Current Status of Testing General Relativity with the Star Orbiting Galactic Center BH Sgr A*

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+ Members in Subaru Proposal/National Grant (Kakenhi)

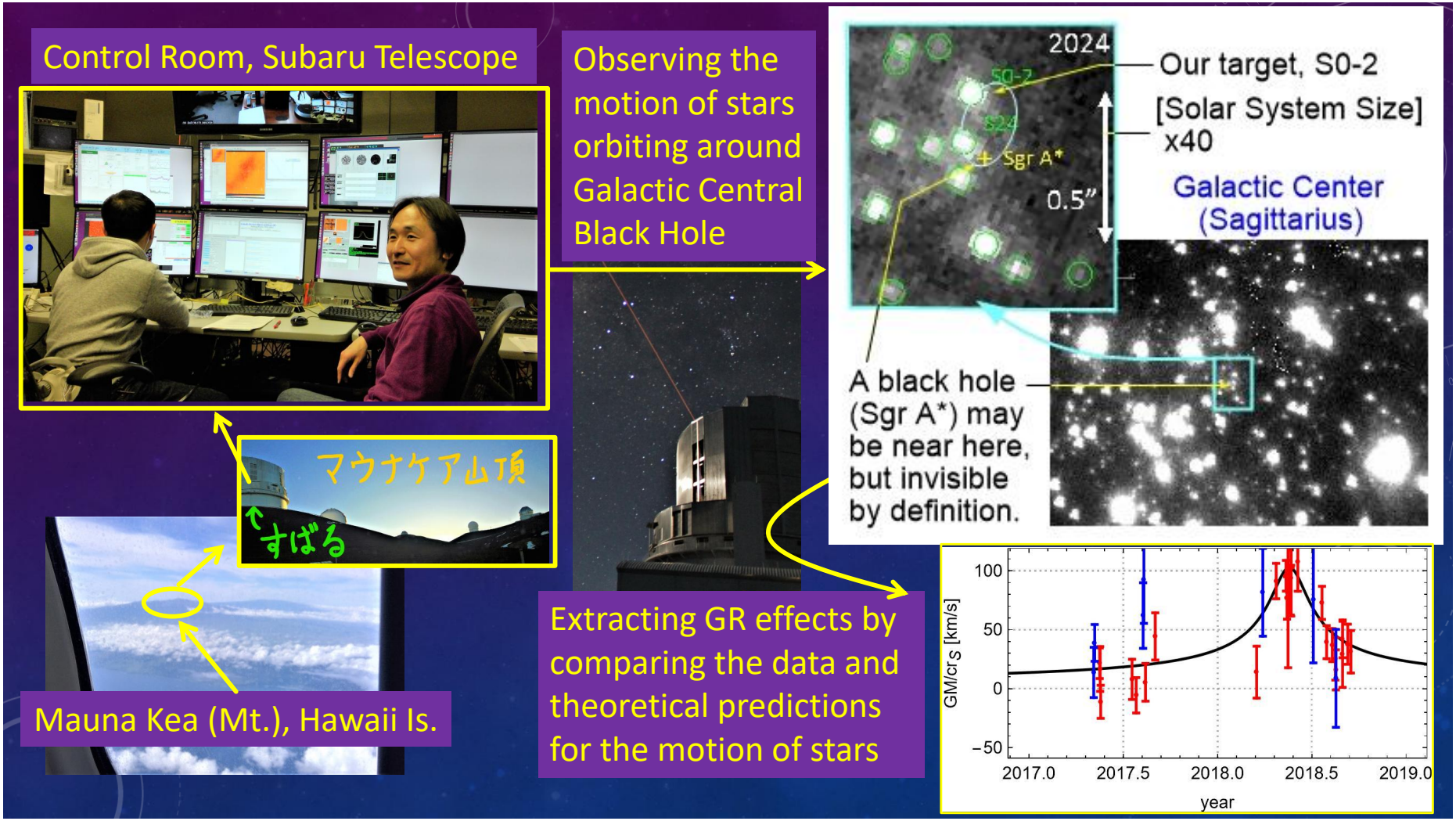
- Subaru Users Meeting 2025FY (NAOJ, mitaka) 2025/10/29 - 31
- Testing Gravity 2025 (SFU, vancouver) 2025/01/30 - 02/01

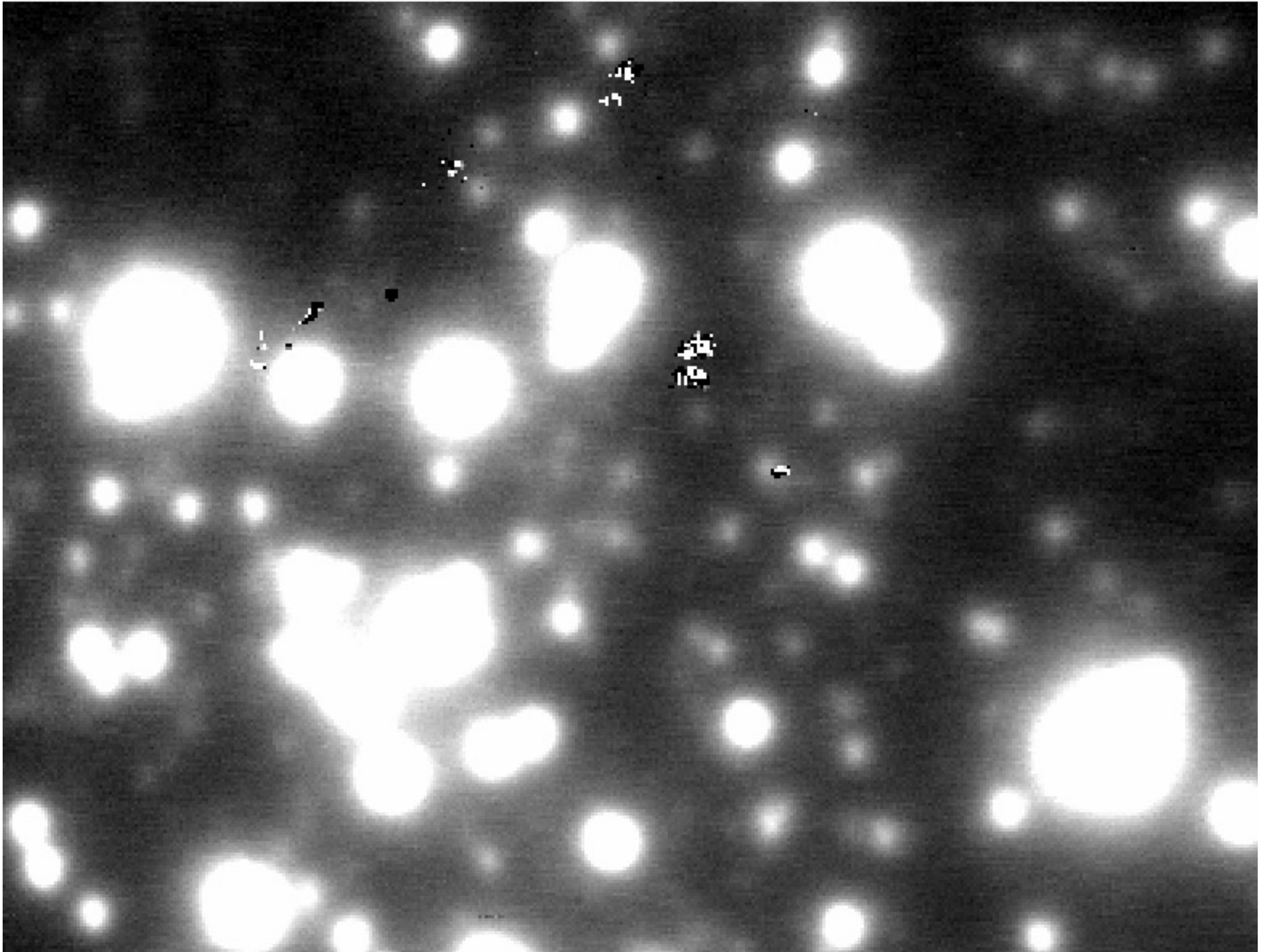
← Prog.Theor.Exp.Phys.2024 093E02 (arXiv:2408.11284)

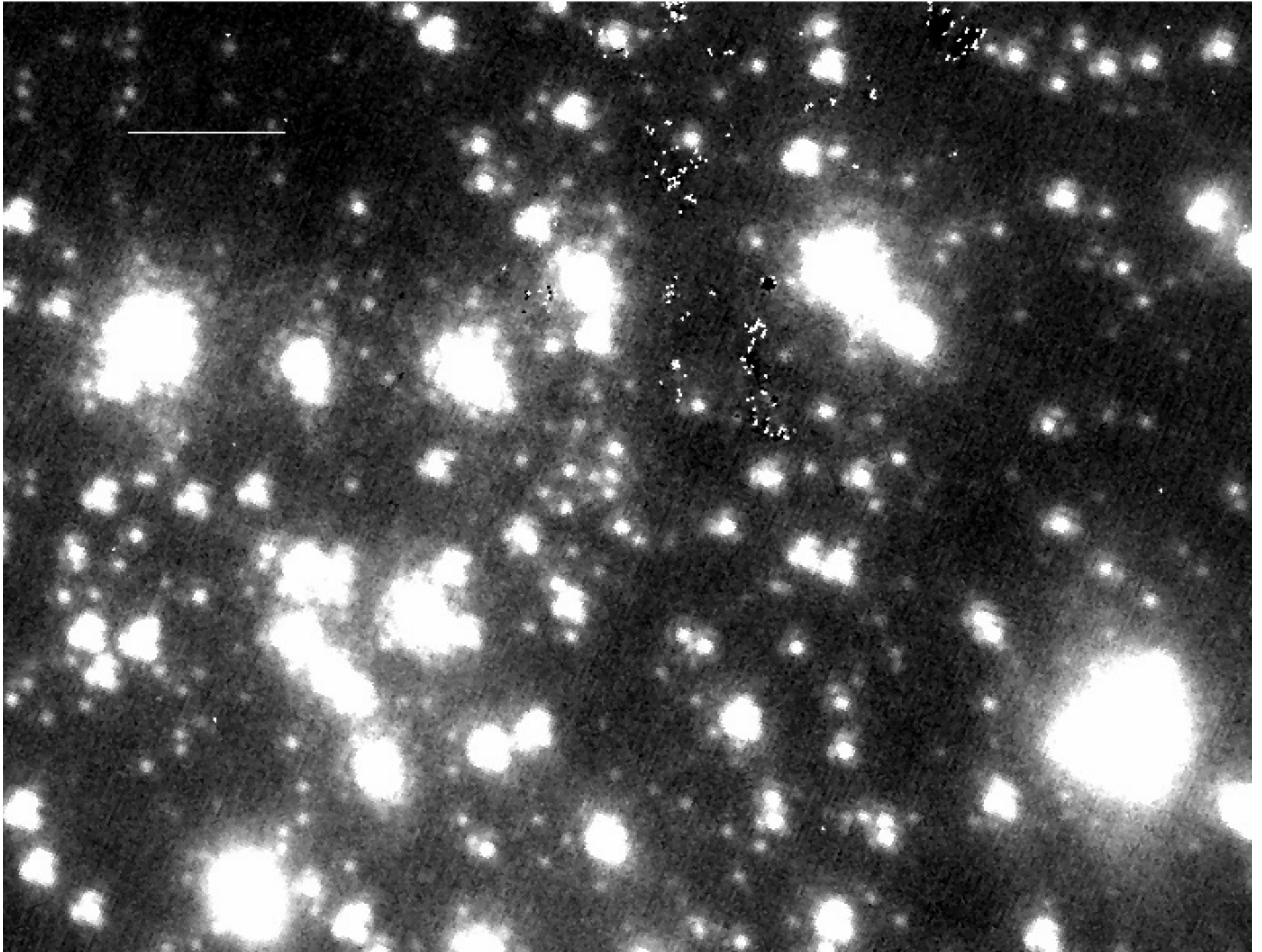
1. Sgr A*and GR

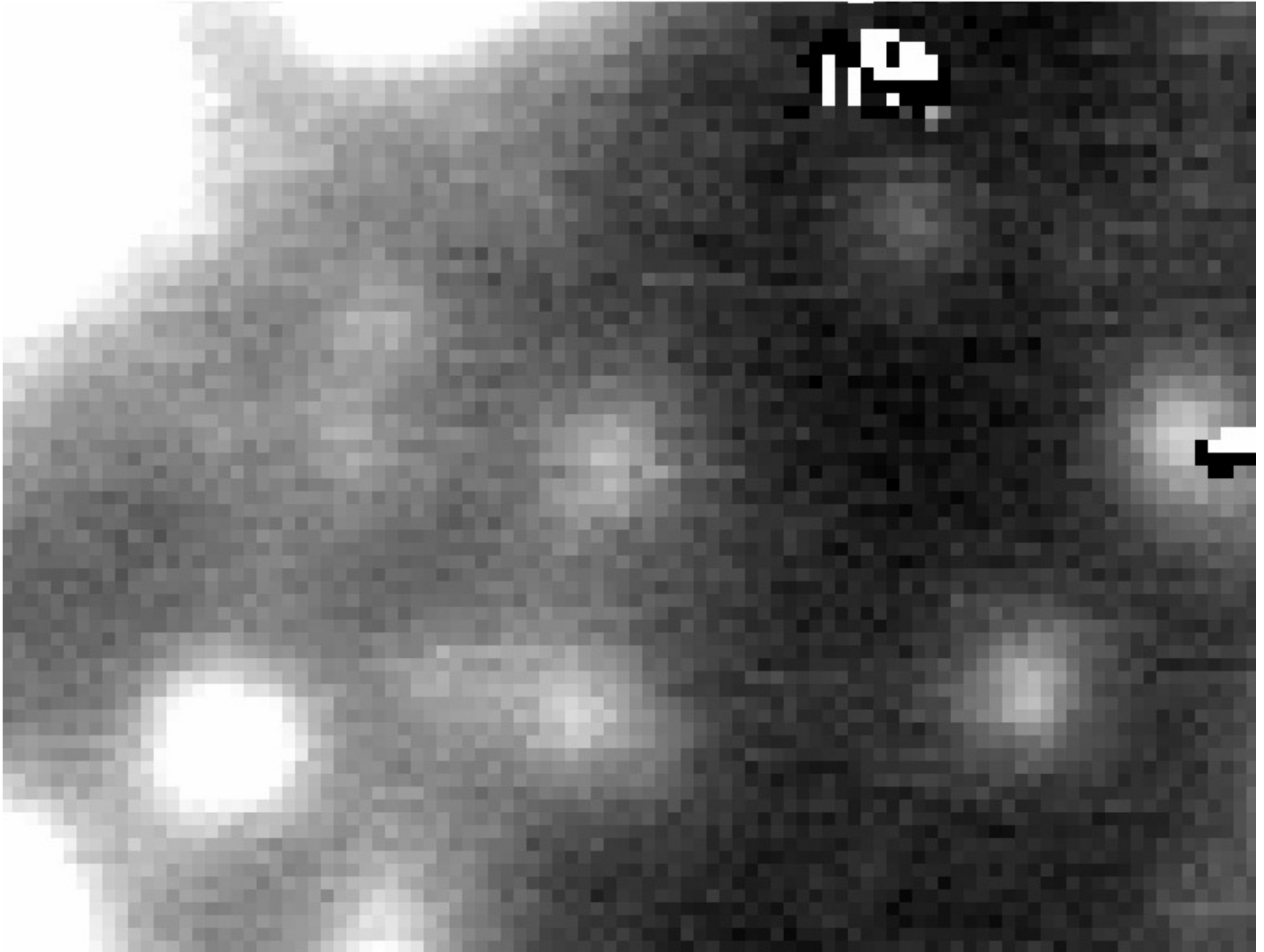
GR = General Relativity

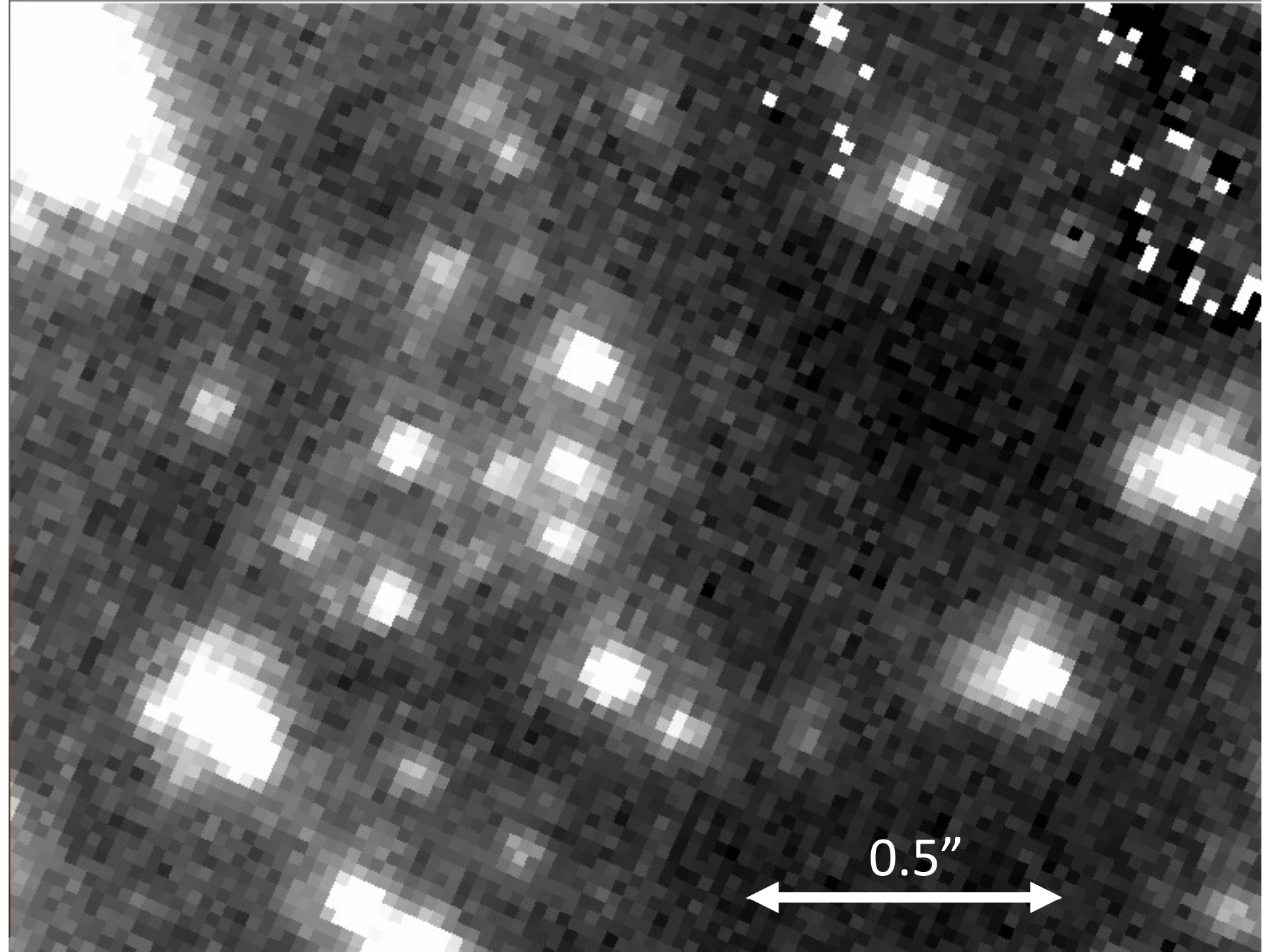
- Subaru GC research **beyond GR** is launched







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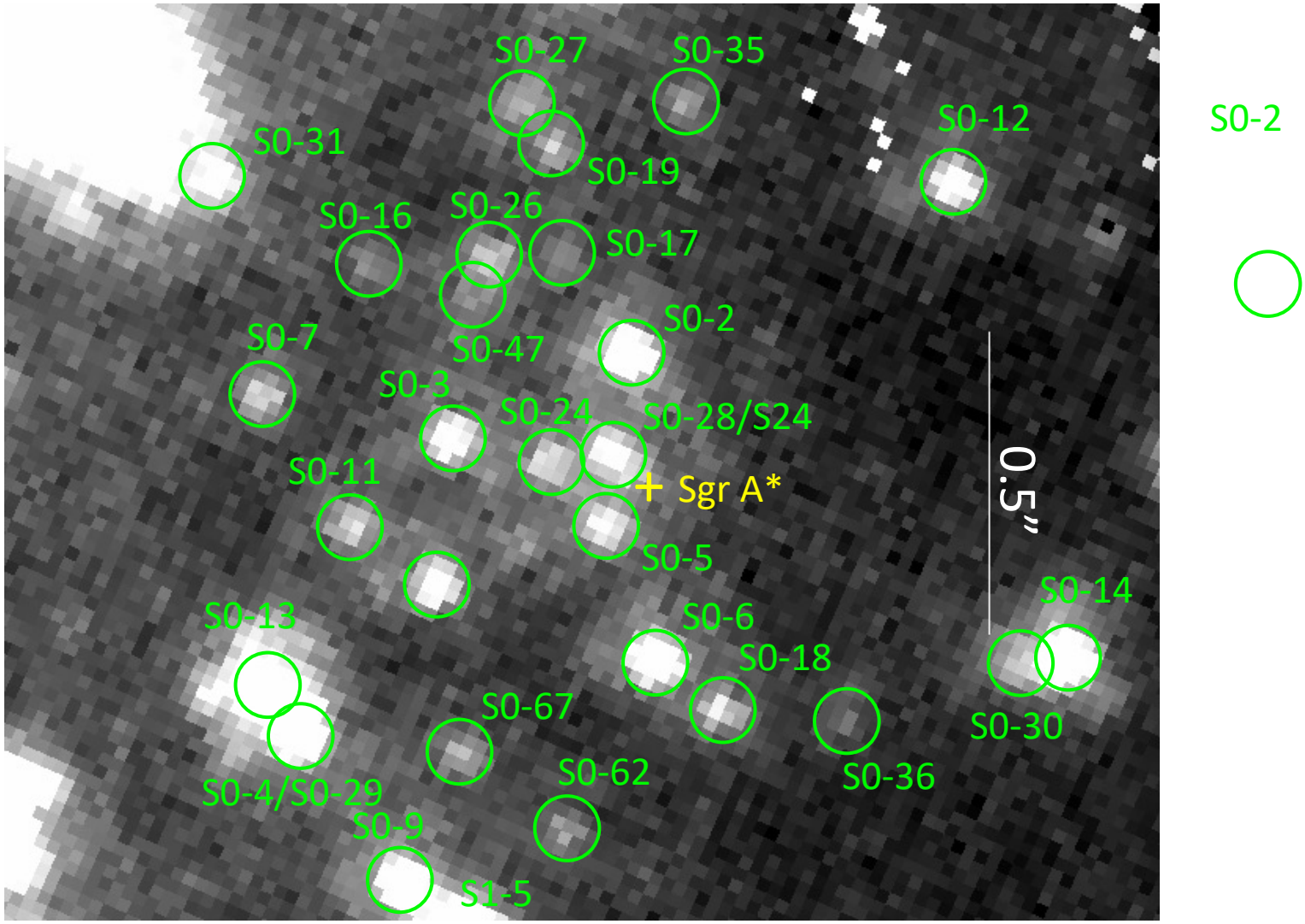
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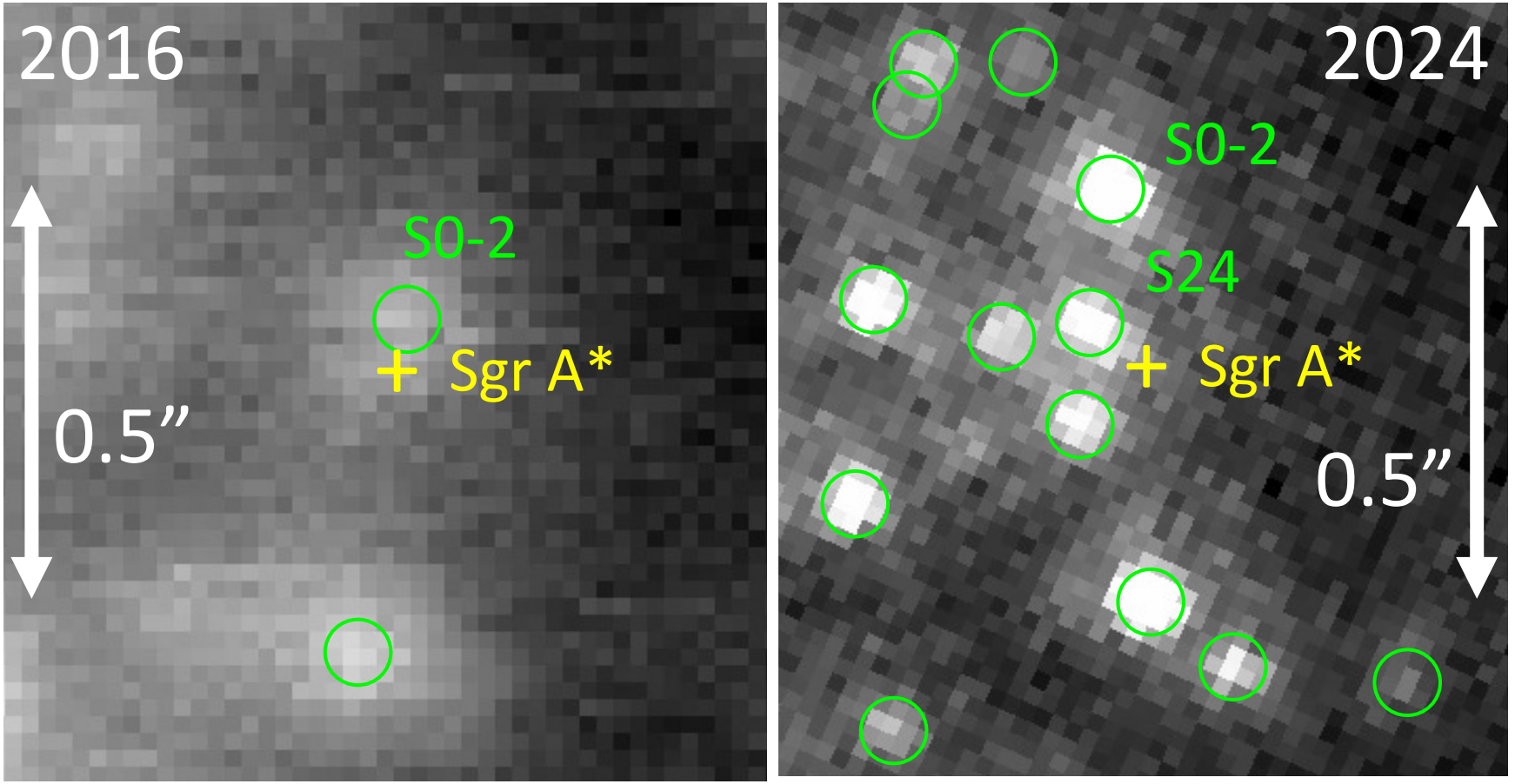
S0-2
S0-3
S0-24
S0-28/S24
S0-11
Sgr A*
S0-5
0.5"
S0-14
S0-13
S0-6
S0-18
S0-67
S0-30
S0-62
S0-36
S0-4/S0-29
S0-9
S1-5
S0-2


2016
S0-2
Sgr A*
0.5”
2024
S0-2
S24
Sgr A*
0.5”

- Note: Some "modified gravity theories" are alive !

  GR is the best candidate, but NOT completely confirmed.

- Fact: Rejection of Newton Gravity

  (GRAVITY collab. A&A 2018 ; Do et al. 2019 Science ;Saida et al. 2019 PASJ)

- Our aim: Which of modified gravity theories is the best-fitting with the data?

→ Method (explained from the next page)

Parametrized Post-Newtonian (PPN) formalism

Saida et al. 2024 Prog.Ther.Exp.Phys (this talk)

→ Is DM concentration at SMBH discovered ???

- Other aim: Origin of stars in GC

Nshiyama et al. 2024 Proc.Jpn.Acad. & 2025 Universe

2. Searching for Gravity Theory

2.1 Taylor expansion of grav. field

- Post-Newtonian (PN) parameter $(V_{\rm p} \simeq 0.03c)$

$$\varepsilon_{\rm p} \sim \frac{GM_{\rm BH}}{c^2 r_{\rm p}} \approx \left(\frac{V_{\rm p}}{c}\right)^2 \sim 10^{-3} \in \boxed{\begin{array}{l}\text{Un-explored region}\\ \text{in } (M_{\rm BH}\,,\,\varepsilon) \text{ space}\end{array}}$$

- PN expansion: Taylor expansion of Grav. Pot. of Kerr BH (BH in GR) $g_{\mu\nu}^{\rm Kerr}$ by ε
- Parametrized PN formalism : Introduce some artificial parameters in the PN expansion of $g_{\mu\nu}^{\rm Kerr}$

2.2 PPN expansion and parameters

- BH's metric tensor / stationary axialsymmetric

$$g_{tt} = -1 + 2\varepsilon(r) + \boldsymbol{A}\,\varepsilon(r)^2 + O(\varepsilon^3) \quad , \; m = GM_{\rm BH}/c^2$$

$$g_{tx} = +2q\frac{x}{r}\,\varepsilon(r)^2 + O(\varepsilon^3) \quad , \; q = \frac{cJ_{\rm BH}}{GM_{\rm BH}^2} \qquad \boxed{\varepsilon(r) = \frac{m}{r}}$$

$$g_{ty} = -2q\frac{y}{r}\,\varepsilon(r)^2 + O(\varepsilon^3)$$

$$g_{tz} = +2q\,\boldsymbol{C_z}\,\frac{z}{r}\,\varepsilon(r)^2 + O(\varepsilon^3)$$

$$g_{ij} = \delta_{ij} + 2\boldsymbol{B}\,\frac{x^i x^j}{r^2}\,\varepsilon(r) + O(\varepsilon^2)$$

The simplest PPN form with A, B, C_z for measuring deviation from GR.

- Kerr (BH in GR) case $\Rightarrow$ $\boxed{A = C_z = 0\,,\; B = 1}$

2.3 Science targets with Subaru/TMT

- One science target with Subaru

  A and B are the effects of Sgr A* mass $M_{\rm BH}$

$\rightarrow$ A, B are 1PN order, in Dec., R.A. and z

- One science target with TMT (Thirty-Meter-Telescope)

  C_z is the effects of Sgr A* spin $J_{\rm BH}$

$\rightarrow$ C_z is 1.5PN order, not detectable with Subaru.

- Currently we consider A and B without C_z.

3. Current Status

- Search the value of 19 + 2PPN parameters:
  - theoretical prediction of S0-2's motion in the PPN from of grav. pot.
  - fitting the theoretical prediction with Obs. Data of S0-2's motion
- 19 parameters, except for $\{A, B\}$ are $\cdots$

- 19 parameters, except for $\{A, B\}$

$M_{\rm BH}$: Mass of Sgr A* (massive BH)

$R_{\rm SgrA}$: Distance to Sgr A*

$\left(\vec{a}_{\rm BH}\right.$: BH spin (magnitude and direction) $\leq$ 1.5PN$\left.\right)$

$\vec{x}_{\rm apo}\,, \vec{v}_{\rm apo}$: S0-2's initial conditions (6 components)

$\vec{v}_{\rm E}$: Our velocity w.r.t. Sgr A*(3 comp.)

$(X, Y)_{\rm Keck}$: Astro. reference point for Keck

$(\dot{X}, \dot{Y})_{\rm Keck}$: Velocity of the ref. point for Keck

$(X, Y)_{\rm VLT}$: Astro. reference point for VLT

$(\dot{X}, \dot{Y})_{\rm VLT}$: Velocity of the ref. point for VLT

$\uparrow$

$(X, Y) =$ (R.A. , Dec.)

NOTE: { Our coordinate origin is at Sgr A*
Assume $\vec{v}_{\rm E}$ and $(\dot{X}, \dot{Y})$ are constant

- We have performed χ^2-fitting.

◇ GR $(A = 0,\ B = 1)$ case (Saida et al. PASJ 2019)

○ $\chi^2_{\mathrm{red(GR)}} = 1.3296$

○ Best-fitting values of some parameters:

$M_{\mathrm{BH}}\,[10^6 M_\odot]$	$R_{\mathrm{Sgr\,A}}$ [kpc]	$T_{\text{S0-2}}$ [yr]	$v_{\mathrm{E}//}$ [km/s]
4.001 ± 0.010	7.987 ± 0.010	16.0486 ± 0.0004	-11.0 ± 1.1

$\rightarrow \dfrac{M_{\mathrm{star}}}{M_{\mathrm{BH}}} \sim 10^{-6} \ll \varepsilon_{\mathrm{p}}$ (M_{star} : mass of other S-star)

Gravity of other stars is negligible in our data.

$\rightarrow$ 2-body-systam of S0-2 & Sgr A* is good.

- ◇ PPN case : $\chi^2_{\rm red(PPN)} = 1.3016 \; < \chi^2_{\rm red(GR)}$
  - ○ Best-fitting values of some parameters:

$M_{\rm BH}\,[10^6 M_\odot]$	:	3.996 ± 0.005
$R_{\rm SgrA}\,[\rm kpc]$	:	7.988 ± 0.004
$T_{\rm S0\text{-}2}\,[\rm yr]$	:	16.0606 ± 0.0003
$v_{\rm E//}\,[\rm km/s]$	:	-9.6 ± 0.5
$A\,[\rm no.dim.]$	:	22.7 ± 1.3
$B\,[\rm no.dim.]$	:	-6.9 ± 0.9

  - ○ Inconsistent with Schwarzschild (BH in GR),
    $A \neq 0\,,\; B \neq 1$ within several σ

◇ $\Delta\chi^2_{\rm red} = 1.33(\mathrm{GR}) - 1.30(\mathrm{PPN}) = 0.03$,

$\rightarrow$ $\Delta\chi_{\rm red} < 1$, χ^2-statistically insignificant

$\rightarrow$ We are writing the numerical code for Hierarchical Bayesian Analysis.

◇ If our PPN result is true, two possibilities arise:

◦ If GR is correct, then the modelling of 2-body-system in "vacuum BH spacetime" is not good.

$\rightarrow$ **Dark Matter concentration at SMBH !?**

◦ If the vac. BH environment is the good model, then a "modified gravity theory" is favored.

4. Summary

our paper

- Using the data of S-stars, Gravity/BH is tested.
- BH's mass effects are in our detection capability.
- 2-body-system is the good model.
- PPN χ^2-fitting is inconsistent with Schwarzschild (BH in GR) case:

  $A = 22.7 \pm 1.3$, $B = -6.9 \pm 0.9$ $\rightarrow$ **DM at Sgr A* ?**
- Hierarchical Bayesian code is under construction.

  with T.Takeuchi , S.Uchida , S.Matsui (Nagoya Univ. Ω-Lab.)