P36 Shapelet representation in phase space

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What is the *Wigner Function Shapelets (WFS)*?

⚠️ Not Wavefront Sensing (WFS) in my poster

WFS is a mathematical representation of the information that a galaxy image possesses, which we will propose (this poster and S.Arai in prep.). Different from the conventional representation, i.e.. the polar shapelets (R. Massey and A. Refregier 2003) with which a galaxy image is decomposed into the infinite series of Laguerre-Gaussian modes labeled by the SU(2) angular and spin modes (j,m) at the two-dimentional configuration space, WFS represents the image in the phase space, where the information of a galaxy image is represented by the Wigner function:

$$W(\boldsymbol{x},\boldsymbol{p})[\psi] = (2\pi\bar{\lambda})^{-2}\int d^2\xi\,\psi(\boldsymbol{x}+\boldsymbol{\xi}/2)\psi^*(\boldsymbol{x}-\boldsymbol{\xi}/2)\exp(-i\boldsymbol{p}\cdot\boldsymbol{\xi}/\bar{\lambda})$$

We aim to treat the galaxy image as a quantised state at a certain resolution λ, representing morphological/symmetric nature of galaxy images. Remarkably, this approach enables us to achieve

- ✅ image propagation is consistently described by Wigner transport, whose limit λ → 0 exhibits a radiative transfer.
- ✅ morphology/symmetry separation with a group-theoretic description is well defined.
- ✅ systematics and noises is characterised by channel interactions in quantum information theory

Formalism for WFS

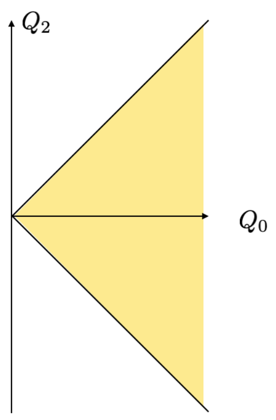
WFS is defined as the cross-Wigner function of the two-dimensional Laguerre-Gaussian (LG) mode function,

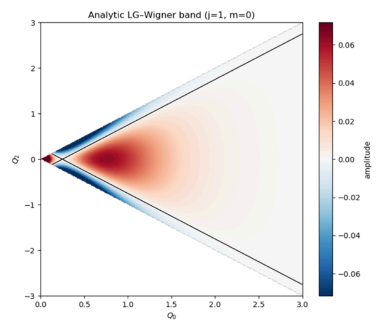
$$W(\boldsymbol{x},\boldsymbol{p})[\psi_a,\psi_b] = (2\pi\bar{\lambda})^{-2}\int d^2\xi\,\psi_a(\boldsymbol{x}+\boldsymbol{\xi}/2)\psi_b^*(\boldsymbol{x}-\boldsymbol{\xi}/2)\exp(-i\boldsymbol{p}\cdot\boldsymbol{\xi}/\bar{\lambda}) \qquad \psi_a \to \Psi^{\rm LG}_{j,m}$$

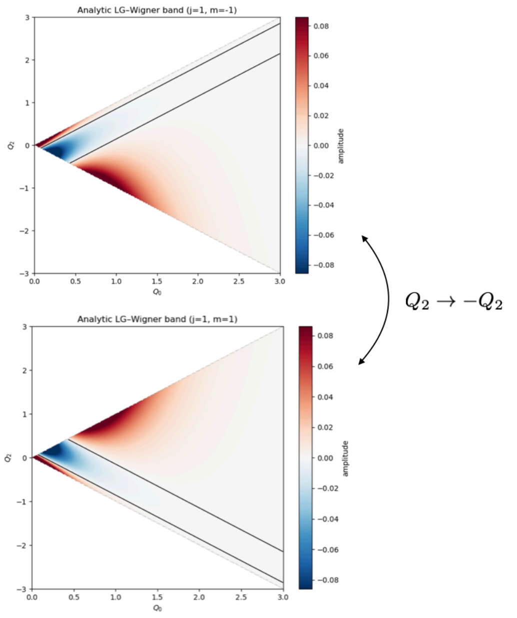
The cross-Wigner function of the LG modes has an analytic expression, given an appropriate choice of canonical variables. As an example, we show the diagonal part of the Wigner function represented by the two U(1)-invariant variables Q_0 and Q_2. The right-most figure shows an example of WFS modes that is computed from a mock image of a spiral galaxy.

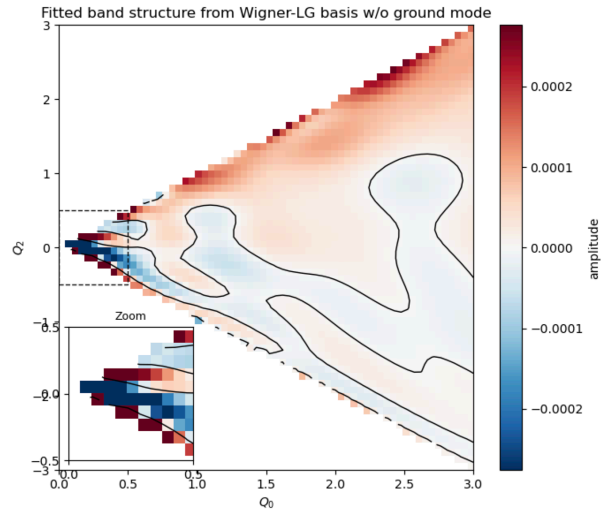
$$W = \sum_{j=0,\frac{1}{2},1,\dots}\sum_{m=-j}^{j} \mathcal{W}_{jm} W^{\rm LG}_{jm} + \text{cross Wigner}$$

$$W^{\rm LG}_{jm}(Q_0,Q_2) \propto (-1)^{2j}e^{-4Q_0}L_{j+m}(4(Q_0+Q_2))L_{j-m}(4(Q_0-Q_2))$$









Applications : WFS ID of systematics (SA in progress)

We employ WFS to extract various information of galaxy images, including cosmological effects (gravitational lensing, parity-violating signature) and observational systematics (point spread functions, AO residuals). Potential usage of WFS is to disentangle patterns of signals from systematics. In this poster for SUM 2026, we will show some preliminary results in which the WFS performs classification of AO residuals. So far, the performance looks good.