Calculation of the effective exposure time in PFS observation

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1 Introduction

This document summarizes the formulation of the effective exposure time (hereafter EET) in the PFS observation. The EET corresponds to how the signal-to-noise ratio (SNR) is degraded (or improved) by the change of the actual observing conditions compared to the nominal ones, which are assumed by users when they submit their proposal, and is used to calculate the completion rate of each target. Under some assumptions and simplification, the EET is defined regardless of the object brightness, therefore can be calculated by using only the change of the observing conditions.

2 The signal-to-noise ratio and exposure time

The PFS spectral SNR per pixel can be written as follows:

$$\frac{S}{N} = \frac{N_* \times \xi \times \eta_{atm} \times \eta_{inst} \times t}{\sqrt{(N_* \times \xi \times \eta_{atm} \times \eta_{inst} + N_{sky} \times \eta_{inst} + N_{dk}) \times t + N_{rd}^2)}}, \tag{1}$$

where N_* is the count rate of objects (stars, galaxies, whatever), N_{sky} is the sky background count rate within the fiber aperture, N_{dk} is the dark current, and N_{rd} is the readout noise. ξ is the fraction of light covered by the fiber aperture, η_{atm} and η_{inst} are the atmospheric and the instrument throughput, respectively, and t is the exposure time. Note that the atmospheric transmission and the fiber aperture effect are applied to only object flux (is this correct?).

Now we think about the case where dark and readout noise can be ignored, and we get

$$\frac{S}{N} = \frac{N_* \times \xi \times \eta_{atm}}{\sqrt{N_* \times \xi \times \eta_{atm} + N_{sky}}} \times \sqrt{\eta_{inst}} \times \sqrt{t}. \tag{2}$$

Then, the required exposure time to achieve a given SNR is calculated as follows:

$$t = (\frac{S}{N})^2 \times \frac{N_* \times \xi \times \eta_{atm} + N_{sky}}{N_*^2 \times \xi^2 \times \eta_{atm}^2} \times \frac{1}{\eta_{inst}}. \tag{3}$$

In the object-limited case ($N_* \gg N_{sky}$), we get

$$t = (\frac{S}{N})^2 \times \frac{1}{N_*} \times \frac{1}{\xi} \times \frac{1}{\eta_{atm}} \times \frac{1}{\eta_{inst}}. \tag{4}$$

In the background-limited case ($N_{sky} \gg N_*$), we get

$$t = (\frac{S}{N})^2 \times \frac{N_{sky}}{N_*^2} \times \frac{1}{\xi^2} \times \frac{1}{\eta_{atm}^2} \times \frac{1}{\eta_{inst}}. \tag{5}$$

Here we assume that $\xi \times \eta_{atm}$ is not significantly lower than 1.

3 Definition of the EET

Since the processing time of DRP is not negligible, which takes several minutes (TBC), it is difficult to evaluate the EET using the flux standard stars in real time. Here we think

Assuming that only seeing, transparency, and sky background level possibly change during the observation, the variable parameters are N_{sky}, η_{atm}, ξ in the above equations, where ξ decreases by increasing seeing size. Then, the nominal exposure time (=900 sec. in the usual case of PFS queue operation) is

$$t_{nom} = (\frac{S}{N})^2_{nom} \times \frac{N_* \times \xi_{nom} \times \eta_{atm,nom} + N_{sky,nom}}{N_*^2 \times \xi_{nom}^2 \times \eta_{atm,nom}^2} \times \frac{1}{\eta_{inst,nom}}. \tag{6}$$

In the same exposure time, observing condition changes and the resulting SNR also changes as

$$(\frac{S}{N})^2_{obs} = \frac{N_*^2 \times \xi_{obs}^2 \times \eta_{atm,obs}^2}{N_* \times \xi_{obs} \times \eta_{atm,obs} + N_{sky,obs}} \times \eta_{inst,obs} \times t_{nom}. \tag{7}$$

The effective exposure is defined as the exposure time to achieve the observed SNR with nominal condition:

$$t_{eff} = (\frac{S}{N})^2_{obs} \times \frac{N_* \times \xi_{nom} \times \eta_{atm,nom} + N_{sky,nom}}{N_*^2 \times \xi_{nom}^2 \times \eta_{atm,nom}^2} \times \frac{1}{\eta_{inst,nom}}. \tag{8}$$

By substituting Eq. 7,

$$t_{eff} = \frac{N_* \times \xi_{nom} \times \eta_{atm,nom} + N_{sky,nom}}{N_* \times \xi_{obs} \times \eta_{atm,obs} + N_{sky,obs}} \times (\frac{\eta_{atm,obs}}{\eta_{atm,nom}})^2 \times (\frac{\xi_{obs}}{\xi_{nom}})^2 \times \frac{\eta_{inst,obs}}{\eta_{inst,nom}} \times t_{nom}. \quad (9)$$

Here we assume that the instrument throughput does not significantly change during the observation run ($\eta_{inst,obs}/\eta_{inst,nom} = 1$). In the object-limited case ($N_* \gg N_{sky}$), we get

$$t_{eff} = \frac{\xi_{obs}}{\xi_{nom}} \times \frac{\eta_{atm,obs}}{\eta_{atm,nom}} \times t_{nom}. \quad (10)$$

In the background-limited case ($N_{sky} \gg N_*$), we get

$$t_{eff} = \frac{N_{sky,nom}}{N_{sky,obs}} \times (\frac{\xi_{obs}}{\xi_{nom}})^2 \times (\frac{\eta_{atm,obs}}{\eta_{atm,nom}})^2 \times t_{nom}, \quad (11)$$

or

$$t_{eff} = (\frac{\sigma_{sky,nom}}{\sigma_{sky,obs}})^2 \times (\frac{\xi_{obs}}{\xi_{nom}})^2 \times (\frac{\eta_{atm,obs}}{\eta_{atm,nom}})^2 \times t_{nom}. \quad (12)$$

Fig.1 and Fig.2 show the change of EET by changing seeing FWHM, transparency, and background level for bright and faint targets, respectively.

4 Calculating the effective exopsure time

4.1 Calculate throughput information from only AG information

In theory, if the instrument throughput is reasonably stable, the total throughput is predictable by using only AG information. ξ and can be calculated by using the seeing size and transparency measured on the AG images. This will be done in future when we get sufficient data to define an empirical relation between AG information and the EET. For now, we show the evaluation of EET based on the reduced data for `SKY` and `FLUXSTD` fibers.

4.2 Calculate throughput information from `FLUXSTD` spectra

In the above formalization, the variable throughput information (η_{atm}, ξ) basically can be calculated by using only information of AG cameras. Perhaps, the observed spectra of flux standard stars (`FLUXSTDs`) provide us more direct measurement of the total throughput information. Here let's think about evaluating the throughput term from the observed spectra of `FLUXSTDs`, which are taken at the same time as the science objects in the same field of view. In this case, the combination of the variable throughput factors (η_{atm}, ξ,

η_{inst}) is simply described by a single parameter $\eta_{tot} = \xi \times \eta_{atm} \times \eta_{inst}$. The EET is simply described by the following equation:

$$t_{eff} = (\frac{\sigma_{sky,nom}}{\sigma_{sky,obs}})^2 \times (\frac{\eta_{tot,obs}}{\eta_{tot,nom}})^2 \times t_{nom}. \tag{13}$$

The total throughput can be calculated by the ratio of the observed spectrum and the intrinsic spectrum of the FLUXSTD,

$$\eta_{tot}(\lambda) = \frac{f(\lambda)_{\text{pfsMerged}}\ [\text{electron/nm}]}{f(\lambda)_{\text{pfsFluxReference}}\ [\text{nJy}]}. \tag{14}$$

Here, we use pfsFluxReference which is an intermediate product generated after spectral typing process of the flux calibration is performed, as the intrinsic spectra of the FLUXSTDs. For each visit, the median value in the reference wavelength rage for each FLUXSTD is then calculated, and the median of them is taken as the observed throughput value, which is used to calculate the EET. The nominal throughput value is calculated in the same way but using ETC with parameters for a nominal condition, where a spectraum of 18 ABmag source flat in f_ν is assumed as the reference source. The wavelength range in the calculation is the same as the one for the observed value.

4.3 Calculate noise level from observed SKY spectra

Assuming that we mostly target faint sources, the noise level of obtained spectra is evaluated by measuring the scatter of sky-subtracted spectra of SKY fibers. In normal operation, a certain fraction of entire fibers is assigned to regions on the sky where no object is nearby (SKY targets). In the commissioning, we usually allocated 400 fibers for SKY targets, but the optimal number of fibers that required to be allocated to SKY targets for the sufficient quality of sky subtraction is still under tested.

The scatter of the SKY spectra is measured as the median of the IQR sigma of fluxes of pfsMerged spectrum of each fiber (in electron/nm) within the reference wavelength range, and then the median is taken as the typical sky noise level for that visit. The nominal value is calculated using a spectrum for a blank target generated in ETC with parameters for a nominal condition in the nominal condition, where the median noise is calculated in the same reference wavelength range.

4.4 The wavelength range to calculate the EET

The wavelength range in which the nominal and actually observed values are measured should be sufficiently wide but clean enough so that the contamination of sky emission lines and atmospheric absorption is low. We carefully choose the range where there is no strong sky line emission and no strong atmospheric feature. The wavelength range of

525 – 555 nm for b-arm, 835 – 860 nm for r-arm/m-arm, and 1040 – 1070 nm for n-arm, which are shown in Fig.3.

5 Verification in the commissioning runs

The validation of tracking the observation progress based on the EET is still on-going in the commissioning. The calculated EET for an example set of visits is shown in Fig.4, where visits in obviously bad condition are excluded. The EET largely depends on the deviation from the nominal values of both noise level and transparency, because it changes with the square of noise and throughput deviation. The peculiar visits showing very short/long EET, where the observing condition is not obviously bad, are still under investigation. Fig. 7 shows an example of the signal-to-noise ratio per pix of observed galaxies in i=21.0 ABmag in different visits (with different condition, therefore the EETs are different), which increases with the increasing EET, but note that the S/N doesn't necessarily increase in proportional to the square root of time perfectly.

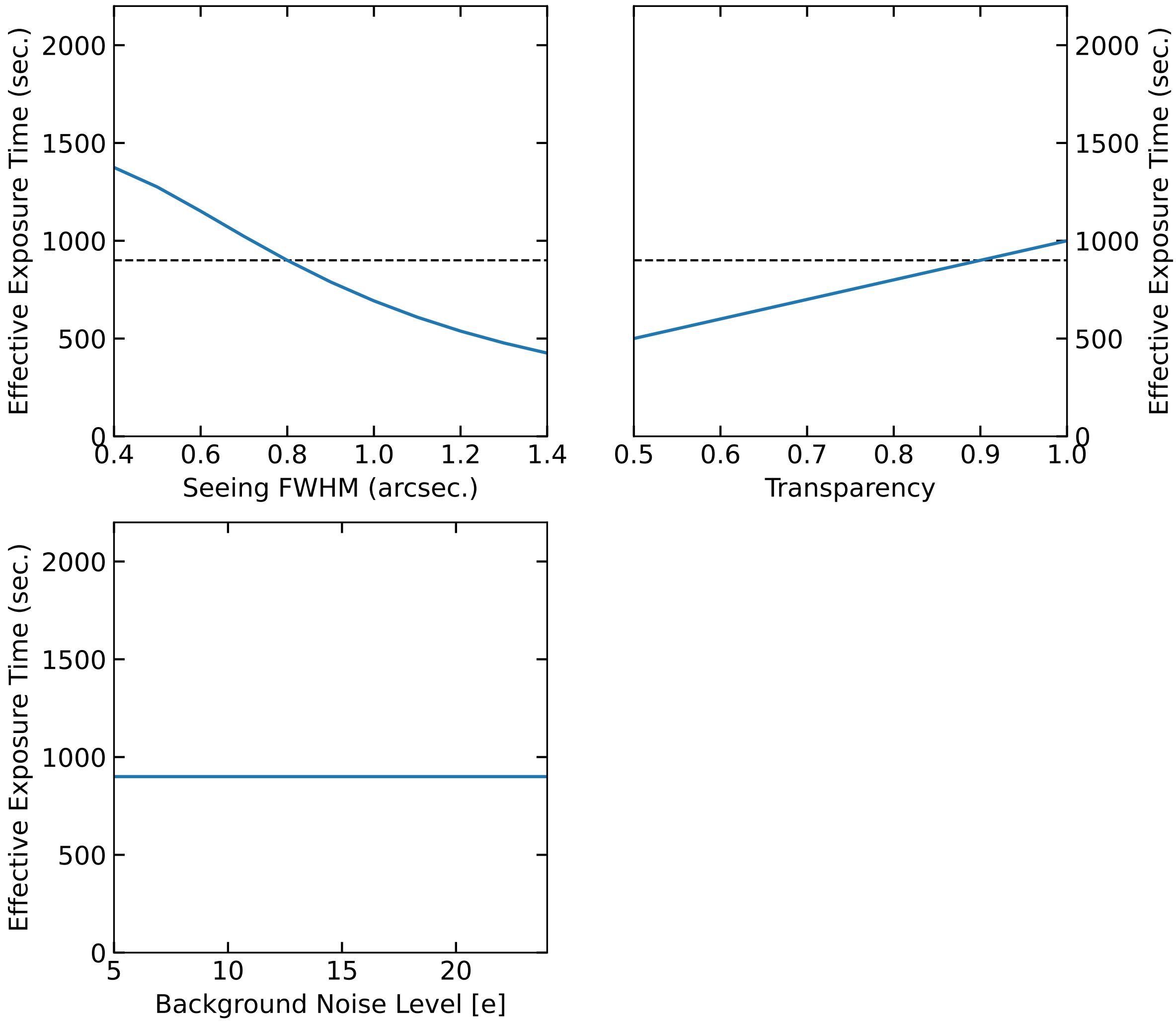

Figure 1: EET of bright targets as a function of seeing FWHM, transparency, and noise background level. The horizontal dashed line indicates the nominal exposure time of 900 sec.

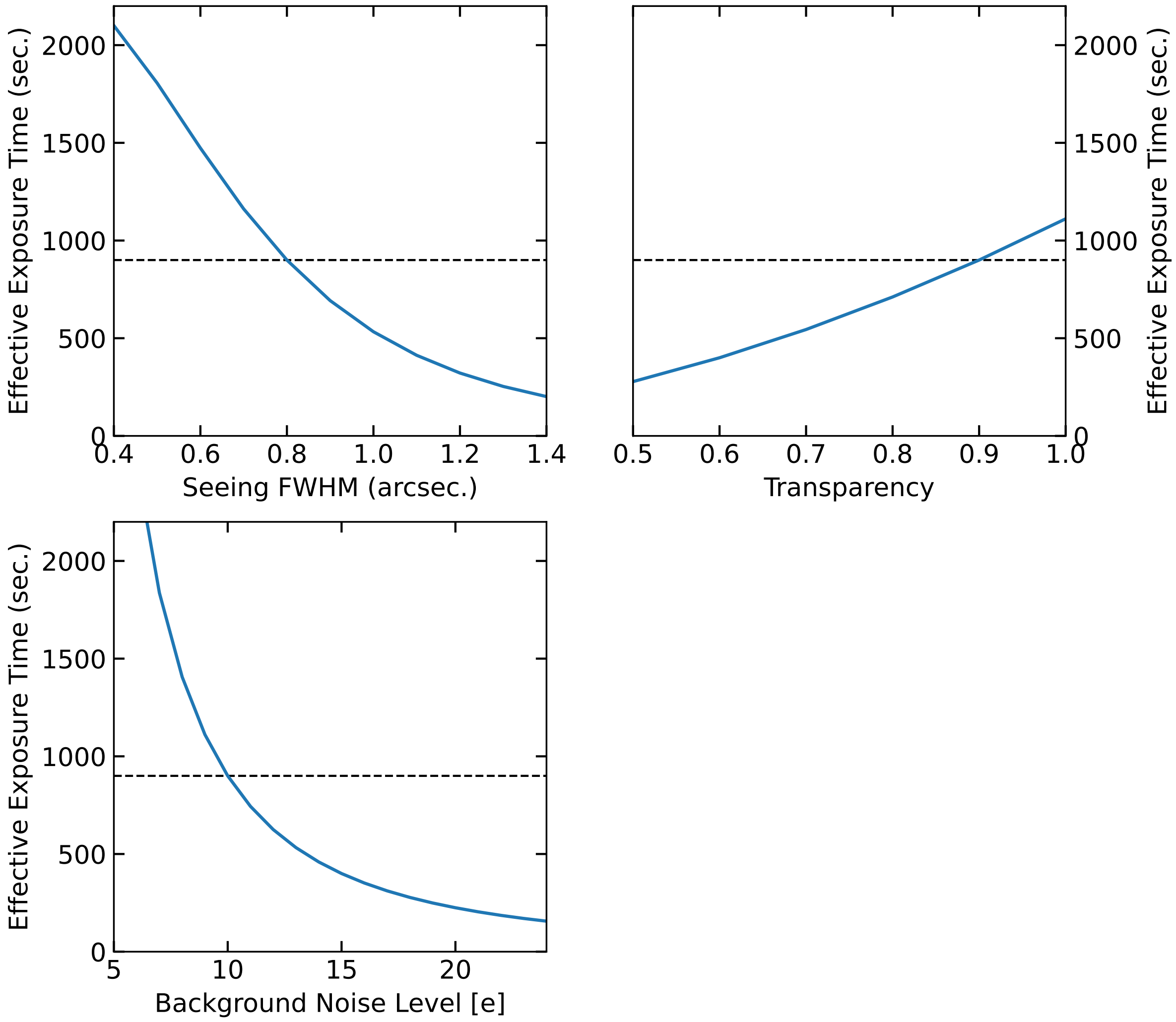

Figure 2: EET of faint targets as a function of seeing FWHM, transparency, and noise background level. The horizontal dashed line indicates the nominal exposure time of 900 sec.

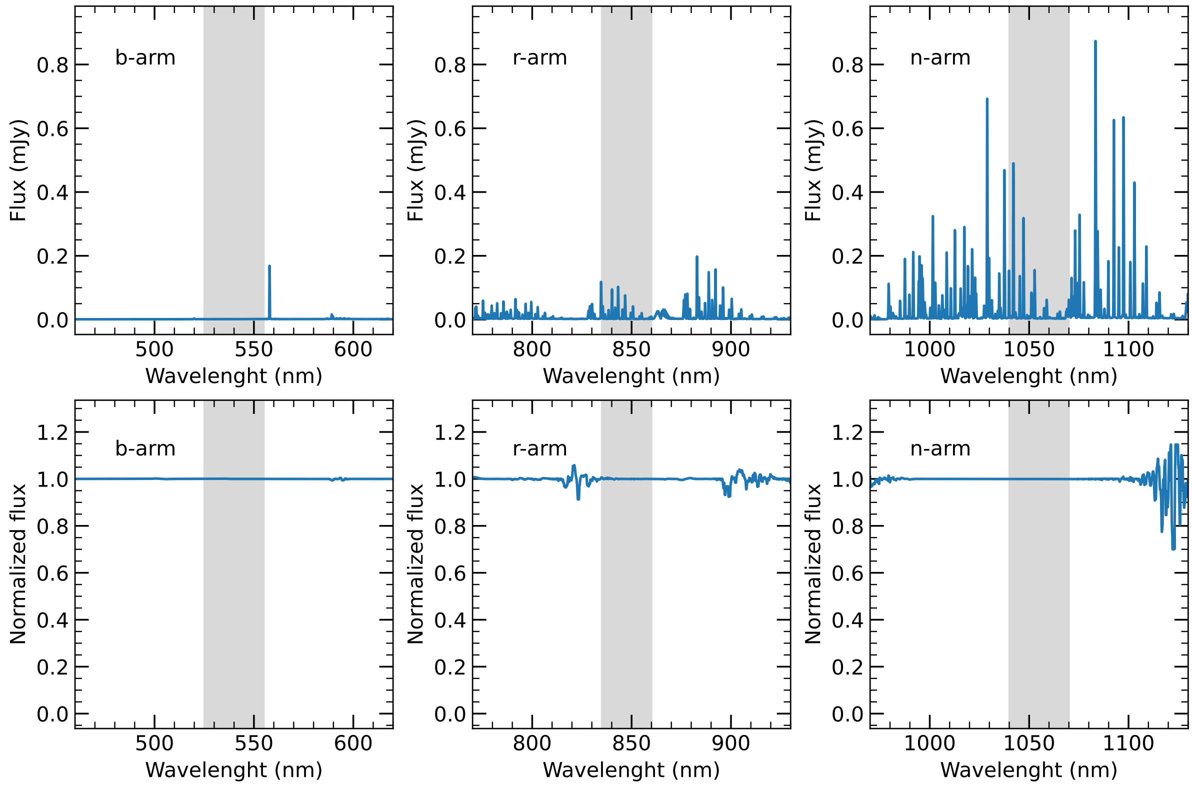

Figure 3: Sky spectrum (top) and the normalized flux standard spectrum (bottom) in each arm. The wavelength range used in each arm is indicated with the shaded region.

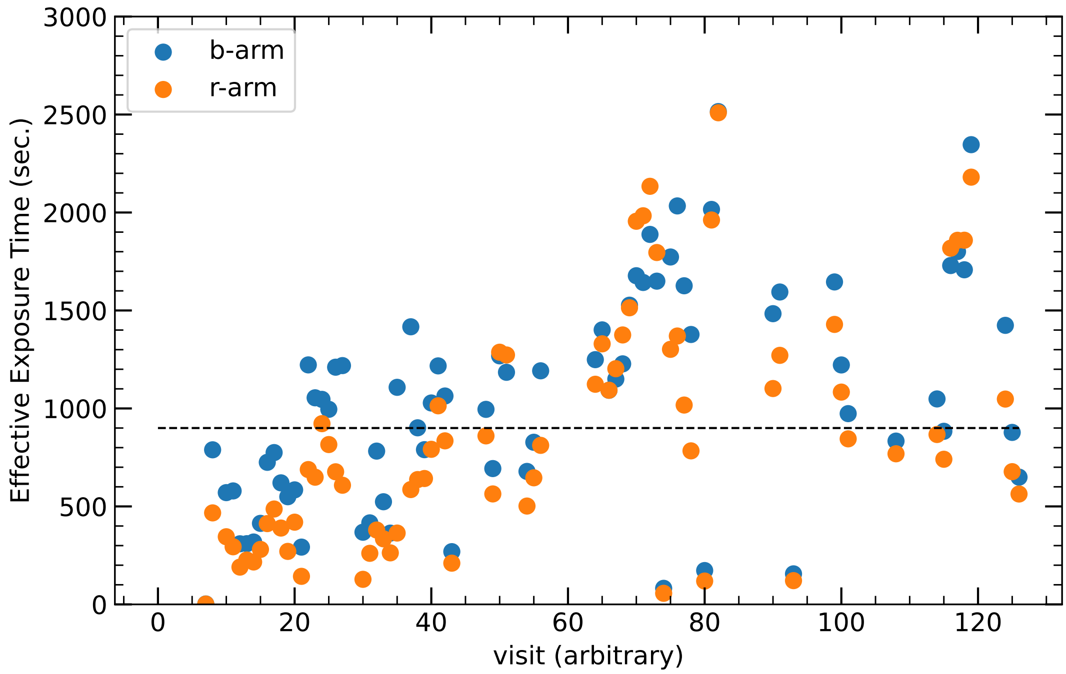

Figure 4: EET as a function of visit during the on-sky commissioning run from 2024 Mar, where visits in obviously bad condition are excluded. Blue and Red are results in b-arm and r-arm, respectively.

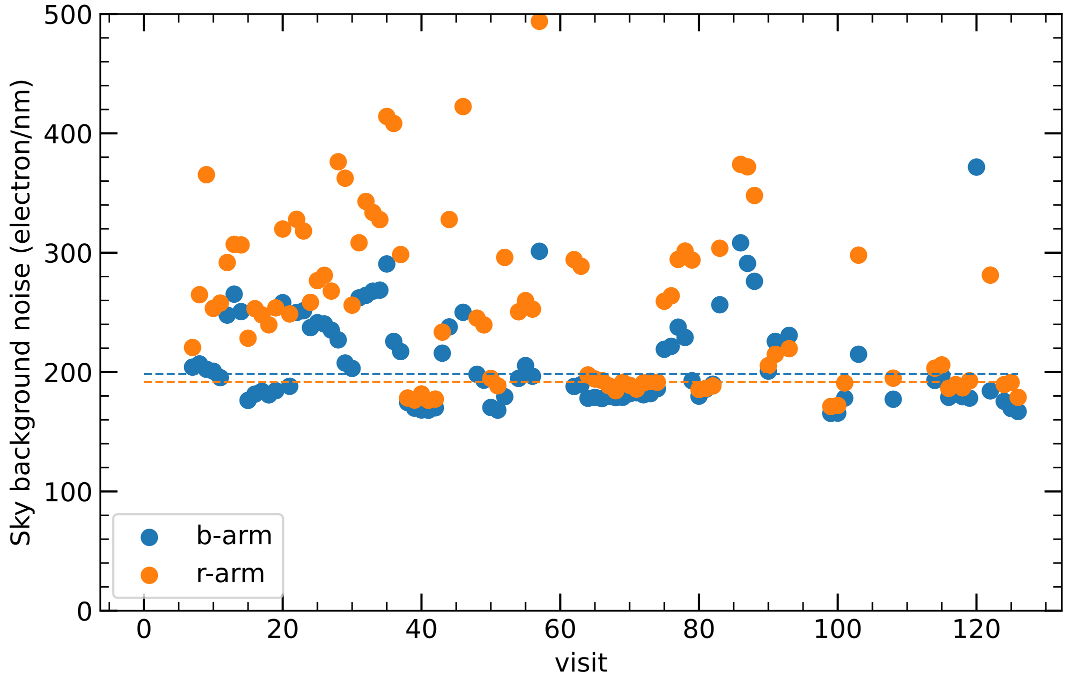

Figure 5: The sky background noise level as a function of visit during the on-sky commissioning run from 2024 Mar, where visits in obviously bad condition are excluded. Blue and Red are results in b-arm and r-arm, respectively. Horizontal dashed lines are the nominal values from ETC calculation.

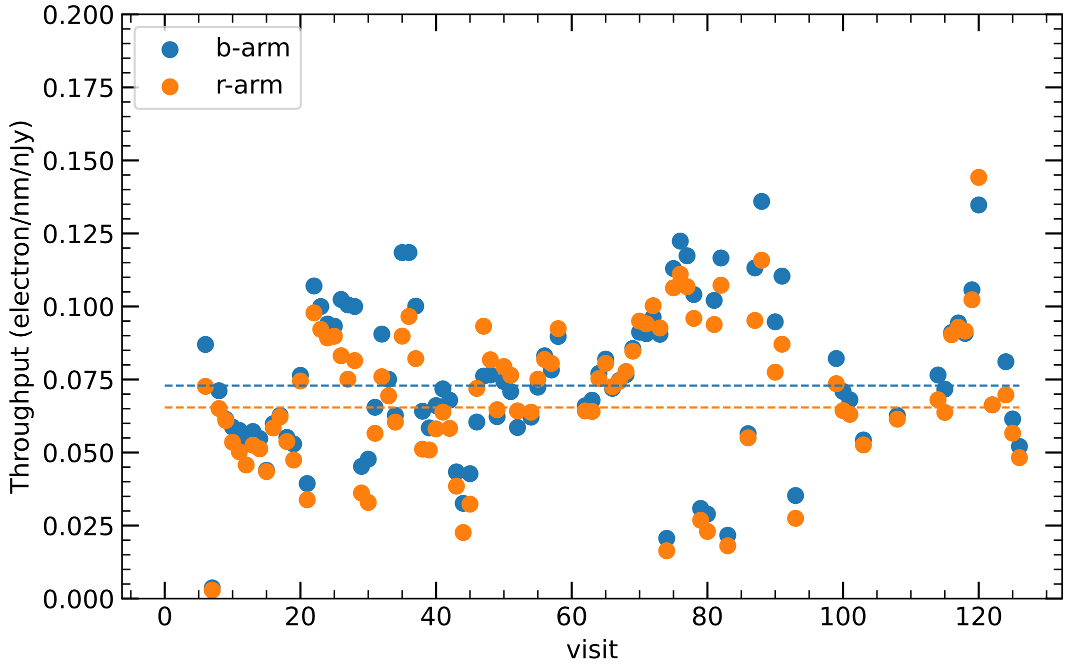

Figure 6: Throughput as a function of visit during the on-sky commissioning run from 2024 Mar, where visits in obviously bad condition are excluded. Blue and Red are results in b-arm and r-arm, respectively. Horizontal dashed lines are the nominal values from ETC calculation.

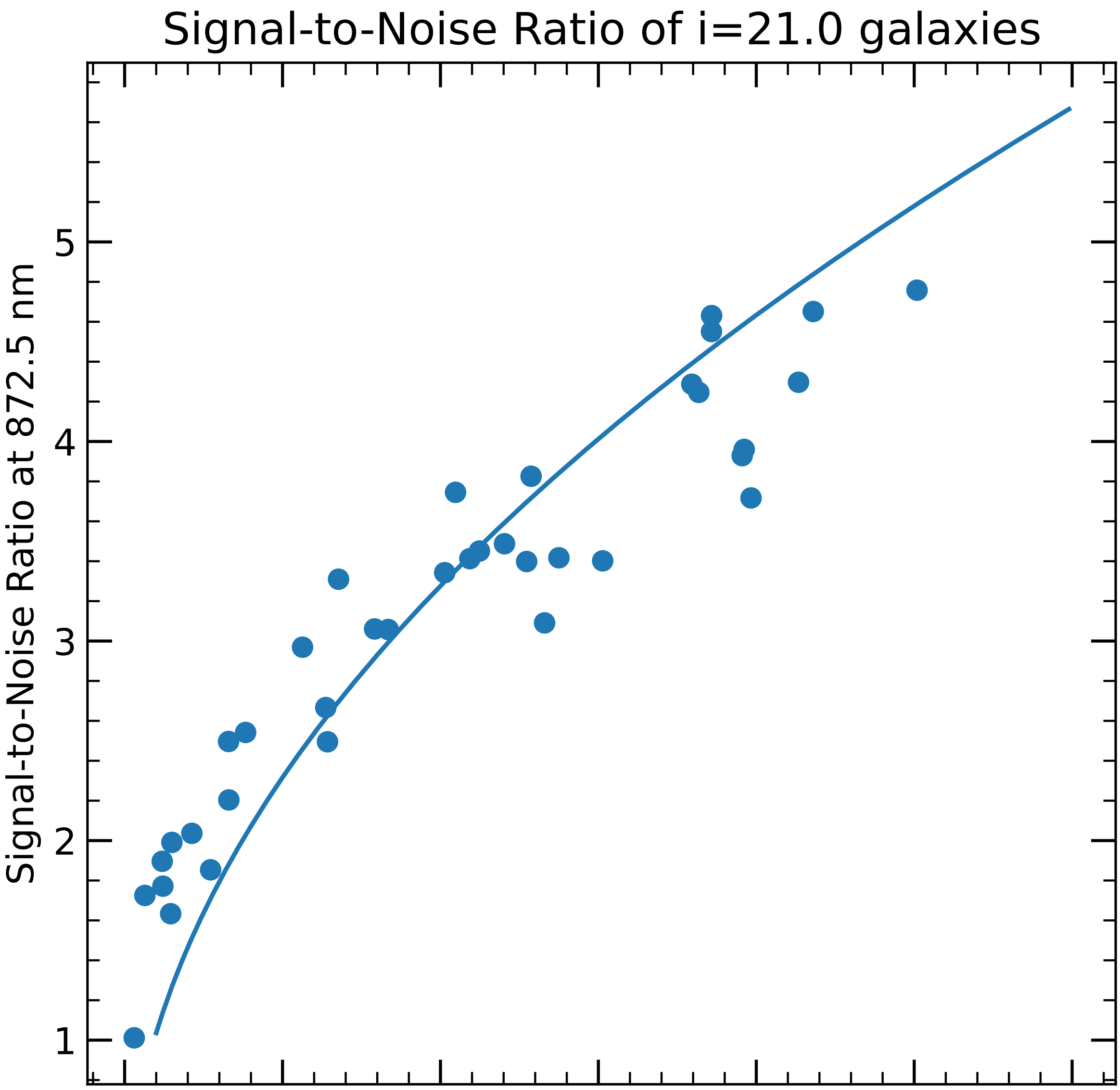

Figure 7: Measured signal-to-noise ratio per pixel of galaxies with 21.0 ABmag in HSC-i2 band and the EET. The solid line indicates the curve of S/N $\propto \sqrt{\mathrm{EET}}$ normalized at EET=900 sec.