obtained by simply adding off-axis parabolas to the nominal PIAA mirror shapes derived for an on-axis collimated beam system. This starting point, for the system considered in this document (90 mm beam diameter, 85 mm off-axis distance 900 mm PIAA M1 - PIAA M2 separation), delivers a beam with a few micron OPD error and a approximately 5% relative error in beam surface brightness.

4.2.2 STEP 1: Definition of the target beam intensity distribution

With an off-axis system, measurement of the surface brightness on a surface can be affected by both the shape of the surface and the angle from which

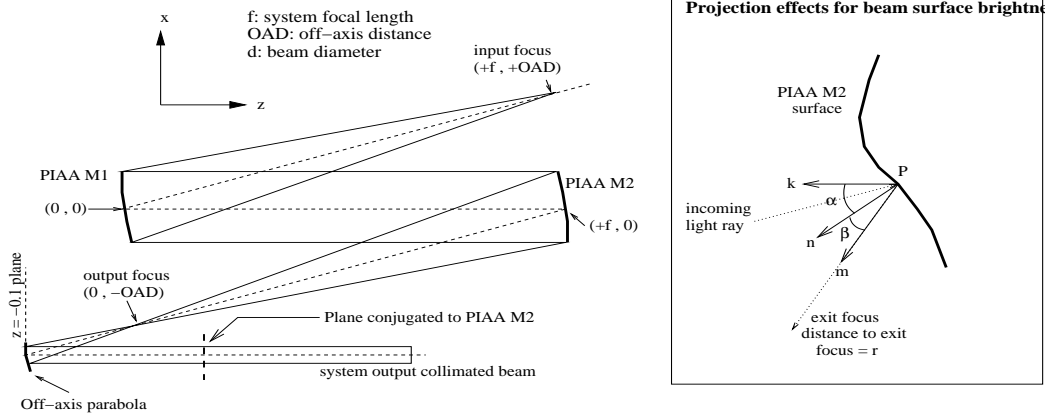


Figure 6: PIAA system geometry used for the design of the off-axis 2D PIAA mirror shapes.

it is viewed, due to projection effects (see figure 6). It is therefore important to simultaneously define both what should be the target beam intensity distribution and how it is measured (on which surface viewed from which angle). This issue is not a problem on slow systems (large F/ratio number), but is significant on the F/10 off-axis system considered here. The geometrical optical design assumed, as shown in fig. 6, assumes that the fast F/10 beam is collimated after the exit focus of the PIAA system with an off-axis parabola (OAP). The full PIAA system is designed to deliver, at the output of the OAP, a perfectly collimated beam with the required apodization profile. Subsequent optics (after the OAP) are assumed to be slow enough so that projection effects after this OAP are small. The system is therefore designed to deliver the desired apodization profile in this collimated beam.

For the optimization of the PIAA mirror shapes, the full PIAA system was modelled, including the OAP at the exit. The intensity distribution and OPD were measured in a $z=\text{cst}$ plane located in the exit collimated beam after the OAP. The location of the plane was chosen to be conjugated to PIAA M2. In this location, the beam intensity is independent of PIAA M2 shape, and is only a function of PIAA M1 shape. PIAA M1 was therefore designed first to produce the appropriate intensity distribution. Then, PIAA M2 can be tuned to remove the residual OPD aberration in the output beam.

4.2.3 Note about projection effects on PIAA M2: Why add an OAP ?

It would be possible to measure beam intensity profile on PIAA M2, although projection effects in an off-axis system configuration need to be taken into account, and would make it more difficult to design the system. Numerically, it is easiest to measure beam intensity distribution by computing the 2-D density of light rays projected $z=\text{cst}$ plane by recording the x and y coordinates of rays as they impact the surface of PIAA M2. This beam surface brightness S_0 is what would be measured on a screen perpendicular to vector k aligned with the z direction on the right panel of fig. 6. The actual surface brightness on the tilted surface of PIAA M2 mirror is $S_1 = S_0 \cos(\alpha)$. As seen from the PIAA system output focus, this corresponds to an apparent surface brightness (intensity by unit of solid angle) equal to :

$$S_2 = \frac{S_1}{\cos(\beta)} = S_0 \frac{\cos(\alpha)}{\cos(\beta)} \quad (5)$$

Due to projection effects, this beam intensity distribution is not equal to the beam intensity distribution in the system output collimated beam. On PIAA M2, the amplitude of projection effects on intensity distribution can be computed from the angle β between the normal to the mirror surface and the direction of the light rays traveling away from the mirror surface towards the output focus (this effect scales as $1/\cos(\beta)$) and the distance between M2 surface and the target focal point.

4.2.4 STEP 2: Evaluation of the response of the system to small perturbations of PIAA M1 shape

As shown in fig. 7, the effect of small changes in PIAA M1 shape is to modify the intensity distribution in the output collimated beam. For the first 100 Zernikes, this effect was computed. The difference between the ideal beam intensity distribution and the one observed at the output the system was then decomposed as a sum of these small perturbations, which can then be used to evaluate how to modify PIAA M1 shape to obtain the correct beam intensity.

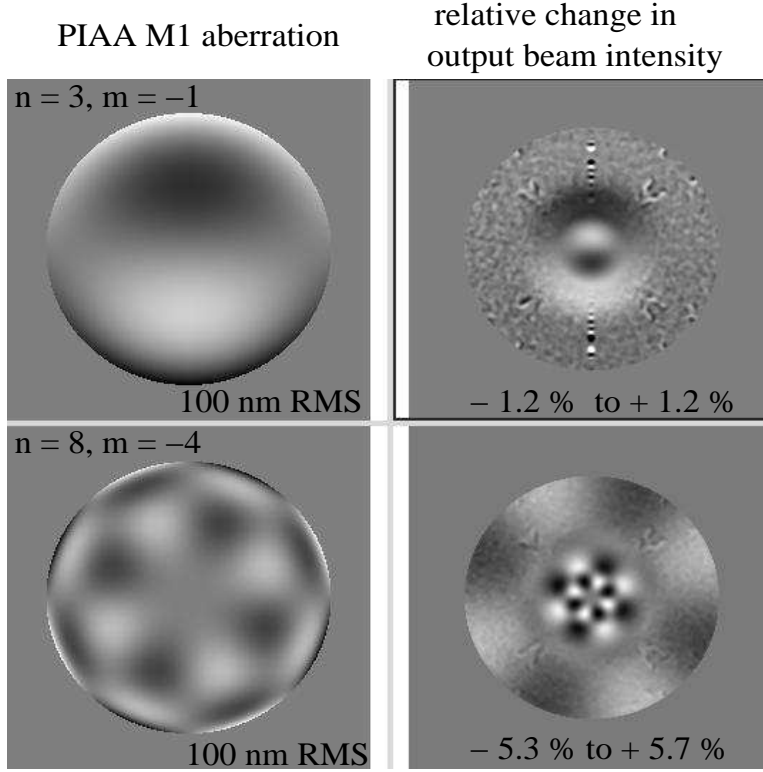


Figure 7: Effect of PIAA M1 surface changes (left) on the intensity distribution on the output beam. (right).

4.2.5 STEP 3: PIAA M1 shape update to reach target beam intensity distribution on PIAA M2 surface

PIAA M1 shape was updated to remove errors in the beam intensity distribution, assuming a perfectly linear relationship between PIAA M1 shape and the output beam intensity distribution. Due to non-linear effects, STEPS 2 and 3 were repeated several times, using the PIAA M1 shape from STEP 3 as the new input for STEP 2.

4.2.6 STEP 4: PIAA M1 shape fit

The PIAA M1 shape modification obtained by the Zernike polynomial fitting process described in STEPS 2 and 3 showed localized edge errors (common problem when fitting a function with Zernike polynomials, which tend to

have large slopes at the edges). This shape was therefore fitted with a more appropriate analytical form which does not have the edge problem.

4.2.7 STEP 5: PIAA M2 shape update

The PIAA M2 shape update is simply obtained from the residual system OPD map.

4.3 PIAA mirror M1 3D shape

All units are in meter (m).

$$M_1(x, y) = f_1(r) + OAP_1(x, y) + OAT_1(x, y) \quad (6)$$

where $r = \sqrt{x^2 + y^2}$.

4.3.1 Term $f_1(r)$

This is the sag of the PIAA M1 mirror for the on-axis focus-to-focus PIAA system. It is provided as an ASCII file (file "PIAAM1_fr.dat" - first column is r in meter, second column is $f_1(r)$). NOTE: The file currently only goes to the nominal edge of the beam ($r = 0.045m$). The term $f_1(r)$ can also be fitted as the sum of a parabola and a series of cosines:

$$f_1(r) = \frac{r^2}{4 \times 0.9} + \sum_{k=0}^{199} a_k \times \cos(k(r/0.045)\pi) \quad (7)$$

This fit has a accuracy of 0.012 nm RMS and a peak error of 0.42 nm. The coefficients a_k are given in the file "PIAAM1_cosfitcoeff_200.dat" (col 1 is k , col 2 is a_k).

4.3.2 Term OAP_1 (Off-Axis Parabola Term)

This is the difference between the base OAP shape and the on-axis parabola shape.

$$OAP_1(x, y) = \frac{(OAD - x)^2 + y^2 - OAD^2}{2(f + \sqrt{f^2 + OAD^2})} - \frac{x^2 + y^2}{4f} \quad (8)$$

where $f = 0.9m$ and $OAD = 0.085m$

4.3.3 Term OAT_1 (Off-Axis Term)

$$OAT_1(x, y) = C_x X + C_{xx} X^2 + C_{yy} Y^2 + C_1 \sin(C_2 R)(1.0 + C_3 R^2) \frac{x}{r} \quad (9)$$

where $X = x/0.045$, $Y = y/0.045$ and $R = \sqrt{X^2 + Y^2}$. The coefficients in this equation are given in the file "PIAAM1-OAT.dat".

4.3.4 Sample file for checking

File "PIAAM1-check.dat" is a list of points for checking the implementation of the equations above. All units are m. Column 1 is x , column 2 is y , column 3 is $M_1(x, y)$, column 4 is $f_1(r)$, column 5 is $OAP_1(x, r)$, column 6 is $OAT_1(x, y)$. The sum of columns 4, 5 and 6 is therefore equal to column 3. A finer grid version of the same file is also available in "PIAAM1-check_fine.dat".

3-D representations of columns 3, 4, 5 and 6 can be found in files "PIAAM1_3Dview_c3.jpg", "PIAAM1_3Dview_c4.jpg", "PIAAM1_3Dview_c5.jpg" and "PIAAM1_3Dview_c6.jpg".

4.4 PIAA mirror M2 3D shape

All units are in meter (m). **WARNING: z is pointing into the substrate in the common x,y,z coordinate system for the full PIAA system.**

$$M_2(x, y) = f_2(r) + OAP_2(x, y) + OAT_2(x, y) \quad (10)$$

where $r = \sqrt{x^2 + y^2}$.

4.4.1 Term $f_2(r)$

This is the sag of the PIAA M2 mirror for the on-axis focus-to-focus PIAA system. It is provided as an ASCII file (file "PIAAM2_fr.dat" - first column is r in meter, second column is $f_2(r)$). NOTE: The file currently only goes to the nominal edge of the beam ($r = 0.045m$). The term $f_2(r)$ can also be fitted as the sum of a parabola and a series of cosines:

$$f_2(r) = -\frac{r^2}{4 \times 0.9} + \sum_{k=0}^{199} b_k \times \cos(k(r/0.045)\pi) \quad (11)$$

This fit has a accuracy of 0.014 nm RMS and a peak error of 0.45 nm. The coefficients b_k are given in the file "PIAAM2_cosfitcoeff_200.dat" (col 1 is k , col 2 is b_k).

4.4.2 Term OAP_2 (Off-Axis Parabola Term)

This is the difference between the base OAP shape and the on-axis parabola shape.

$$OAP_2(x, y) = -\frac{(OAD + x)^2 + y^2 - OAD^2}{2(f + \sqrt{f^2 + OAD^2})} + \frac{x^2 + y^2}{4f} \quad (12)$$

where $f = 0.9m$ and $OAD = 0.085m$

4.4.3 Term OAT_2 (Off-Axis Term)

This term has been fitted as a sum of cosine/sine waves up to a spatial frequency of 25 cycles per aperture.

$$OAT_2(x, y) = -\frac{1}{2} \sum_{i,j} [C_{ij} \cos(iX + jY) + S_{ij} \sin(iX + jY)] \quad (13)$$

where $i = 0...24$, $j = -24...24$, $X = x/0.045$ and $Y = y/0.045$. This surface fit is good to 1.5 nm RMS, but most of the error is at spatial frequencies greater than 25 cycles per aperture. The fit is therefore very good ($<<nm$) for low spatial frequencies. The coefficients C_{ij} and S_{ij} can be found in the file "PIAAM2_OAT25.dat", where column 1 is i , column 2 is j , column 3 is C_{ij} and column 4 is S_{ij} .

4.4.4 Sample files for checking

File "PIAAM2_check.dat" is a list of points for checking the implementation of the equations above. All units are m. Column 1 is x , column 2 is y , column 3 is $M_2(x, y)$, column 4 is $f_2(r)$, column 5 is $OAP_2(x, r)$, column 6 is $OAT_2(x, y)$. The sum of columns 4, 5 and 6 is therefore equal to column 3. A finer grid version of the same file is also available in "PIAAM2_check_fine.dat".

3-D representations of columns 3, 4, 5 and 6 can be found in files "PIAAM2_3Dview_c3.jpg", "PIAAM2_3Dview_c4.jpg", "PIAAM2_3Dview_c5.jpg" and "PIAAM2_3Dview_c6.jpg".

5 Description of Numerical Simulations used

5.1 Full 2-D diffraction propagation without approximations

This code is quite slow and has been used for final check of a few PIAA configuration. It was also used to verify the validity of approximations made by other faster propagation codes.

The routine is “PIAAWFCFRESNEL_PROPAGATE_EXACT2D()” in the module “PIAA_WFC_Fresnel1D” performs a 2D diffraction propagation without approximation. Its execution time is long and is function of both the sampling on PIAA M1 and the number of point onto PIAA M2 on which the complex amplitude needs to be computed. The input amplitude and phase maps can be modified (arrays “A0array” and “phi0array” in the code). The input of this routine is a “pup.prof” amplitude profile file, which needs to be in the running directory. The output is the ASCII file “ex2d.log”.

5.2 Fast 2-D diffraction propagation using Fresnel approximation

This code decomposes the PIAA mirror surfaces into individual planes. It performs a geometrical projection of the complex amplitude on the 3D surfaces of PIAA M1 and PIAA M2 onto a series of parallel planes. Standard Fresnel propagation using FFTs is used to propagate between these planes. This code is significantly faster than the exact 2D propagation, but adopting the Fresnel approximation leads to some errors. This code however accurately simulates the chromaticity of the PIAA unit, and the errors arising from the Fresnel approximation can be numerically compensated for by comparison with both the raytracing code ($\lambda = 0$ case) and the full 2D exact propagation code. The routine is in module “PIAA_WFC_Fresnel”.

5.3 Fast 1D diffraction propagation using Fresnel approximation

A 1D version of the 2D Fresnel approximation code described above was written and used for fast chromaticity optimization. This code is called many times by routine “PIAA_WFC_FRESNEL_run1D” in the module “PIAA_WFC_Fresnel1D”

to perform the PIAA chromaticity optimization.

5.4 2D binary apodizer code

This can be turned on within the routine “PIAA_WFC_FRESNEL_testOPTpoint1D” in module “PIAA_WFC_Fresnel1D” by changing the if statement in: “if(0==1)
// make 2D pupil & PSF + make binary apodizer”.

5.5 2D raytracing code

This can be called in module “PIAAgeom” within the “run_PIAA_lab_experiment_raytrace_simul_fin” routine. This code is used to compute the corrective terms to be applied to the PIAA mirror shapes when an off-axis focus-to-focus configuration is adopted. Zernike polynomials are applied to the mirror shapes until the output of the 2D raytracing code matches expectations.