Simple χ^2 analysis of orbiting stars around Sgr A* to constrain the dark mass distribution

Yohsuke Takamori (NIT, Wakayama College)

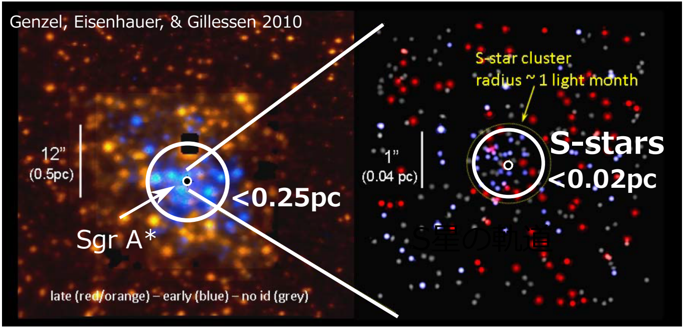
YT, S. Nhisyama, T. Ohgami, H. Saida, R. Saitou, & M. Takahashi arXiv:2006.06219

Subaru Users Meeting FY2020

Introduction

- Sgr A* is the central compact radio source in the Galaxy. We have found orbiting stars around Sgr A* which are called S-stars. Their dynamical system is well explained by considering the two-body problem. The mass of Sgr A* is estimated as $M_{\mathrm{SgrA*}} \sim 4 \times 10^6 M_\odot$ (supermassive black hole!).
- S-stars are useful probes to investigate the environment around Sgr A*.

Central region of the Galaxy

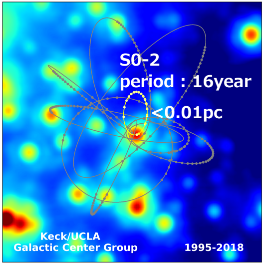


Our project with Subaru/IRCS

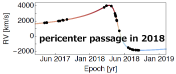
Subaru proposal: S14A, S15B, S16A, S17AB, S18A(Intensive), S18B, S19A

published papers: Nishiyama+18, Do+19, Saida+19, Hess+20

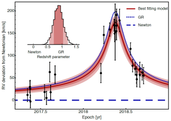
- **General relativistic signal** (Do+19, Saida+19)
- **Test of physical constant** (Hees+20)



Time variation of the redshift of photons from S0-2

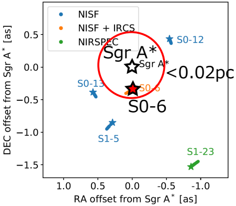


Relativistic signal

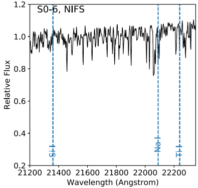


Do+19

S0-6's motion



S0-6's spectrum



Variation of fine structure constant $\frac{\Delta\alpha}{\alpha} = (1.0 \pm 1.2) \times 10^{-4}$

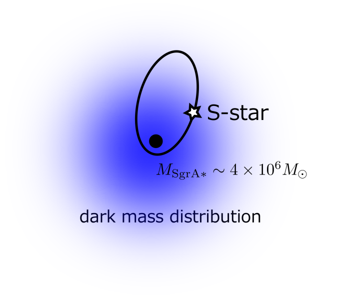
Hees+20

- **Dark mass distribution around Sgr A* (this poster)**
- **Test of theory of gravity**
- **Quasi-black hole**

Constraint on a dark mass distribution around Sgr A*

Constraint on the total amount of a dark mass distribution $M_{\rm ext}$ within S0-2's orbit $(r <\sim 0.01\,{\rm pc})$ (1σ)

- **Gillessen+09 (VLT)**
  $M_{\rm ext} < 0.04 M_{\rm SgrA*}$ (less than 4 %)
- **Gillessen+17 (VLT)**
  $M_{\rm ext} < 0.01 M_{\rm SgrA*}$ (less than 1 %)
- **Do+19 (Keck+Subaru)**
  $M_{\rm ext} < 5.5 \times 10^3 M_\odot$ (less than 0.14 %)
- **Gravity collab. 20 (VLT)**
  $M_{\rm ext} < 0.001 M_{\rm SgrA*}$ (less than 0.1 %)



The total mass of the dark extended mass within $0.01\,{\rm pc}$ is less than $1\,\%\,(10^4 M_\odot)$ of the central surpermassive black hole.

Our view points

- **The observational data of S0-2 during the pericenter passage would play an important role to give the strong constraint.**
  $4\,\%(2009) \to 1\,\%(2017) \to 0.1\,\%(2020)$
- **So far, the constraint is obtained by an orbital fitting method. One should search best-fit parameters more than 14. It is computationally demanding.**
  - orbital parameters for a S-star (6 parameters)
  - Sgr A*'s parameters (7 parameters) (position (3), motion (3), and mass (1))
  - parameters of a dark mass model (more than 1)
    (e.g., the total mass of the dark mass)

Aims of our work

- **Clarify the role of the spectroscopic data of S0-2 during the pericenter passage in 2018 obtained by Subaru/IRCS.**
- **Suggest a computationally less demanding method to give a constraint for the dark mass distribution.**

Simple χ^2 analysis

From the previous studies, it is clear that $M_{\rm ext}(r < 0.01\,{\rm pc})/M_{\rm SgrA*} = \eta \ll 1$.

- **Assumption**
  The parameters of S0-2 and Sgr A* is determined by the two-body problem.
- **Method**
  Calculate χ^2 with respect to the observational data of S0-2:

$$\chi^2(\eta) = \sum_{i=1}^{N} \frac{(f_i - f_{\rm model}(t_i))^2}{\sigma_i^2}$$

f_i : observational data of S0-2 (astrometry and spectroscopy)

σ_i : uncertainty of the observation

$f_{\rm model}(t_i)$: theoretical value at the observational time t_i

Because $\eta \ll 1$, it should be that $\chi(\eta \neq 0)/\chi(0) - 1 = \delta\chi^2_{\rm n} \ll 1$.
We can determine the upper value of η by $\delta\chi^2_{\rm n} = 1$. It means that we allow a dark mass model within a perturbation from the two-body problem.

Comparison between the orbital fitting method and our method (1)

We calculate $\delta\chi^2_{\rm n}$ for the orbital-fitting model of S0-2 as the two-body system obtained by the previous studies (Boehle+16, Gillessen+17, Do+19).

- **Mass function with a dark mass distribution**

$$M(r) = M_{\rm tot}(1 - \eta + \eta \mathcal{M}_{\rm ext}(r))$$

  The mass of the central black hole is given by $M_{\rm tot}(1-\eta)$.

- **post-Newtonian equation of motion for S0-2 with a dark mass distribution**

$$\frac{d^2\vec{r}}{dt^2} = -\frac{GM(r)}{r^3}\vec{r} + \frac{GM(r)}{c^2 r^3}\left(\frac{4GM(r)}{r} - v^2\right)\vec{r} + \frac{4GM(r)\vec{r}\cdot\vec{v}}{c^2 r^3}\vec{v},$$

  where $\vec{r}$ and $\vec{v}$ are the potion vector from the central black hole and the velocity of S0-2, respectively.

Comparison between the orbital fitting method and our method (2)

We consider two models for the dark mass distribution:

- **power-law model (Do+19)**

$$M(r) = \begin{cases} M_{\mathrm{tot}} \left(1 - \eta + \eta \left(\frac{r}{r_{\mathrm{c}}}\right)^{3/2}\right) & (r \le r_{\mathrm{c}}) \\ M_{\mathrm{tot}} & (r > r_{\mathrm{c}}) \end{cases}$$

$r_{\mathrm{c}} = 0.011\,\mathrm{pc}$

- **Plummer model (Mouwad+05)**

$$M(r) = M_{\mathrm{tot}} \left(1 - \eta + \eta \frac{\int_0^r \rho(\xi)\xi^2 d\xi}{\int_0^{r_0} \rho(\xi)\xi^2 d\xi}\right)$$

$$\rho(r) = \left\{1 + \left(\frac{r}{r_{\mathrm{c}}}\right)^2\right\}^{-5/2}$$

$r_0 = r_{\mathrm{apo}} \sim 0.01\,\mathrm{pc},\ r_{\mathrm{c}} = 0.015\,\mathrm{pc}$

Comparison between the orbital fitting method and our method (3)

S0-2's position vector $\vec{r} = (x, y, z)$ and velocity $v = (v_x, v_y, v_z)$

Theoretical model of the observations:

- **Astrometry** (x : Dec., y : R.A.)

$$x(t_{\rm obs}) = -\frac{x(t_{\rm em})}{R_0} + x_0 + v_{x0}(t_{\rm obs} - t_0)$$
$$y(t_{\rm obs}) = \frac{y(t_{\rm em})}{R_0} + y_0 + v_{y0}(t_{\rm obs} - t_0)$$

  (x_0, y_0) and (v_{x0}, v_{y0}) are the position and the velocity of Sgr A* on the sky plane. R_0 is the distance to the Galactic Center. t_0 is the origin of time for astrometry.

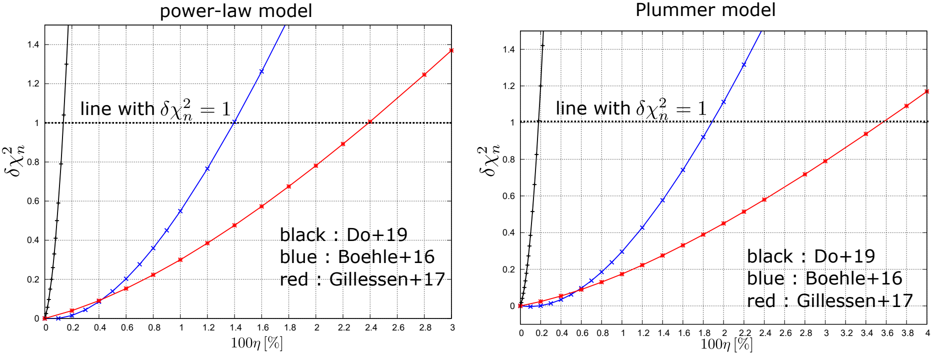
- **Spectroscopy** (redshift of photons from S0-2)

$$cZ(t_{\rm obs}) = v_z(t_{\rm em}) + \frac{v^2(t_{\rm em})}{2c} + \frac{GM(r(t_{\rm em}))}{cr(t_{\rm em})} + v_{z0}$$

The emission time $t_{\rm em}$ relates the observational time as $t_{\rm em} = t_{\rm obs} - z(t_{\rm obs})/c$.

Comparison between the orbital fitting method and our method (4)

In Boehle+16 and Gillessen+17, the upper value of 100η is $\sim 1\,\%$.
In Do+19, it is $\sim 0.1\,\%$.



The upper value of 100η is obtained at the cross point of the line with $\chi_\mathrm{n}^2 = 1$.
For Boehle+16, and Gillessen+17, it is a few %. For Do+19, it is ~0.1%.
Our method can give the comparable constraint to the previous studies.

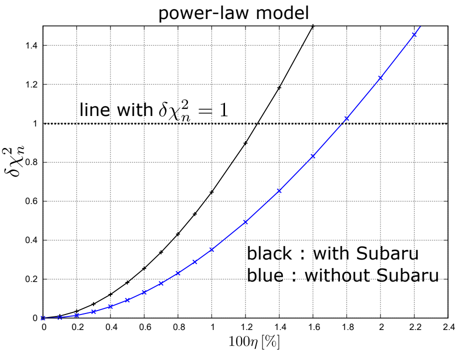
Role of the spectroscopic data of S0-2 during the pericenter passage in 2018 (1)

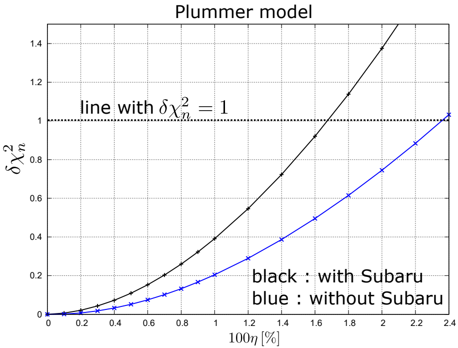
- In our previous study (Saida+19), we have obtained the orbital fitting model of S0-2 by the least-χ^2 fitting as the two-body system. In Saida+19, we use the open data in Boehle+16 (astrometry and spectroscopy), Gillessen+17 (astrometry and spectroscopy), and the data from Subaru/IRCS (spectroscopy).

- We re-obtain the orbital fitting model of S0-2 by the least-χ^2 fitting without Subaru's data as the two-body system.

- We calculate $\delta\chi^2_{\mathrm{n}}$ for the orbital model with Subaru and without Subaru. Then, we compare the upper value of η between both the models.

Role of the spectroscopic data of S0-2 during the pericenter passage in 2018 (2)



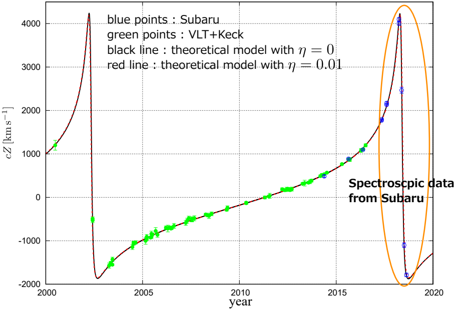


The upper value of η using the orbital model in Saida+19 is smaller than that using the orbital fitting model without Subaru.

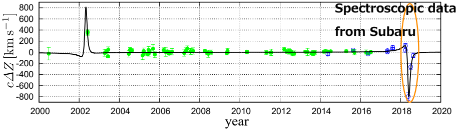
Role of the spectroscopic data of S0-2 during the pericenter passage in 2018 (3)

We find that the upper value of η is strongly limited due to the steep variation of the redshift during the pericenter passage in 2018.

Time variation of the redshift of photons from S0-2



$$c\Delta Z = cZ(\eta = 0) - cZ(\eta = 0.01)$$



points : residual between the observed data and the theoretical model with

black line : residual between the theoretical model with $\eta = 0$ and with $\eta = 0.01$

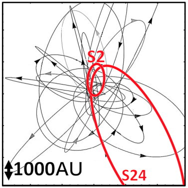
Spectroscopic observations for a S-star at an appropriate time during its pericenter passage give a strong constraint on the dark mass distribution within its orbit.

Summary

- S-star's motions are well explained by the two-body system. It means that an extended mass component around the central supermassive black hole is quite less than the mass of the black hole.
- We suggest a computationally less demanding method to constrain the dark mass distribution around Sgr A*. Our method can give the comparable constraint to the orbital fitting method.
- The spectroscopic data of S0-2 during their pericenter passage in 2018 obtained by Subaru/IRCS can give a strong constraint on the dark mass distribution.
- Observing S-stars at an appropriate time during their pericenter passage, we can give a strong constraint on the dark mass distribution within its orbit.

Future target : S24

- S24 would experience the pericenter passage in 2024.
- The distance of the pericenter is ∼0.004 pc (within S0-2!).
- S24 is a late-type star.
  → the number of absorption line is more than 30
  → the estimated uncertainty of the redshift is $\sim 2\,\mathrm{km\,s^{-1}}$



Spectroscopic observations of S24 during the pericenter passage could give a constraint on the dark mass distribution within 0.01 pc